MINIMUM REDUNDANCY ARRAY CONSIDERED FOR THE PHASE CORRECTION

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1 INTRODUCTION

Aperture synthesis is one of the most important techniques in radio astronomy for high resolution observation of galactic and extragalactic radio sources(Fomalont and Wright, 1974). Essential to the technique is the measurement of amplitude and phase of the complex visibility function of a radio source. In many cases, the measurement of visibility phase presents difficulties due either to instabilities in the instrumentation or to irregularities in the ionosphere and the troposphere(Hinder and Ryle, 1971). These difficulties are especially serious for mm-wave aperture synthesis and very long baseline interferometry(VLBI).

Some statistical methods to eliminate the phase error due to the above effect from all the visibility phases are very powerful and successful(Schwab, 1980). But such a method requires many simultaneous spacings in the array so that it is not so effective in the array which has smaller number of antennas.

If the array has sufficient number of redundant spacings, one can correct the phase error by using the closure relation of the phase(Jennison, 1958; Rogstad, 1968), assuming only one visibility phase. This method is simple but requires the special array configuration such as the uniform array and the compound array.

In this paper, we consider a new type of the minimum redundancy array(MRA) which has sufficient number of redundant spacings for the above phase correction. The MRA is the array which achieves maximum resolution for a given number of antennas and is obtained by reducing the number of redundant spacings present in the array (Moffet, 1968; Ishiguro, 1980). The new type of the MRA(here after to be called MRA 2) will be advantageous for the design of the array configuration which has a number of stations but a few antennas such as the Nobeyama mm-wave interferometer (Ishiguro et al., 1984), with which the visbility functions of all spacings are measured by rearranging the antennas on the stations.

2 PHASE CORRECTION BY USING THE CLOSURE RELATION

Jennison(1958) suggested first that it is possible to obtain the relation which does not involve the antenna based phase error terms, by forming the algebraic sum of three phases around a closed loop of three spacings. Consider two antennas denoted by i and j. In absence of the noise of each correlator output, the visibility phase measured at the spacing between these antennas can be written as,

$$\varphi_{ij}=\psi_{ij}+\theta_i-\theta_j, \tag{1}$$

where ψ_{ij} is the true visibility phase for this spacing and θ is the phase error associated with each antenna. When φ_{ij} is summed around a closed loop of three antennas i, j, and k, the closure relation,

$$C_{ijk}=\varphi_{ij}+\varphi_{jk}-\varphi_{ik}=\psi_{ij}+\psi_{jk}-\psi_{ik}, \tag{2}$$

is obtained, where C_{ijk} is called the closure phase. This quantity is widely used in VLBI observations (Rogers et al., 1974; Readhead and Wilkinson, 1978).

Figure 1 shows an example of the MRA-2 of 5 stations. We will show how to correct the phase error by the redundancy in this array. Suppose that the visibility phase for the unit spacing is known. Since $\psi_{12}=\psi_{23}$, the phase, ψ_{13} can be determined by substituting the phase for the unit spacing into the closure relation of stations 1, 2, and 3. One can obtain the phase, ψ_{14}, using the closure relation of stations 1, 3, and 4 and the phase, ψ_{13}. All remaining phases can be determined in same way.

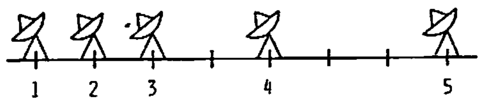

Figure 1. An example of the MRA-2 of 5 stations.

3 THE MRA-2'S FOR A LARGE NUMBER OF STATIONS

We have found the MRA-2's for a large number of stations with the aid of a computer. Results for N≦16 are shown in Table 1.

Figure 2 shows the redundancies of the MRA-2's, the MRA's obtained by Ishiguro's Method-1 (Ishiguro, 1980), and the minimum redundancy compound arrays. The redundancy is defined as the ratio of the number of unique spacings with the number of possible pairs of stations. The redundancies of MRA-2's are superior to those of the compound arrays and approach those of the normal MRA's as the number of stations increases. They are equal for N>15 stations. It is found that the redundancies of MRA's do not change whether the phase correctability is included or not.

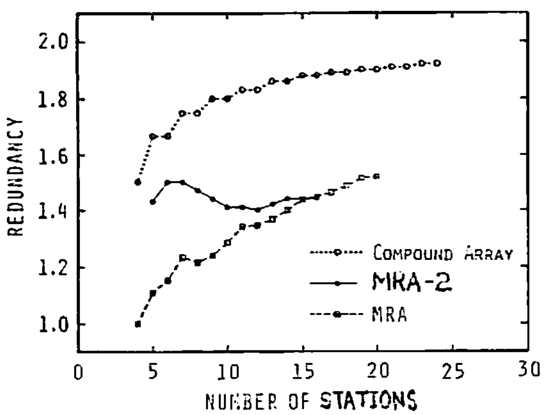

Figure 2. Redundancies of optimum MRA-2's.

N	L	R	Array Configuration
5	7	1.43	.1.1.2.3.
6	10	1.50	.1.1.2.3.3.
			.1.1.1.3.4.
7	14	1.50	.1.1.2.4.5.1.
			.1.1.1.3.4.4.
8	19	1.47	.1.1.7.4.1.2.3.
			.1.1.3.3.7.2.2.
			.1.1.2.2.5.7.1.
9	25	1.44	.1.1.10.4.1.2.3.3.
			.1.1.10.4.3.1.2.3.
			.1.1.11.2.2.2.3.3.
			.1.1.11.3.3.2.2.2.
			.1.1.5.7.3.4.2.2.
			.1.1.3.3.3.10.2.2.
			.1.1.2.4.7.5.4.1.
10	32	1.41	.1.1.2.4.7.7.5.4.1.
11	39	1.41	.1.1.2.4.7.7.7.5.4.1.
			.1.1.1.14.4.5.3.3.3.4.
			.1.1.13.1.11.3.3.2.2.2.
12	47	1.40	.1.1.1.17.6.5.4.4.3.3.2.
13	55	1.42	.1.1.3.9.5.5.16.4.3.4.2.2.
			.1.1.1.21.6.5.4.4.4.3.3.2.
14	63	1.44	.1.1.7.1.4.23.2.2.2.11.3.3.3.
			.1.1.7.1.24.1.5.3.3.11.2.2.2.
			.1.1.1.25.6.5.4.4.4.4.3.3.2.
			.1.1.1.3.20.8.8.2.4.4.4.5.2.
			.1.1.1.3.16.12.4.8.2.4.4.5.2.
			.1.1.1.1.24.6.6.3.2.5.5.4.4.
			.1.1.1.1.24.5.6.5.4.4.3.4.3.
15	73	1.44	.1.1.5.5.22.9.5.4.4.9.4.2.1.1.
			.1.1.1.1.29.6.6.3.2.5.5.5.4.4.
16	83	1.45	.1.1.1.1.34.6.6.3.2.5.5.5.5.4.4.

N: Number of stations.
L: Maximum spacing.
R: Redundancy.

Table 1. Optimum MRA-2's. Points and numbers represent the positions of stations and the spacings between them, respectively.

4 THE PERFORMANCE TEST OF THE MRA-2

In this section, we show the performances of the MRA-2 obtained in the section 3 through a computer simulation of the aperture synthesis by use of model brightness distributions. The number of stations used in the array is 16. We constructed the model brightness distribution consisting of several Gaussian components (Figure 3).

Figure 4 shows a result of the simulation. In this example, the phase error at each antenna is 20° rms and the noise level in each correlator output is 0.5% rms of total flux. Figure 4 (a) shows the synthesized map without the phase correction. The synthesized map after the phase correction by the closure relation is shown in Figure 4 (b). From this example, it is clear that the good synthesized map can be obtained with the MRA-2.

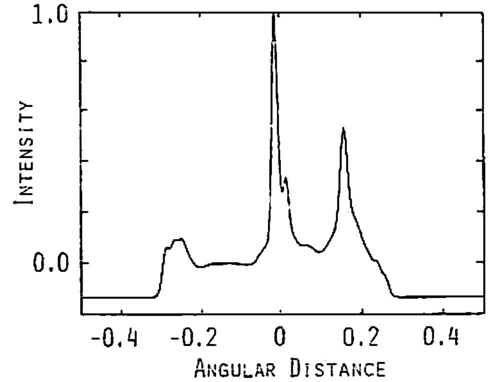

Figure 3. Fourier transform of the visibility function of the model truncated at the maximum spatial frequency obtainable with the MRA-2 of 16 station's. Scale of the intensity is arbitrary.

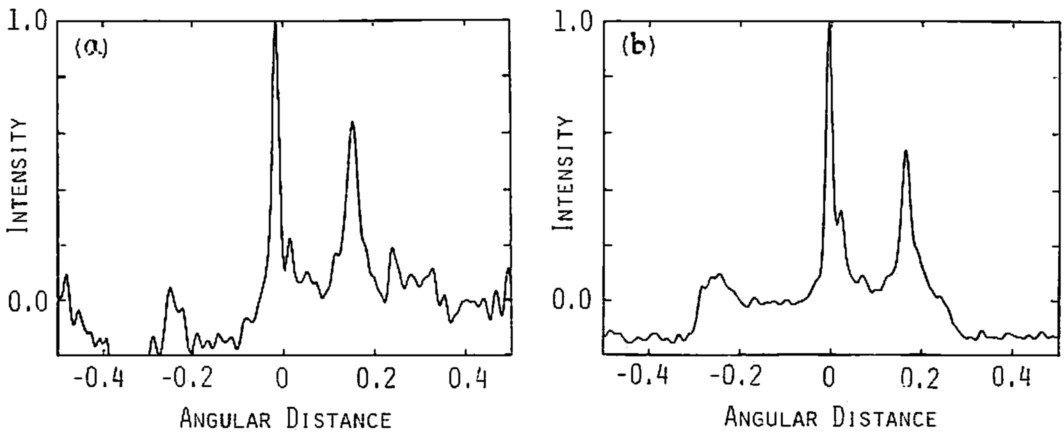

Figure 4. (a) Synthesized map without the phase correction. (b) Synthesized map with the phase correction. Scale of the intensity is arbitrary.

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