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SYNTHESIS OF MONOPULSE ARRAY DIFFERENCE PATTERNS

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Three methods are presented in this paper for synthesizing a difference pattern produced by a monopulse array. The first method is to determine an optimum set of excitations required to maximize the difference gain for a given squint angle, element spacing, and the number of elements in the array (Fig. 1). The success of this synthesis is based on the fact that the difference gain can be expressed as a ratio of two quadratic forms:

$$G_d = \frac{4\pi\, \Delta^2(u_m)}{\int_0^{2\pi}\int_0^{\pi} \Delta^2(u) \sin\theta \, d\theta \, d\varphi}$$

$$= \frac{[I']^T [R][I']}{[I']^T [Q][I']} \quad , \qquad (1)$$

where

$$\Delta(u) = 4 \sum_{i=1}^{N} I_i \sin(b_i u_s) \sin(b_i u)$$

= difference pattern function, (2)

$$[R] = [V][V]^T \quad , \qquad (3)$$

$$[V] = \begin{bmatrix} \sin b_1 u_m \\ \sin b_2 u_m \\ \vdots \\ \sin b_N u_m \end{bmatrix} \quad , \qquad (4)$$

$[V]^T$ = transpose of $[V]$,

$$[I'] = \begin{bmatrix} I_1 \sin b_1 u_s \\ I_2 \sin b_2 u_s \\ \vdots \\ I_N \sin b_N u_s \end{bmatrix} \quad , \qquad (5)$$

Q_{mn} = typical element in $[Q]$

$$= \tfrac{1}{2}\left[\frac{\sin(b_m - b_n)\pi}{b_m - b_n} - \frac{\sin(b_m + b_n)\pi}{b_m + b_n} \right], \qquad (6)$$

I_i = amplitude excitation of the i^{th} pair of elements,

$b_i = 2\, d_i/\lambda$, $u = \pi \cos\theta$,

$u_s = \pi \cos\theta_s$, θ_s being the squint angle,

$u_m = \pi \cos\theta_m$,

and θ_m is the position of the main beam in the difference pattern. The method developed previously for optimizing the sum gain can then be applied to determine the required excitation matrix (for a given θ_s):[1]

$$[I'] = [Q]^{-1} [V] \quad , \qquad (7)$$

yielding

$$(G_d)_{max} = [V]^T [Q]^{-1} [V] = N - \frac{\cos(N+1)u_m \sin N u_m}{\sin u_m} \quad , \qquad (8)$$

where u_m satisfies

$$\Delta'(u_m) = 0 \quad . \qquad (9)$$

The second approach is to employ a perturbation technique to reduce the sidelobe level in a difference pattern from a known monopulse array[2]. Since there are ordinarily many sidelobes, the question always arises as to which one or ones should be reduced and by how much. In actual application, it is quite possible that we may prefer reducing the level for certain sidelobes more than for others. Indeed, it is not unusual that the level of a few sidelobes may be sacrificed in order to achieve other advantages. Before anticipating any special applications, our approach here is to weigh uniformly all the sidelobes in a difference pattern by equalizing them to an approximately same level through successive perturbations.

The third method is to apply Haar's theorem to the synthesis of a difference pattern which will approximate a desired pattern in the minimax sense[3]:

$$\max_{0 \le u \le \pi} |f(u) - \Delta(u)| = \min. \quad , \qquad (10)$$

where f(u) is a specified pattern, and $\Delta(u)$ is the synthesized pattern. The solution so obtained not only provides precise information concerning the required amplitude excitations and the minimum possible deviation between the synthesized and desired patterns with respect to an element spacing, but can also be unique. Specific results for $f(u) = u \exp(-Au^2)$, where A is a positive constant used to control the pattern shape, will be discussed.

Mathematical derivations involved in this paper will be given at the Symposium. Many numerical examples for the difference gain and patterns with approximately equal sidelobes will be presented as a function of array parameters.

References:

[1] Cheng, D. K. and F. I. Tseng, Gain optimization for arbitrary antenna arrays, IEEE Trans. on Antennas and Propagation, Vol. AP-13, No. 6, pp. 973-974, November, 1965.

[2] Ma, M. T., Note on nonuniformly spaced arrays, IEEE Trans. on Antennas and Propagation, Vol. AP-11, No. 4, pp. 508-509, July, 1963.

[3] Ma, M. T., Another method of synthesizing nonuniformly spaced antenna arrays, IEEE Trans. on Antennas and Propagation, Vol. AP-13, No. 5, pp. 833-834, September, 1965.

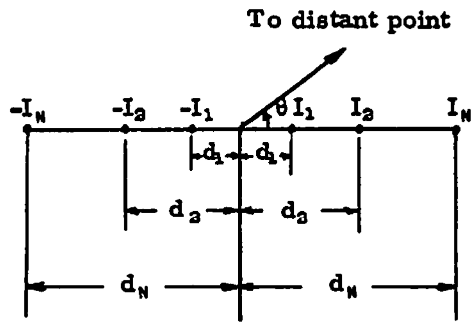

Fig. 1. A nonuniformly but symmetrically spaced difference array.