

## Optimizing Signal Transmission in a MIMO System Operating in Rayleigh and Ricean Fading Channels

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### 1. Introduction

Recently, multiple input multiple output (MIMO) wireless communication systems have drawn a considerable attention of the wireless communication community. In contrast to conventional wireless systems, a MIMO system includes  $n_T$  and  $n_R$  number of antenna elements at a transmitter and a receiver respectively, which are both being greater than 1. The use of such a multiple antenna system both at the transmitter/receiver sites leads to increased transmission capacity. It has been shown that the average channel capacity divided by  $n$ , where  $n = \min(n_T, n_R)$ , approaches a nonzero constant. This constant is related to the average signal to noise ratio (SNR) if the fades between pairs of transmit and receive antenna elements are independent and identically distributed (i.i.d.) [1]. The assumption of the i.i.d. fading has been made in many works concerning estimations of the capacity of a Rayleigh fading channel with or without optimizing power transmission [1, 2]. It has to be noted that in real propagation environments the fades are not independent due to the finite spacing between antenna elements and because of the scattering angle spread [3, 4]. As the MIMO technology aims to serve a broad range of wireless communication systems starting from personal to wide area communication, with examples such as HIPERLAN II, IEEE 802.16 [5], 3GPP[6], the modeling of the MIMO channels requires considering a more general fading environment than the one described by the Rayleigh distribution. This paper addresses such a more general situation by assuming a Ricean fading channel, which can be characterized by different values of the Ricean K-factor.

Our attention focuses on determining the input signal covariance at the transmitter that optimizes the capacity in both Rayleigh and Ricean cases and compare them with the non-optimized cases of beamforming and independent transmission. In order to properly carry out the intended investigations, our considerations begin by taking into account the Ricean K-factor while determining mutual information of a transmission channel. By assuming that the duration of each data stream is short enough so that the channel can be regarded as stationary during data transmission, the measure of mutual information should adequately represent channel capacity in this quasi-static fading environment. Having assuming this, we focus our attention on the mutual information changes as we vary the input signal covariance by adapting the signal power at the transmit antennas in order to maximize transmission capacity.

The paper is structured as follows. In section 2 we introduce a MIMO channel model which includes a Ricean fading channel and explain the meaning of all the associated parameters. Having done this, in section 3 we present a solution for the maximum capacity by applying a method of Lagrange multiplier. Section 4 includes simulation results and discussion of the obtained capacity for different modes of transmission, as given in terms of bits per second per Hz., versus a function of K-factor and the transmit antenna cross correlation. Finally section 5 provides conclusions on the paper's findings.

### 2. MIMO channel model

Our MIMO channel model is an extension of the model presented in [2, 4]. The new model assumes the Ricean case of transmission channel. Also it assumes the presence of correlation between antenna elements in the transmit array. The channel properties are represented by a matrix  $H$ , which is composed of a matrix  $H^{LOS}$  representing the line of sight channel and a matrix  $H^{NLOS}$  representing a non line of sight channel. When  $H^{LOS}$  is equal to zero,  $H$  represents the Rayleigh fading channel. In almost all publications on MIMO systems  $H^{NLOS}$  is modeled as an i.i.d. complex Gaussian random matrix. Here, to complete the information about our model we define the properties of the matrix  $H^{LOS}$ .

In the case of the line of sight propagation, which takes place between two uniform linear arrays without scattering,  $H^{LOS}$  depends on a distance between a transmitter and a receiver, the plane-wave direction of arrival ( $\alpha$ ) and the spacing between antenna elements at a transmitter and a receiver ( $d_t$  and  $d_r$ ). As a result, the normalized covariance matrix of  $H^{LOS}$  can be written as

$$H^{LOS} H^{LOS\dagger} = \begin{bmatrix} 1 & e^{-j2\pi(d_t/\lambda)\sin(\alpha)} & \dots & e^{-j2\pi((n_t-1)d_t/\lambda)\sin(\alpha)} \\ e^{-j2\pi(d_r/\lambda)\sin(\alpha)} & 1 & \vdots & \vdots \\ \vdots & \vdots & 1 & e^{-j2\pi(d_t/\lambda)\sin(\alpha)} \\ e^{-j2\pi((n_r-1)d_r/\lambda)\sin(\alpha)} & e^{-j2\pi((n_r-2)d_r/\lambda)\sin(\alpha)} & \dots & 1 \end{bmatrix} \quad (1)$$

where the symbol  $\dagger$  means conjugate and transpose operation of matrix. The covariance matrix of  $H^{LOS}$  can be simplified when the antenna element spacing in both arrays is taken as half-wavelength ( $d_t=d_r=\lambda/2$ ). Under such assumption, the matrix depends only on the plane-wave direction of arrival. In particular when  $\alpha$  is very small or close to 90 degrees all elements of the covariance matrix are approximately the same and equal to one. In this case, the matrix has rank of one. It means that every channel from each transmit antenna to each receive antenna is completely dependent. Assuming that at a given instant of time both transmitter and receiver perfectly know the channel [7], the mutual information ( $C$ ) of MIMO system can be represented by (2):

$$C = \log_2 \det(I + H^{LOS} pQH^{LOS\dagger} + H^{NLOS} pQ\Sigma H^{NLOS\dagger}) \quad (2)$$

The derivation of this expression follows directly from [1] via inferring mutual independence between  $H^{LOS}$  and  $H^{NLOS}$ . In order to simplify further considerations, we assume that  $n_t=n_r=n$ . Also we assume that for the non line of sight channel, antenna correlations occur only at the transmitter end. The correlations are described by a covariance matrix ( $\Sigma$ ). Both  $\Sigma$  and  $Q$  need to be normalized. This is done by introducing the following constraint:  $\text{Tr}\{\Sigma\}=\text{Tr}\{Q\}=n$ , where  $\text{Tr}\{\}$  is trace operation of matrix and  $Q$  is a transmitted signal covariance matrix. By using a singular decomposition value technique, we can simplify (2) as shown in (3).

$$C = \log_2 \det(I + pQ\Lambda^{LOS} + pQ\Sigma\Lambda^{NLOS}) \quad (3)$$

where  $p$  is the SNR per transmit antenna,  $\Lambda^{LOS}$  is a diagonal eigenvalue matrix for a line of sight channel and  $\Lambda^{NLOS}$  is a diagonal eigenvalue matrix of a non line of sight channel. Using the above assumptions and necessary derivations, the Ricean K-factor ( $K$ ) can be derived and is given by (4).

$$K = \frac{\text{Tr}\{\Lambda^{LOS}\}}{\text{Tr}\{\Sigma\Lambda^{NLOS}\}} \quad (4)$$

### 3. Optimization

In order to obtain the maximum information capacity of transmission, the transmitted signals have to be optimized at the transmitter. This can be done by considering the covariance matrix  $Q$  which has to take into account channel variations. We solve this problem using the method of Lagrange multiplier by maximizing (3) under the constraint that power is normalized:  $\text{Tr}\{Q\}=n$ . The optimizing equation is then written as

$$\frac{\partial \zeta(q_i)}{\partial q_i} = 0; \quad \zeta(q_i) = C + L(n - \text{Tr}\{Q\}) \quad (5)$$

where  $q_i$ ,  $s_i$ ,  $\lambda_i^{LOS}$ ,  $\lambda_i^{NLOS}$  are the eigenvalues of matrix  $Q$ ,  $\Sigma$ ,  $\Lambda^{LOS}$ ,  $\Lambda^{NLOS}$  respectively and  $L$  is a Lagrange multiplier for power constraint. Here we do not show full derivations. Instead we provide the final solution, which is given as follows.

$$\begin{aligned} \text{if rank of } \Lambda^{LOS} = 1; & \quad q_1 = \frac{1}{L \ln(2)} - \frac{1}{p\lambda_1^{LOS} + ps_1\lambda_1^{NLOS}} \\ & \quad q_i = \frac{1}{L \ln(2)} - \frac{1}{p\lambda_i^{LOS} + ps_i\lambda_i^{NLOS}} \quad 2 \leq i \leq n \\ \text{if rank of } \Lambda^{LOS} \neq 1; & \quad q_i = \frac{1}{L \ln(2)} - \frac{1}{p\lambda_i^{LOS} + ps_i\lambda_i^{NLOS}} \quad 1 \leq i \leq n \end{aligned} \quad (6)$$

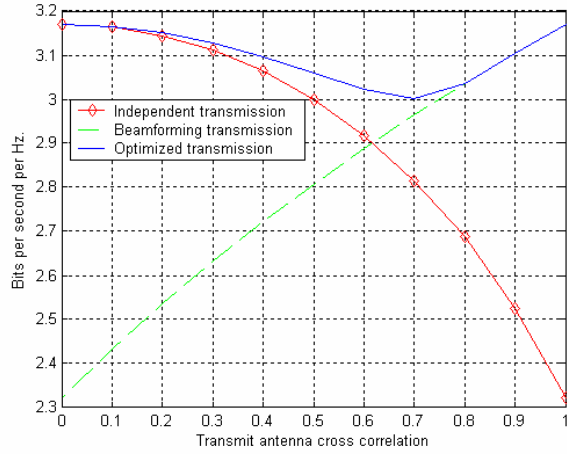


Fig.1. Comparison of mutual information for different mode of transmission as the function of transmit antenna cross correlation, rank of  $\Lambda^{LOS}=1$ ,  $K=0$ .

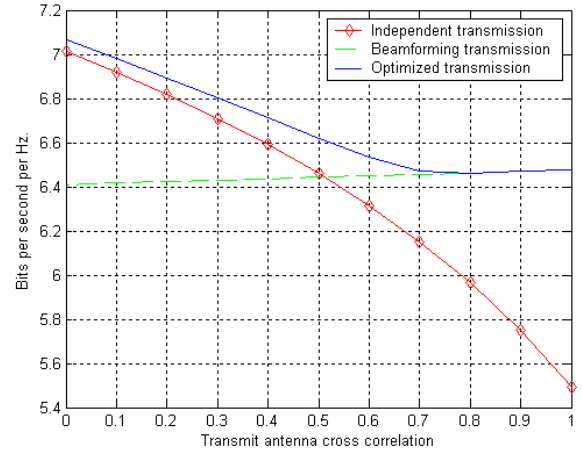


Fig.2. Comparison of mutual information for different mode of transmission as the function of transmit antenna cross correlation, rank of  $\Lambda^{LOS}=1$ ,  $K=10$ .

#### 4. Simulation Results and Discussion

In this section, we focus our considerations on the case of an  $n=2$ -element transmit and receive array MIMO system because of ease of interpreting the obtained results. In this case, the transmit antenna cross correlation is given by  $(s_1-s_2)/2$ . It becomes apparent that when the cross correlation is near zero both transmit antennas are almost independent. Here, we consider three modes of transmission according to the eigenvalue of matrix  $Q$ . The first mode represents independent transmission, which occurs when both transmit antennas send independent signals ( $q_1=1, q_2=1$ ). In the second mode, called the beamforming transmission mode, the correlated antennas form a beam pattern ( $q_1=2, q_2=0$ ). The third mode concerns the optimized transmission following the solution given in (6).

Fig.1 presents the results for all the three modes of transmission as a function of transmit antenna cross correlation when  $K=0$  and the rank of  $\Lambda^{LOS}=1$ . This is the case of a Rayleigh fading channel. It can be seen that the optimized capacity is a non-monotonic function of antenna cross correlation. At high cross correlation the beamforming mode is very effective. In turn, transmission of multiple data streams (independent mode) is very effective at low cross correlation. However, both beamforming and independent modes are less effective for intermediate cross correlation levels, as compared with the optimized mode. Using beamforming transmission at low cross correlation or applying independent transmission at higher cross correlation is inefficient. One can see that switching at an appropriate cross correlation level (around 0.78 in this figure) from the beamforming transmission at high cross correlation to independent transmission at lower cross correlation can be a good strategy. However, if there are some errors in selecting appropriate cross correlation levels, the capacity can drop more than 25% in comparison with the optimized capacity. Another shortfall of this strategy is that the switching mechanism increases complexity at both transmitter and receiver ends. Therefore, the optimized transmission seems to be the best choice for the Rayleigh fading channel.

The results shown in Fig.2 concern the Rician fading channel as the factor  $K$  is increased from 0 to 10. The results for the case of independent transmission mode show similar trends as those of Fig.1. However for the beamforming transmission mode, the capacity is not increased when cross correlation becomes high. The reason for this is due to the fact that in the Rician fading channel with  $K=10$ , the amount of power in the line of sight channel is 10 times of that in the non line of sight channel. This assumption results in a strong correlation between signals at each receive antenna. As a result, the beam forming occurs even when the transmit antenna cross correlation is low. The results shown in Fig.2 reveal that the optimized transmission is still the best choice to maintain the highest capacity at all cross correlation levels.

The results presented in Fig. 3 and 4, concern the case when the Rician  $K$ -factor is equal to 1. One has to note that when  $K=1$ , the power in the line of sight channel is the same as in the non line of sight channel. This situation occurs in a real environment, for example in an indoor wireless communication

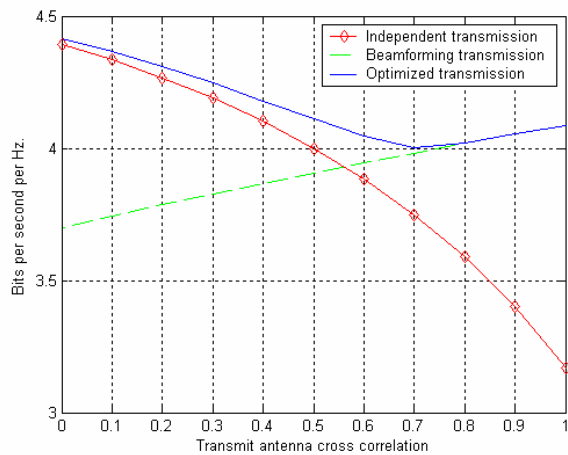


Fig.3. Comparison of mutual information for different mode of transmission as the function of transmit antenna cross correlation, rank of  $\Lambda^{LOS}=1$ ,  $K=1$ .

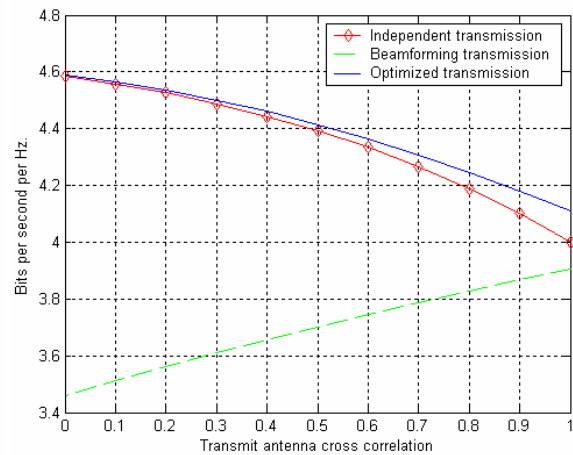


Fig.4. Comparison of mutual information for different mode of transmission as the function of transmit antenna cross correlation, rank of  $\Lambda^{LOS}=2$ ,  $K=1$ .

situation when both the transmitter and the receiver are not obstructed but due to reflections the environment becomes rich in multipath. Fig. 3 reveals that the results for the three assumed modes of transmission are similar to those in Fig.1 but the results presented in Fig.4 show different trend. Here in Fig. 4, the rank of  $\Lambda^{LOS}=2$ , which is the full rank of a  $2 \times 2$  matrix. This means that the two line of sight channels at each receive antenna are fully independent. In this case, the channel features the line of sight components, which are uncorrelated. As seen, the beamforming transmission in this fading environment is inefficient. In turn, the independent transmission mode can achieve high capacity even at a high cross correlation level. At the same time, the optimized mode of transmission gains nearly the same capacity as independent transmission. This result indicates that for this case it is not necessary to use optimization because independent transmission achieves high capacity for all values of transmit cross correlation levels.

## 5. Conclusions

In this paper, we have shown how to calculate mutual information for a MIMO system operating under the assumption of Rayleigh and Rician fading channels. The derived formulas have formed the basis for optimizing signals at the transmit end to achieve maximum transmission capacity. We have provided the solution to this optimization problem using a method of Lagrange multiplier. Our theory has been demonstrated in an example of a  $2 \times 2$  array MIMO system, whose performance has been assessed via computer simulations. Three modes of operation of this MIMO system including independent, beamforming and optimized transmission have been considered. Our simulated results have shown that the optimized signal transmission achieves the highest capacity compared with the two remaining modes of transmission.

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