OPTIMISATION OF THE PERFORMANCE INDICES OF THE MONOPULSE ARRAYS WITH CONSTRAINTS ON SUM AND DIFFERENCE MODE SIDELOBES

N. BALAKRISHNAN
DEPT. OF AEROSPACE ENGINEERING
INDIAN INSTITUTE OF SCIENCE
BANGALORE - 560 012, INDIA.

1. INTRODUCTION

The amplitude comparison monopulse employs two overlapping patterns that are squinted by an angle ϕ_{sq} on either side of the boresight direction. The pattern through which the power is transmitted to the target is called the sum pattern and is obtained by transmitting in-phase power through both the squinted beams. For angular tracking, the reflected power from the target is received through a difference mode pattern which is obtained by combining the signals from the squinted beams out of phase [1].

The slope of the difference pattern normally determines the angular accuracy of tracking. However, in view of the AGC action [2] either the slope-sum ratio (also known as the Difference Mode angular sensitivity, DMAS) or slope gain product is taken as a performance index for optimisation. Often, the sum mode directivity is also considered for optimisation if the target range is of interest.

The optimisation of performance indices with constraints only on the sum pattern sidelobes will often yield a difference mode pattern with very large sidelobes. This may lead to increased noise in the difference channel. Hence, the performance maximisation in monopulse systems have to be carried out with specific constraints on both the sum and difference mode sidelobes. The sidelobe levels are normally expressed, as a fraction of the sum mode radiation peak.

This paper describes a numerical formulation using a multivariable search method for optimising the performance indices of the monopulse arrays with constraints on sum and difference mode sidelobe levels. The problem of constraining the sidelobe levels has been tackled by using a penalty method called Created Response Surface Technique (CRST). This method converts the constrained optimisation problem into a series of unconstrained ones. Numerical examples have been worked out to illustrate the application.

2. FORMULATION

2.1 Performance Indices:

The sum mode pattern of an N-element symmetrical array of progressively phased elements is given by [2]

$$\Sigma(\phi) = 2\delta I_0 + 4\sum_{i=1}^{M} I_i[\cos(\beta x_i \cos\phi).\cos(\beta x_i \sin\phi_{sq})] \quad (1)$$

where I_i and x_i are the currents and positions of the i^{th} element. δ is a switching function, being 1 for arrays with odd number of elements and 0 for even number of elements.

The sum mode directivity can be obtained as

$$D_s = \frac{|\Sigma(\emptyset)|^2_{max}}{\int_0^{\pi/2} \Sigma^2(\emptyset)\sin\emptyset d\emptyset} \qquad (2)$$

The slope-sum ratio or DMAS is given by

$$DMAS = \left.\frac{\Delta'(\emptyset)}{\Sigma(\emptyset)}\right|_{\emptyset=\pi/2} = \frac{4\sum_{i=1}^{M} I_i \beta x_i \sin(\beta x_i \sin\emptyset_{sq})}{\beta x_M[2\delta I_o + 4\sum_{i=1}^{M} I_i \cos(\beta x_i \sin\emptyset_{sq})]} \qquad (3)$$

where x_M is the distance of the last element from the centre of the array and $\Delta'(\emptyset)$ is the slope of the difference mode pattern.

2.2 Sidelobe level constraint and modified objective function

Each sidelobe is constrained to be less than a given value. Mathematically,

$$\psi_p(\bar{I},\bar{x},\bar{\alpha};\emptyset) \leq S_p \;, \; p = 1,2,\ldots,m \qquad (4)$$

where m is the number of sidelobes, ψ_p is the actual level of the p^{th} sidelobe and S_p is its constraint value. For this a penalty method viz. CRST is used. In this method, a new objective function is defined as

$$\sigma = -G_o + \delta R \sum_{i=1}^{M} (\psi_p - s_p)^2 \qquad (5)$$

G_o is the performance index to be maximised.

where R is a positive constant whose initial value is normally taken as unity, δ is a switching function, being 1 when the constraint is violated and 0 when not. Thus the penalties are levied only when the sidelobe constraints are violated. The procedure now consists of carrying out a minimisation of eqn. (5). The value of R is increased by a constant factor and with the solution of the previous iteration as the starting point, (5) is minimised again. This iteration is carried out till no further reduction of (5) is possible. It can easily be verified that when the constraints are all satisfied the objective function value δ will be independent of the penalty. Thus this method converts the original constrained problem into a series of unconstrained ones, with each iteration descending down a created response surface [3].

The penalty method described above has fuzed the directivity and the constraint expressions into one. There is no explicit expression available for the sidelobe level and hence it is not possible to evaluate its derivatives with respect to the parameters of minimisation. Hence, a search method, namely, the simplex method of Nelder and Mead [4] is used. This method is comparable in speed for this class of objective functions to the derivative methods while possessing a distinct advantage of warranting no information about the derivatives of the objective function. The simplex method sets up 'n+1' points called simplex in an n-dimensional space. It gropes towards the minimum by flipping or contracting the simplex. The logic used is based on an evaluation of the function at each corner of the simplex. It may be noted here that the simplex method is intrinsically more resistant to convergence to local minimum than most other methods by virtue of its having (n+1) starting points, with the result that the probability of one them being close to the global minimum is higher.

3. NUMERICAL EXAMPLES AND DISCUSSIONS

Several examples have been worked out to check the validity of the formulation. Only a few are reported here. A 21-element half-wavelength spaced array has been optimised-though the method is equally applicable to unequally spaced arays-for squint angles of 1° - 4° in steps of 1°. A constraint of 0.2 (-13.9794 db) has been placed both on the sum and diff. mode sidelobes. First, the DMAS of the monopulse array has been considered as a performance index to be maximised. The currents in the individual elements are varied. The results are included in Table I. The table also includes the computed D_s, D_d and slope gain product at the optimum DMAS point. The results corroborate the well-known facts that DMAS increases with increasing squint angle while the slope-gain product is optimum around $\phi_{sq}=3^\circ$. This squint angle, as is to be expected is around half the beam width of a 21-element equally spaced equally excited (6.37°).

Table I also includes, the results of optimising the DMAS at $\phi_{sq}=3^\circ$ but with constraints of 0.1 (-20 db) and 0.05 (-26.02 db) on the sum and difference mode sidelobes. It is clear from the table that the DMAS decreases with increasing severity of the sidelobe constraints.

The optimisation of D_s has also been performed for $\phi_{sq}=3^\circ$ with the sidelobe constraints of 0.2, 0.1 and 0.05. The results are included in Table-II.

It is apparent from the results in Tables I and II that there exists a certain amount of trade off between DMAS and D_s. However, when the sidelobe constraints are severe, the DMAS and D_s optimisation yield nearly the same results.

The slope gain product optimisation has also been performed by including the squint angle as a variable. It has been found that the optimum value of the squint angle, for a sidelobe constraint of 0.2, has been 2.82°. The optimum squint angle increased to 2.86° when the sidelobe constraint is increased to 0.1.

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TABLE-I

DMAS optimisation of 21-element half-wavelength-spaced array

ϕ_{sq}	sidelobe constraint	optimum DMAS	computed D_s	computed D_d	slope-gain product
1	0.2	0.2268	19.2437	9.8456	4.3929
2	0.2	0.4475	17.9907	12.0032	8.0514
3	0.2	0.7107	13.4070	11.9125	9.5279
4	0.2	0.8682	8.8177	11.8978	7.6559

TABLE-II

Optimisation of sum mode directivity D_s for a 21-element half wavelength spaced array at $\phi_{sq}=3^\circ$.

Sidelobe constraint	Optimum D_s	computed DMAS	computed D_d	slope-gain Product
0.2	14.0068	0.6029	12.2962	8.4452
0.1	13.8558	0.5254	12.2444	7.2811
0.05	13.4373	0.4599	12.1031	6.1792