

A COMPUTER AIDED DESIGN METHOD FOR OPTIMIZING PHASED-ARRAY ANTENNA GEOMETRY

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1. INTRODUCTION A new technique is presented for determining the optimal face tilt and element packing geometry for a planar, phased-array antenna with scan limits specified in earth's coordinates. The technique, although very simple to implement, is instructive in that it quickly shows, in one display, the relationships between the required azimuth and elevation scan limits, the element packing geometry, the grating lobe locations, grating lobe scan boundaries, the maximum off-axis scan angle, and the array face tilt angle. Previously reported array design methods insured that radiating grating lobes could not be formed if the antenna's main beam was confined to the design scan area [1], but they did not take advantage of the irregular shape of the grating lobe scan boundary. The face tilt angle was chosen to balance the maximum off-axis scan requirement [2]. The application of the technique described in this paper clearly illustrates that the previously reported techniques were only optimal for special cases and, in general, did not minimize the total number of elements.

2. THEORY From [3] the array factor for a uniformly illuminated, triangularly-packed array is

$$AF(k_x, k_y) = \left\{ 1 + \exp \left\{ j 2 \pi [ \Delta x (k_x - k_{x0}) + \Delta y (k_y - k_{y0}) ] \right\} \right\} \tag{1}$$

$$\sum_{m = \frac{-M}{2}}^{\frac{M}{2} - 1} \sum_{n = \frac{-N}{2}}^{\frac{N}{2} - 1} \left\{ \exp \left\{ j 2 \pi [ 2 m \Delta x (k_x - k_{x0}) + 2 n \Delta y (k_y - k_{y0}) ] \right\} \right\}$$

where  $k_x = \sin \theta \cos \phi$ ,  $k_y = \sin \theta \sin \phi$ ,  $k_{x0} = \sin \theta_0 \cos \phi_0$ ,  $k_{y0} = \sin \theta_0 \sin \phi_0$ , and the subscript 0 denotes the scan position of the main beam. An examination of Equation 1 shows that the second factor on the right side of the equation has maxima points at

$$k_x - k_{x0} = \frac{p}{2 \Delta x}, p = 0, \pm 1, \pm 2, \dots$$

and

(2)

$$k_y - k_{y0} = \frac{q}{2\Delta x}, q = 0, \pm 1, \pm 2, \dots$$

However, when these values are substituted into the first factor on the right side,

$$\left| 1 + \exp \left\{ j 2 \pi [ \Delta x (k_x - k_{x0}) + \Delta y (k_y - k_{y0}) ] \right\} \right| = 1 + (-1)^{p+q} \quad (3)$$

Therefore, the array factor has maxima points only at points where  $(p + q)$  is even. The values of  $k_x$  and  $k_y$  where

$$k_x - k_{x0} = \frac{p}{2\Delta x}, p = 0, \pm 1, \pm 2, \dots \quad (p + q) \text{ even} \quad (4)$$

$$k_y - k_{y0} = \frac{q}{2\Delta x}, q = 0, \pm 1, \pm 2, \dots$$

represent points on the  $(k_x, k_y)$  plane where the power radiated by all elements in the array add in phase. The value of  $k_x$  and  $k_y$  associated with  $p = q = 0$ , is the location of the main beam and values for all other  $p$  and  $q$  terms represent the location of grating lobes.

Since  $k_x = \sin\theta\cos\phi$  and  $k_y = \sin\theta\sin\phi$  and  $\theta = \sin^{-1} (k_x^2 + k_y^2)^{\frac{1}{2}}$ , where  $\theta$  is the off-axis scan angle of the grating lobe, values of  $k_x$  and  $k_y$  where  $(k_x^2 + k_y^2) > 1$  do not represent real radiating far-field angles. The boundary at which these grating lobes form in real space is the unit circle or

$$k_x^2 + k_y^2 = 1 \quad (5)$$

Substituting Equation (2) into Equation (3) yields

$$\left( k_{x0} + \frac{p}{2\Delta x} \right)^2 + \left( k_{y0} + \frac{q}{2\Delta y} \right)^2 = 1 \quad (6)$$

for  $(p + q)$  even.

Equation (6) defines the grating lobe scan boundaries in terms of  $k_{x0}$  and  $k_{y0}$  which are the sine-space coordinates of the beam scanning direction and  $\Delta x$  and  $\Delta y$  which are the spacing between elements in wavelengths. Any value of  $k_{x0}$  and  $k_{y0}$  that causes the left side of Equation (6) to become less than unity will result in the formation of grating lobes at real far-field angles. Equation (4) represents unit circles centered at the locations of grating lobes in the  $k_x k_y$  plane when the array main beam is scanned to broadside (i.e.,  $k_{x0} = k_{y0} = 0$ ).

The usefulness of Equation (6) is shown in Figure 1 which is a sine-space plot with the primary unit circle centered at the origin and the secondary unit circles centered at the six grating lobes nearest the primary unit circles. The area of the primary unit circle that is not intersected by secondary unit circles represents the sine-space over which the main beam can be scanned without the formation of real grating lobes. The dashed line represents the grating lobe scan boundary.

3. RESULTS Equation (6), along with the required transformations between earth's coordinates and array coordinates were implemented on a Personal Computer to generate the displays like those shown in Figures 2 through 4. The program is interactive. Element geometry, scan limits and face tilt are inputted and the display is generated. The design that requires the fewest number of elements to support a given required scan boundary (RSB) is the one that minimizes the area inside the GLB that is outside the RSB. This is accomplished by plotting the RSB and GLB on the same sine-space plot for several different face tilt angles and choosing the one that provides the best "fit" of the GLB around the RSB. Figure 2 shows a design in which the tilt angle is chosen to equalize the maximum off-axis scan for all extreme scan positions and the element geometry is chosen to allow the array to scan the cone of angles containing the extreme scan points (i.e., the GLB is tangent to the circle through the extreme scan points at 6 points). This design obviously does not minimize the unused area inside the GLB. A design where the tilt angle remains the same but the GLB is allowed to be tangent to the RSB is shown in Figure 3. This reduces the unused area inside the GLB, but it is still not minimized. Finally, Figure 4 shows the design that minimizes the area inside the GLB. Here the face is tilted back until the GLB and RSB are tangent at 6 points and therefore the extreme scan angles are not equalized. This design required 12.5% fewer elements for the same aperture area and RSB than the first design and 5.0% fewer elements than the second design.

4. CONCLUSIONS A computer aided design technique has been developed for quickly designing the geometry of planar phased-array antennas. The array can be designed to either minimize the maximum scan losses or to minimize the total number of array elements. In general, the two designs are not the same. The technique makes it possible to efficiently use the space inside the GLB by matching the GLB and RSB as closely as possible even for unusual scan requirements. An additional advantage of the method is that all the information required to determine the optimum tilt angle, element geometry, grating lobe position, GLB, element area, and scan losses are contained on one single display.

#### 5. REFERENCES

1. M.I. Skolnik (ed.), Radar Handbook, McGraw Hill, New York, 1970, Chap. 11.
2. P. J. Kahrilas, Electronic Scanning Radar Systems (ESRS) Design Handbook, Artech House, 1976, Chap. 7.
3. L. E. Corey, "A Graphical Technique for Determining Optimal Array Antenna Geometry," accepted for publication in the IEEE Transactions on Antennas and Propagation.

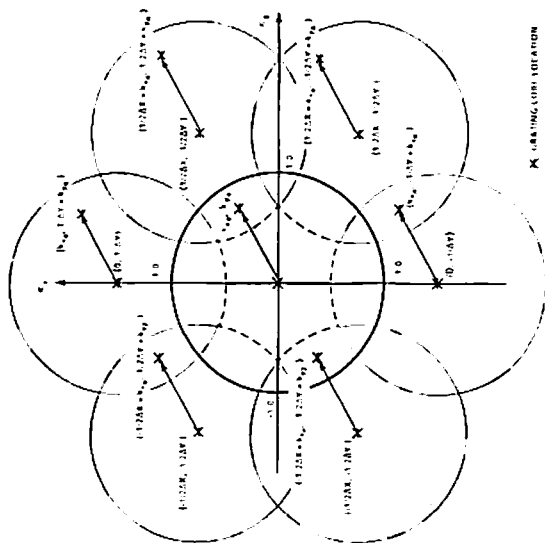


FIGURE 1. MIGRATION OF THE GRATING LOBE WITH SCANNING

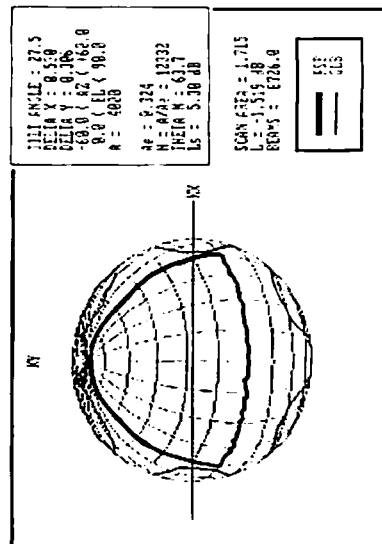


FIGURE 2. GLB TANGENT TO CIRCLE PASSING THROUGH EXTREME SCAN POINTS

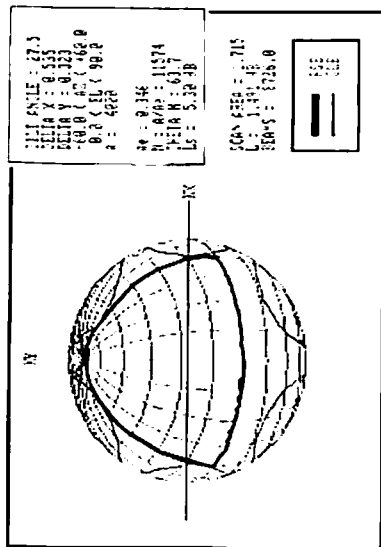


FIGURE 3. MINIMUM OFF-AXIS SCAN DESIGN

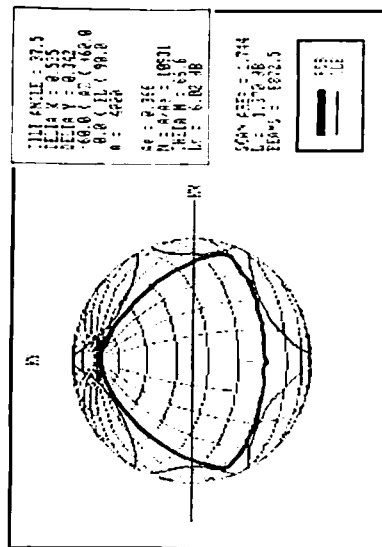


FIGURE 4. MINIMUM ELEMENT NUMBER DESIGN