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POLYNOMIALS FOR ARRAY SYNTHESIS

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I. Introduction

Chebyshev polynomials are often used to design arrays for optimum patterns. The Dolph-Chebyshev pattern is optimum in the sense that for a given sidelobe level the beamwidth is minimum, or conversely, for a given beamwidth the sidelobe level is minimum. The sidelobes of this pattern are all equal. Although the Dolph-Chebyshev pattern is, in this way, mathematically unique, its sidelobe behavior is not flexible enough for many applications. For example, in order to minimize the external antenna noise effect for the case of non-uniform temperature distribution, the sidelobes of the array pattern should be unequal.[1] Therefore, it is useful to provide a synthesis procedure which can give a more flexible sidelobe behavior[2] and still is optimum in the Dolph-Chebyshev sense. Such a technique is discussed in this paper.

II. Modified Chebyshev Polynomials

A class of optimum polynomials which includes the Chebyshev polynomials as a subclass and is suitable for array synthesis will be given. For our purpose, the pattern of an array is considered to be optimum if for a given peak sidelobe level the beamwidth is minimum, or conversely for a given beamwidth the peak sidelobe level is minimum. The problem so stated does not have a unique solution as the peak sidelobe can occur anywhere outside the main beam and in the visible range. It becomes uniquely defined when the sidelobe envelope is specified to within an arbitrary constant. For the Dolph-Chebyshev patterns, the sidelobe envelope is a constant line.

Let $P_n(z)$ be the array polynomial and $Q_m(z)$ the sidelobe envelope, where $n >> m$. The array polynomials $P_n(t)$ which have all the following properties will be denoted as $P_{n,m}(z)$:

(i) All zeros of $P_n(z)$ are real, distinct and in a finite interval (a, b).

(ii) All zeros of $P_n(z) + Q_m(z)$ and $P_n(z) - Q_m(z)$ in (a, b) are of multiplicity 2.

(iii) $[P_n^2(z) - Q_m^2(z)]$ has, in addition to (n-1) double zeros, two single zeros at a and b.

There exists an algebraic procedure of characterizing these polynomials. In particular, it can be shown that m = 0, $P_{n,o}(z)$ reduces to the Chebyshev polynomials.

For array applications, we let $(a, b) = (-1, 1)$ and $Q(z)$ be symmetric about $z = 0$. We then have

$$P_{n,m}(z) = z\, P_{n-1,m-1}(z) \qquad (1)$$

Furthermore, we shall work out examples for optimum patterns with monotonically decreasing near-in sidelobes. This is achieved by letting

$$P_{n,m}(z) = cz^m T_{n-m}(z) \qquad (2)$$

Note that zeros of $P_{n,m}(z)$ are readily determined and maxima of $P_{n,m}(z)$ are given by the solution of the equation

$$(n-m)\tan[(n-m)\alpha] + \tan\alpha = 0 \quad (3)$$

where $z = \cos\alpha$.

III. Small Arrays

The polynomials described in II can be used to generate optimum array patterns in the following manner. The $P_{N,m}(z)$ polynomials is used with $2N+1$ elements and the transformation

$$z = a\cos\psi + b \qquad (4)$$

where $\psi = kd\sin\theta$, is used to relate the variable z for the polynomial and the variable ψ of the array pattern. For spacings $d \geq \lambda/2$, DuHamel[3] has shown that

$$a = \frac{z_o + 1}{2} \quad b = \frac{z_o - 1}{2} \qquad (5)$$

where $z_o = z(\theta=0) = a + b$ is determined either by the sidelobe level or by the beamwidth of the array pattern.

The identification of the polynomial $P_{N,m}(z)$ with the array pattern $E_{2N+1}(\theta)$ gives the set of current coefficients.

IV. Large Arrays

When the number of array elements is large, we have the following asymptotic expression relating the beamwidth $\Delta\theta$, the sidelobe level R and the effective aperture length $\overline{L}$

$$\left(\frac{1}{2}\frac{\overline{L}}{\lambda}\Delta\theta\right)^2 = \ln(\sqrt{2}/G)\ln(2\sqrt{2}GE_o^2) \quad (6)$$

where $\overline{L} = 2(N-m)d$

$$\ln G = m[\ln z_o - \ln z(\theta = \frac{\Delta\theta}{2})]$$

$$R = 20\log_{10} E_o$$

When m = 0, the above expression reduces to the known result for Dolph-Chebyshev pattern[4]

$$\Delta\theta = 0.18\left(\frac{L}{\lambda}\right)^{-1}(R+4.5)^{1/2} \qquad (7)$$

where L = 2Nd.

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References

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