DETERMINATION OF THE OBJECT'S POSITION IN A STRATIFIED MEDIUM VIA INVERSION OF THE NULL-FIELD APPROACH

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Abstract. *A 2-D inverse scattering problems is considered. The position of a homogeneous dielectric cylinder embedded into a plane-stratified media is determined using nonlinearised inversion of the null-field integral equations. Crossections of the hypersurface for arising nonlinear functional are presented and discussed.*

Introduction

The paper presents a treatment of a 2-D inverse scattering problem of parameters retrieval for a permeable object embedded into a stratified medium [1]. Statement of the problem and its solution are quite similar to those by Kleinman a.o. [2]. However employment of the null-field approach provides us with the integral equations which are defined on the boundary of the object's crossection. The latter fact leads to a considerable simplification of the problem especially in the case of a homogeneous object. Here constitutive parameters as well as the shape of the object are supposed to be known a priori. The retrieval of permittivity was studied in [1]. A futher generalization of the approach for determination of both permittivity and geometry can be easily achieved by including appropriate parameters into the optimization procedure. We show that within our approach the number of said parameters is much less then that in [2].

Statement of the problem and general formalism

A regular medium filling the whole space x, y, z is assumed to be arbitrarily stratified and is characterized by permittivity distribution $\varepsilon(z)$, $-\infty < z < +\infty$. A scatterer inserted in such a medium (see fig. 1) has the form of a cylinder oriented along the x axis, with S_p being its' cross section (of finite diameter) in the yz plane. Its' permittivity has a constant value of ε_p. The permeability equals 1 everywhere.

It is presumed that the scatterer is wholly contained within a uniform layer $0 < z < -h$ of the host medium which may be otherwise arbitrarily stratified. However for the simplicity of numerical treatment upper and lower half spaces (z>0, z<-h) are supposed to be uniform as well. This means that the function $\varepsilon(z)$ takes the constant values ε_e (z>0), ε_c (z<-h), ε_s (0<z<-h).

Scalarization of the problem was achieved under assumption that the scatterer is illuminated by a TM-polarized plane monochromatic ($e^{-i\omega t}$)wave coming from the upper half space. An amplitude of the initial field $U_{IN}(\vec{r})$ is assumed to be known in the whole regular medium (with no scatterer).

Let us consider a group of expressions produced by the null-field approach for the prescribed scattering configuration, which include integral equations

$$\int_L \left[G(\vec{r},\vec{r}\,') \frac{\partial U^+_{TOT}(\vec{r}\,')}{\partial N'} - U^+_{TOT}(\vec{r}\,') \frac{\partial G(\vec{r},\vec{r}\,')}{\partial N'} \right] dL' = -U_{sc}(\vec{r}), \qquad (1)$$

$$\vec{r} \in S_E, \quad \vec{r}\,' \in L,$$

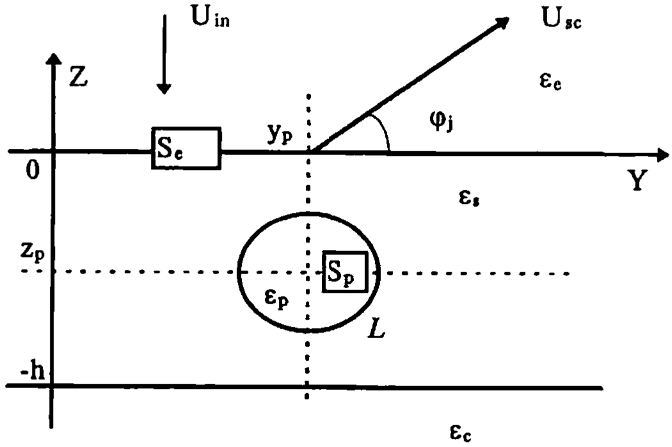

Fig. 1.
Sketch of the problem (circular cylinder embedded into a three-layered medium).

$$\int_L \left[G(\vec{r},\vec{r}\,')\frac{\partial U^-_{TOT}(\vec{r}\,')}{\partial N'} - U^-_{TOT}(\vec{r}\,')\frac{\partial G(\vec{r},\vec{r}\,')}{\partial N'} \right] dL' = -U_{IN}(\vec{r}), \quad (2)$$

$$\vec{r} \in S_P, \quad \vec{r}\,' \in L,$$

$$\int_L \left[\chi_\sigma(\vec{r}\,')\frac{\partial U^-_{TOT}(\vec{r}\,')}{\partial N'} - U^-_{TOT}(\vec{r}\,')\frac{\partial \chi_\sigma(\vec{r}\,')}{\partial N'} \right] dL' = 0 \quad (3)$$

$$\vec{r}\,' \in L.$$

appended by the following boundary conditions:

$$U^-_{TOT}(\vec{r}) = U^+_{TOT}(\vec{r}),$$
$$\frac{\partial U^+_{TOT}(\vec{r})}{\partial N} = \frac{\partial U^-_{TOT}(\vec{r})}{\partial N}, \quad \vec{r} \in L. \quad (4)$$

Where: $U^{+(-)}_{TOT}(\vec{r}) = U_{IN}(\vec{r}) + U_{SC}(\vec{r})$. For explanation of notations in (1)-(4) the reader is referred to fig.1. We note that $\chi_\sigma(\vec{r})$ in (3) are solutions to a homogeneous Helmholtz equation

$$\left[\Delta + k_P^2(\vec{r})\right]\cdot \chi_\sigma(\vec{r}) = 0,$$
$$\vec{r} \in S_P, \quad (5)$$

each of the sets $\{\chi_\sigma(\vec{r}\,')\}$ and $\left\{\frac{\partial \chi_\sigma(\vec{r}\,')}{\partial N'}\right\}$ is supposed to be complete on $L^2(L)$, and $G(\vec{r},\vec{r}\,')$ denotes the Green's function of the "host" medium (in absence of the scatterer).

As it was shown in [1], one can solve the problem of z_p and y_p evaluation simply by searching for solutions and kernels of equation (1) in the corresponding compact set(s). The searching process is controlled by properly choosing the values of these parameters. A global minimum of the metric difference functional,

$$F(a_i) = \sum_{j=1}^{N} \| \; \tilde{U}^j_{sc} \; | - | \; U^j_{sc} \; \|, \tag{6}$$

corresponds to a correct estimate for unknown parameters (here a_i is the guess value, and j is the number of measurement in some informational domain). Note that since our inverse problem is posed in nonlinearised form there is no restrictions on the electrical contrast of the medium and accuracy of the first estimate.

Numerical treatment

As we deal with the problem of minimizing a nonlinear multidimensional functional the key question is the form of a hypersurface produced by this functional. Obviously direct computation of (6) for each case under consideration is too expensive. Our numerical experiments were organized in the form of a multidimensional database filled with the scattered field values, computed for a number of measurement points. Each set of scattered field values corresponds to a set of appropriate parameters. So when minimizing (6) one has to deal only with the database seeking process. This leads to a considerable reduction of CPU time needed for the evaluation of (6).

Here we suppose that the measurements are obtainable on a line segment placed in the upper half-space 10λ above the plane boundary $z=0$. The length of the segment is 2λ. To enchance experimental applicability of our solution we "measure" the magnitude rather then complex amplitude of the scattered field.

As a first step the value of y_p is determined. Fig. 2 presents the values of $F(y_p)$ calculated for the case where a circular cylinder placed in a two layered medium (two semi-infinite half-spaces). The upper one is free space, and the lower is characterized by $\varepsilon_s = \varepsilon_c = 2.0 + i3.0$. A circular cylinder of λ/5 radius and $\varepsilon_p = 3.0$ is placed at a depth $z_p = 0.7\lambda$. We seek for a minimum of $F(y_p)$to retrieve the position of the cylinder along OY axis. To be even more realistic we "measure" the scattered field at a number of equidistant points spaced with a period $D_{ms} = \lambda$. The behavior of this functional was uncertain up to the spatial period $D_{ms} = \lambda/10$. The procedure of minimizing $F(z_p)$ for the same configuration and constitutive parameters is illustrated at Fig. 3. These figures presents crossections of the two-dimensional functional $F(y_p, z_p)$. It is clearly seen that a problem of length nonuniqueness arises in z_p evaluation for unifrequency probing of a three-layered medium with an inclusion, especially in the case where the size of an inclusion is less then the wavelength in the host medium, see Fig. 4.

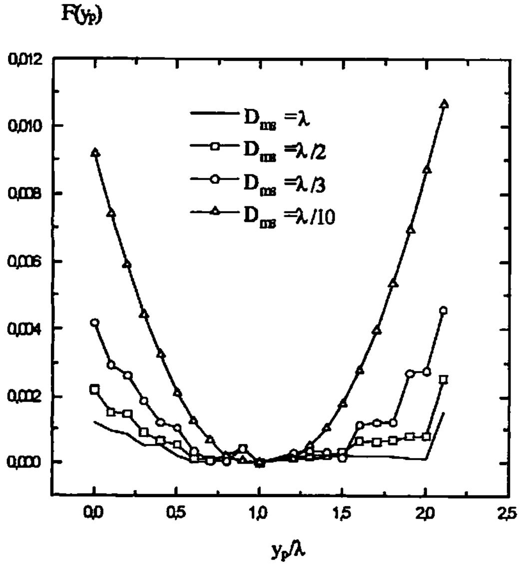

Fig. 2

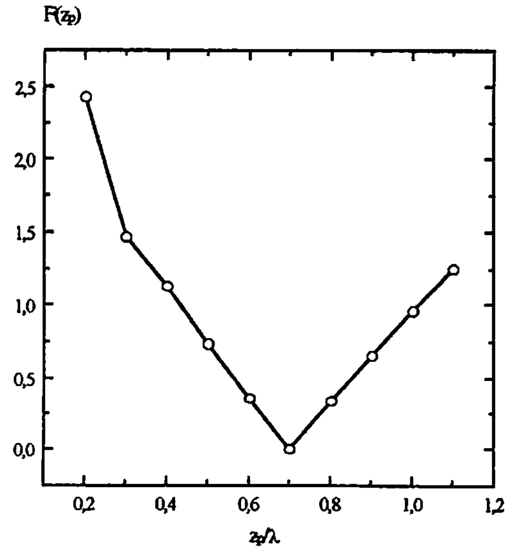

Fig. 3

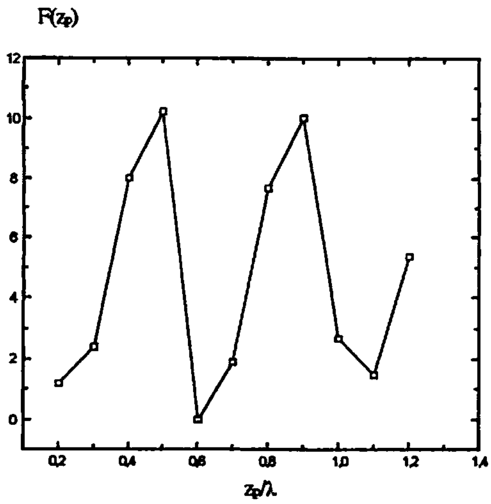

Fig. 4

Literature

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