PROCEEDINGS OF ISAP '92, SAPPORO, JAPAN

PREDICTION OF RADIATION FROM ARBITRARY CROSS-SECTION HORNS USING A FINITE DIFFERENCE TECHNIQUE

A D Olver and S Sanyal
Queen Mary and Westfield College, University of London
London E1 4NS, U.K.

INTRODUCTION

This paper presents a study of the radiation characteristics of arbitrary cross-section open-ended waveguides or narrow flare angle horns using a finite difference technique to predict the aperture fields. This enables the radiation patterns of horns with any arbitrary cross-section to be computed.

The radiation patterns from waveguides or horns with rectangular, circular or elliptical cross-section are obtained by a two part process [1]. First the aperture fields are computed by solving the modal propagation equation for the required geometry (which might need to be done by iteration if the waveguide is inhomogeneously filled). Secondly the radiation characteristics are computed by solving the radiation integral with the appropriate boundary conditions. When the cross-section of the aperture is arbitrary and does not fit one of the standard coordinate systems, an analytical equation for the mode functions cannot be derived and another technique must be used to obtain the aperture fields. The finite difference technique is a suitable method that can be implemented with relative ease on computers. The technique is described here, together with examples of its use to predict ridged and cross shaped horns.

FINITE DIFFERENCE TECHNIQUE

The waveguide modes of an arbitrary cross-section waveguide are the eigenfunctions and the eigenvalues of the two dimensional Helmholtz equation

$$\nabla^2\varphi(x,y) + k_c^2\varphi(x,y) = 0 \tag{1}$$

where k_c is the cut-off wavenumber and $\varphi = E_z$(or H_z) with the boundary condition $\varphi = 0$ for TM modes (or $\partial\varphi/\partial n = 0$ for TE modes).

In the finite difference technique, Equ.(1) is solved at discrete points in the waveguide. If a square mesh of side h is fitted in the waveguide cross-section, then the finite difference replacement of Equ.(1) is the matrix eigenvalue equation

$$(A - \lambda)\varphi = B\varphi = 0 \tag{2}$$

where $\lambda = k_c h^2$.

At each grid point, Fig. 1, a general five-point operator equation expresses the relationship between the fields at the grid point and the fields at surrounding points,

$$b_{gc}\varphi_c + b_{gf}\varphi_f + b_{gg}\varphi_g + b_{gh}\varphi_h + b_{gk}\varphi_k = 0 \tag{3}$$

where the φ's are the values of $\varphi(x,y)$ at the end of the corresponding arm of the operator and the coefficients b are the elements of matrix $\boldsymbol{B}$. These depend on the location of node g in the mesh and on the orientation of the corresponding arm of the operator.

Equation (3) can be used for dominant modes. For higher order modes, the problem needs to be reformulated as a positive semi-definite matrix equation

$$\boldsymbol{C}\varphi = 0 \tag{4}$$

where $\boldsymbol{C} = \boldsymbol{B}^T\boldsymbol{B}$. The coefficients of C are given by an algorithm for a 13 point operator consisting of five overlaid 5 point operators [2]. The boundary conditions at the exact physical boundary can be satisfied by using a development of the basic finite difference scheme which has operators with unequal length arms to the grid [3].

RADIATION CHARACTERISTICS

The eigenvalues are used to obtain the transverse electric and magnetic field components from the derivatives of the mode function. The electric field for TE modes is

$$E_t = \frac{j\omega\mu}{k_c^2}\left[\hat{u}_y\frac{\partial\varphi}{\partial x} - \hat{u}_x\frac{\partial\varphi}{\partial y}\right] \tag{5}$$

The fields for the other components take on similar forms. Accurate computation of the φ derivatives near the boundary requires the values of φ at appropriate locations in the boundary. The far-field radiation characteristics are predicted by the standard vector Huygens formula [4] using numerical integration of the transverse fields over the aperture.

COMPUTER IMPLEMENTATION

The technique outlined above has been implemented in a computer program which fits a mesh to a specified shape of waveguide and then iteratively solves Equ.(4). The mesh fitting is straight forward in the inner region of the waveguide. Around the edges, it is complicated by the need to use modified forms of Equ.(3) to take account of the appropriate boundary conditions. After the mesh has been established, iteration is used to converge on the eigenvalues for the waveguide. The iterations are stopped when the residuals left after substituting the eigenvalues into Equ.(2) are less than 0.001.

A typical starting mesh size of 13 by 13 is used. For most shapes of waveguide a final square mesh of 23 by 23 points has been found to be adequate. Larger meshes give potentially more accurate eigenvalues, but at the expensive of significantly increased computer time. A typical waveguide takes between 200 and 1000 iterations to converge to the required accuracy. Convergence in waveguides with re-entrant corners is not directly possible for TE modes. To overcome this problem it has been found necessary to insert more accurate trial field values. Symmetry in waveguides can be utilised to reduce the computational effort but then mixed boundary conditions must be used.

RESULTS AND DISCUSSION

The program was validated by comparison of the transverse fields and radiation patterns of rectangular and circular open-ended waveguides computed using the finite difference technique and computed in closed form. These gave results which were in good agreement with the analytical methods.

A symmetric ridged horn and radiation patterns are shown in Fig. 2. The co-polar pattern and the cross-polar pattern are both computed in the 45° plane using the standard definition for horns. The wavelength in this and the other examples is $a/2.25$ where a is the maximum waveguide cross-section.Symmetry was not used in the computations of the fields in this example. In fact the fields are symmetrical about the axes even though the lines plotted in the sketch of the waveguide do not appear symmetrical. This is due to the way in which the iterations proceed and the automatic values chosen for the contour lines. The technique is as easy to apply to non-symmetrical structures as is demonstrated in Fig. 3 which shows the fields and patterns for an asymmetric ridged horn. The effect of the asymmetry on the radiation pattern is to increase the cross-polar level in the 45° plane to -12 dB.

A cross-guide, as shown in Fig. 4, radiating a TE mode. This has a field pattern and radiation characteristics which are similar to those of a circular waveguide. The peak cross-polar level is slightly higher due to the greater field curvature. The examples clearly demonstrate the potential of the finite difference technique and show that it is a powerful method of predicting the radiation patterns of arbitrary cross-section horns.

REFERENCES

1. Rudge, A.W. et. al. (Eds) *The Handbook of Antenna Design*, Chapter 4, IEE Books, London, 1984.

2. Beaubien, M.J. & Wexler, A., IEEE Trans MTT-16, Dec 1968, pp.1007-1017.

3. Beaubien, M.J. & Wexler, A., IEEE Trans MTT-18, Dec 1970, pp.1132-1149.

4. Silver, S. (Ed): *Microwave Antenna Theory and Design*, IEE Books, London, 1984, pp.334-6.

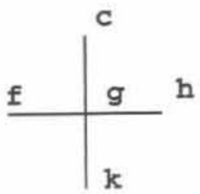

Fig 1. Five point operator

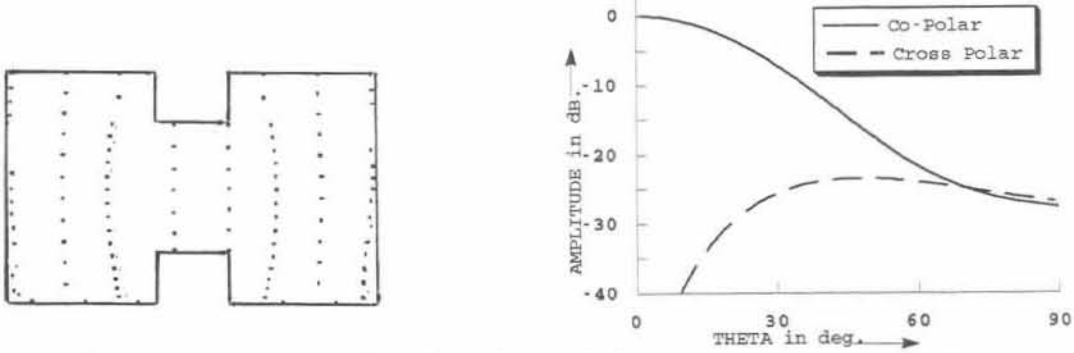

Fig 2. Symmetrical ridged horn and pattern

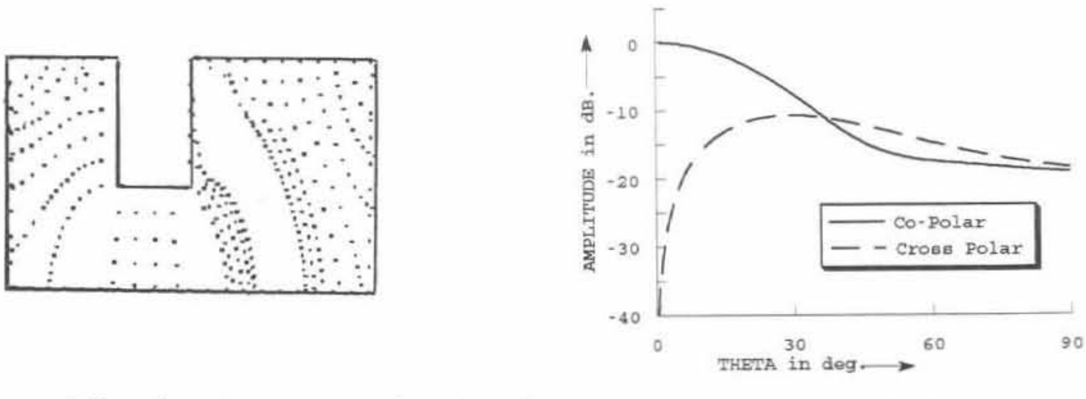

Fig 3. Asymmetrical ridged horn and pattern

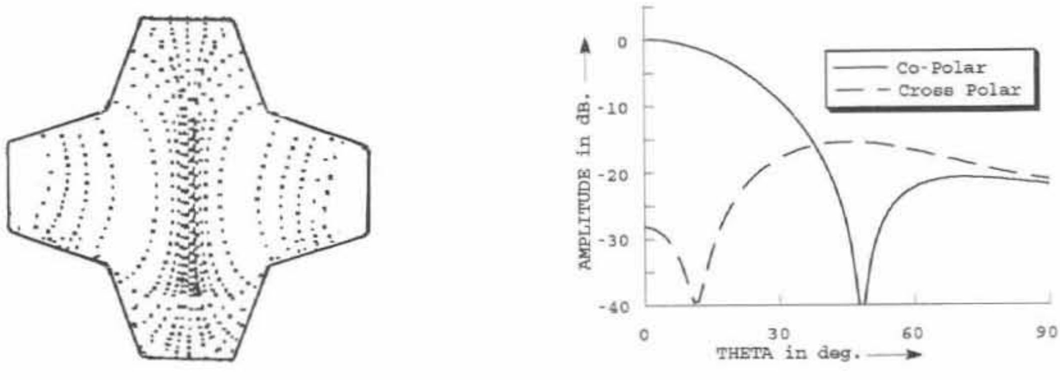

Fig 4, Cross horn and pattern