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The problems of the mutual coupling of a finite planar waveguide array (FPWA) hollowing through a perfectly conducting plane have been investigated by means of the several methods.¹⁻³ In this paper we describe a theoretical study of these problems based upon the variational technique. We derive the exact stationary expression for the mutual admittance of FPWA when the higher-order modes of all the guides are considered at the apertures.⁴ For numerical examples, we analyze the linearly distributed FPWA and show the values of the mutual admittances in the case of two guides (Fig.1), the element patterns (Fig.2), and the rotation of the polarization angle of the far field radiated from FPWA excited by the linearly polarized waves in the case of two square guides (Fig.3).

Each guide of FPWA is assumed to be identical and its dimension ($a \times b$) is chosen so that only the fundamental (or dominant) mode can propagate. Using the equation of the continuity condition for the tangential (x - y) magnetic fields at the apertures ($z=0$), we have the following stationary expression, Eq.(1), for the mutual (or self) admittance. Where $K_z = k \times E_z$ is the tangential electric field on S_z ; (i) of the superscript denotes

the field when only the dominant mode voltage in the i -th guide is applied and

$$Y_{ij} = \sum_{k=1}^N \left[\frac{j}{2\pi\omega\mu} \sum_{l=1}^N \left(\int_{S_k} K_z^{(i)} \cdot G \cdot K_z^{(j)} dS_k dS_l \right) \right. \\ \left. + \sum_{mn} \left\{ Y_{mn}^H(K_z^{(i)} \cdot h_{mn}^{kH} dS_k) (K_z^{(j)} \cdot h_{mn}^{kH} dS_l + Y_{mn}^E(K_z^{(i)} \cdot h_{mn}^{kE} dS_k) \right. \right. \\ \left. \left. \times (K_z^{(j)} \cdot h_{mn}^{kE} dS_l) \right\} \right] \\ \times \frac{1}{Y_{i0}^H} \left[\left(\int_{S_i} K_z^{(i)} \cdot h_{i0}^{kH} dS_i \right) \left(\int_{S_j} K_z^{(j)} \cdot h_{j0}^{kH} dS_j \right) \right]^{-1} \quad (1)$$

all the other guide dominant mode voltages are equal to zero at the apertures; h_{mn}^{kH} and h_{mn}^{kE} are the orthonormal modal functions of the TE and TM (with respect to z) waves, respectively; Y_{mn}^H and Y_{mn}^E are their admittances, respectively; Σ' denotes the sum of the modes except for the dominant mode; and $G = (i\mathbf{l}\mathbf{l} + j\mathbf{j}\mathbf{j} + k^{-2}\nabla_z \nabla_z) \exp(-jk\rho) / \rho$, where $\rho^2 = (x_k - x_l)^2 + (y_k - y_l)^2$.

The element pattern of E_θ or H_θ when only the r -th guide is excited and the other guides are terminated by the matched load, respectively, is given by the equation (2). In this equation, $\sigma_{mn} = 2$ if (m,n) equals the dominant mode number and $\sigma_{mn} = 1$ if (m,n) otherwise; ϵ_m is called the Neumann factor; δ_{rl} is the Kronecker delta function; S_{rl} is the scattering coefficients; $M_{mn}^{i(1)}$ and $N_{mn}^{i(1)}$

are equal to the TE and TM mode voltages normalized by that of the dominant mode which may be called the yielding (or distribution) factors, respectively; Γ_m^{\pm}

$$F_m^{(j)}(\theta, \phi) = \frac{\sum_{l=1}^N \sum_{m=1}^{\infty} \frac{E_m^{(j)} E}{(8Z/\pi)(1+\Gamma_m^{\pm}) \sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}} e^{j(m\pi n - l\pi/2 + k \sin\theta \cos\phi a \cos\psi + \nu \sin\theta \sin\phi)}$$

$$\times \sum_{l=1}^N (S_{12} + S_{21}) \left[\left(\frac{m}{a} M_{mn}^{(j)} + \frac{n}{b} N_{mn}^{(j)} \right) \{ S(X_m^+) - (-1)^m S(X_m^-) \} \right] \sin\phi$$

$$- \left(\frac{n}{b} M_{mn}^{(j)} - \frac{m}{a} N_{mn}^{(j)} \right) \{ S(X_m^+) + (-1)^m S(X_m^-) \}$$

$$\times \{ S(\gamma_m^+) - (-1)^m S(\gamma_m^-) \} \cos\phi \quad (2)$$

is the reflection coefficient at the aperture for the single guide; N is the total number of the guides; and $S(Z) = \sin Z/Z$, $X_m^{\pm} = (m\pi \pm k a \sin\theta \cos\phi)/2$, and $\gamma_m^{\pm} = (n\pi \pm k b \sin\theta \sin\phi)/2$.

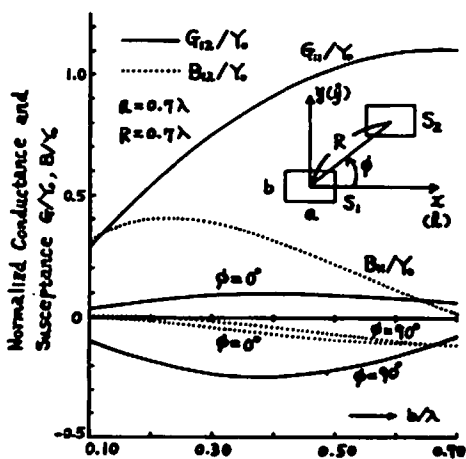


Fig.1 Admittances of two guides

For simplicity, we show the numerical results evaluated by the dominant mode approximation in Fig.1—Fig.3.

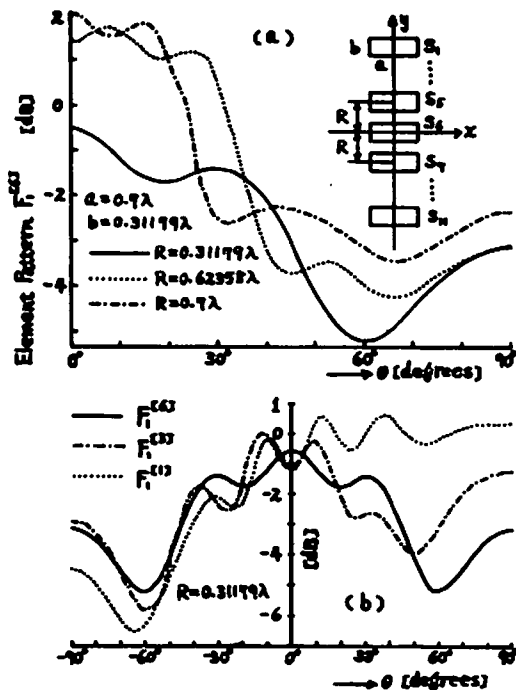


Fig.2 Element Patterns of 11-Element Array

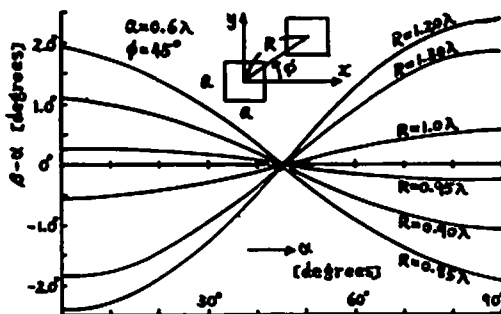


Fig.3 Rotation of Polarisation Angle of Far Field

- 1 G.V.BORGIOTTI, IEEE Trans., AP-16, No.3, May 1968
- 2 R.J.MAILLOUX, IEEE Trans., AP-17, No.6 November 1969
- 3 C.P.WU, IEEE Trans., AP-18, No.3, May 1970
- 4 Y.SUGIO and T.MAKIMOTO, Tech. Rept. of Radiation Science Research of Japan, October 1970