APPLICATIONS OF DETERMINISTIC ERROR ANALYSIS IN ARRAY ANTENNAS.

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Introduction

Statistical error analysis has been investigated thoroughly in antenna theory (ref. 1). Deterministic analysis however has often been neglected due to the crude bounds obtained (worst case) or due to the lack of analytical expressions. In this paper we intend to show that well applied deterministic error analysis methods have useful applications in array theory.

Complex error analysis

A complex number, known to be in a small domain in the complex plane, used as argument of a function is mapped onto a domain that can be computed from the original domain and the knowledge of the function. One of the simplest choices for the domain is a small rectangle parallel with real and imaginary axisses (fig. 1). The mapped domain corresponding with the function can be surrounded by a domain of the original shape. This is illustrated in fig. 1 for the multiplication; a new rectangle surrounds completely the octogonal mapped domain. One can show that for the rectangular error case, the computation time for computing both the maximal and minimal bounds together with the function itself lies between 3 and 4 times the time required for the mere function evaluation in the case that the function is +, -, x, : or exp (). In practice, a small extra error is induced by the conversion of the errors given in polar form (decibels and degrees) to the rectangular ones, and by the eventual inverse conversion at the end of the computation.

Illustrations

One example that has been experimentally verified is a secondary radar antenna (ref. 2), composed of 16 pairs of monopoles. Both elements of a pair are fed with the same amplitude and phase. It was our aim to obtain the requirement of a - 24 dB sidelobe level by using a - 32 dB, 16-element Chebychev distribution. D. Fraeyman, to whom the authors express their gratitude for realising and measuring the network of the array, found errors of $\pm$ 0.2 dB and $\pm$ 5 ° on the 8 center outputs and $\pm$ 0.4 dB and $\pm$ 4° on the other 8 outputs. The worst case pattern corresponding with those errors applied on the theoretical distribution is shown in fig. 3 (solid curve). In practice, those errors do not occur on all elements;

hence, the measured pattern (dotted, fig. 3) and the pattern computed with the measured excitation coefficients (dash-double dotted, fig. 3), are well below the worst case pattern. However, the fact that the latter 2 curves do not coincide is due to installation errors introduced by the mismatch of the monopoles and by the bending of the cables connecting the network outputs to the monopoles. If we estimate those installation errors at $\pm$ 0.1 dB and $\pm$ 2°, we can check that the curve of the worst case error computed with the measured excitation coefficients combined with the small installation errors (dash-dotted, fig. 3) approaches the measured pattern.

Another example shows a $cosec^2$ pattern with 51 $\lambda_o/2$ spaced elements, that has to be realised between elevation angles of 1° to 20° (fig. 2). For the synthesis, a Tychonov regularisation between θ' = 0° to 70° and 91.5° to 180° has been used (ref. 3). To check the stability of the regularisation procedure, we proceed by introducing small errors on the computed excitation coefficients. In the case of errors equal to $\pm$ 0.1 dB and $\pm$ 2° the difference between the computed and the worst case error patterns remain within $\pm$ 3 dB over the desired range, and is very small in the critical direction (1°), demonstrating the effectiveness of the regularisation procedure.

Conclusion

Deterministic error analysis can yield practical bounds for many array problems. In this paper it has been demonstrated for two examples from the field of radar antennas.

References

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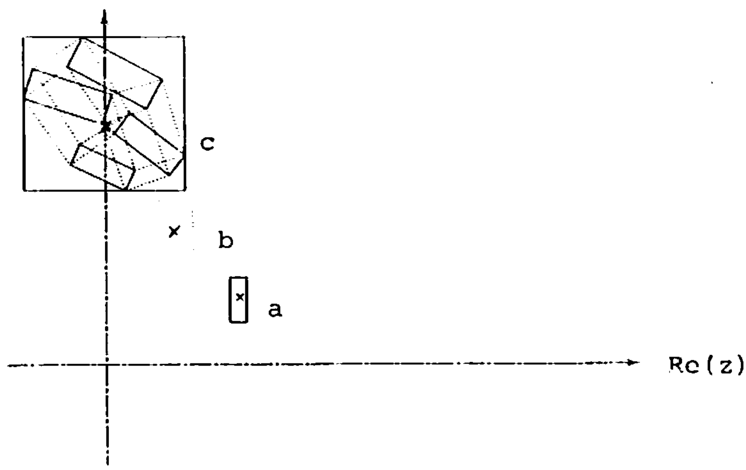

Fig. 1 : Deterministic error bounds for the multiplication c=axb in the complex plane.

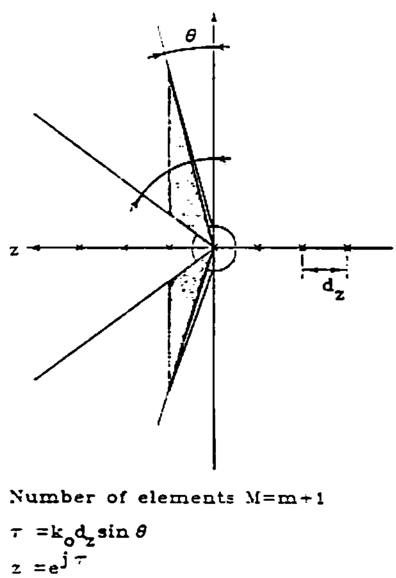

Fig. 2 : Configuration for the array analysis problem.

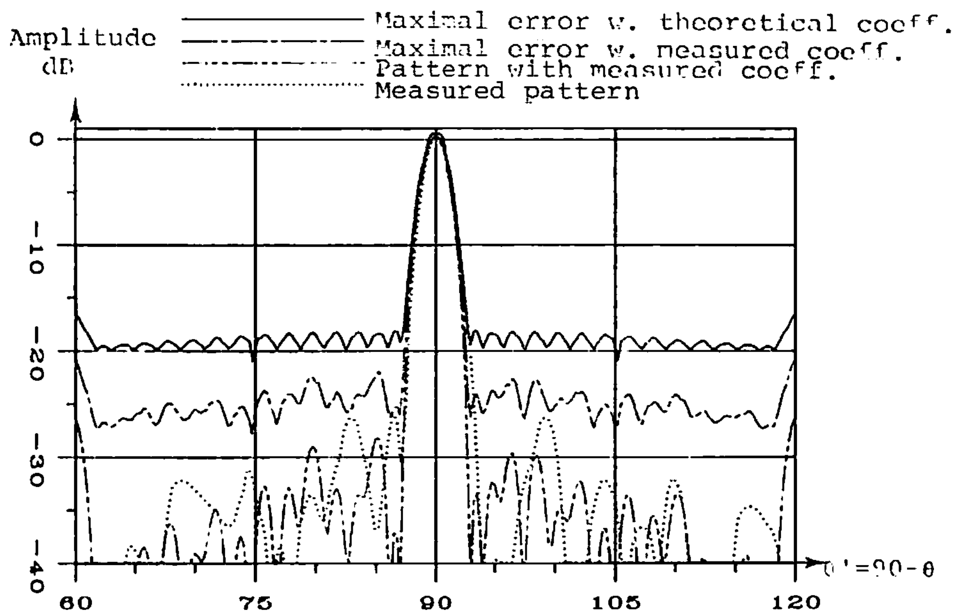

Fig.3 : Comparison of error bounds and experiment for a 32 element secundary radar (spacing 26cm, 26.5 cm,etc. f=1.09 GHz; aim= -32 dB Chebychev distribution)

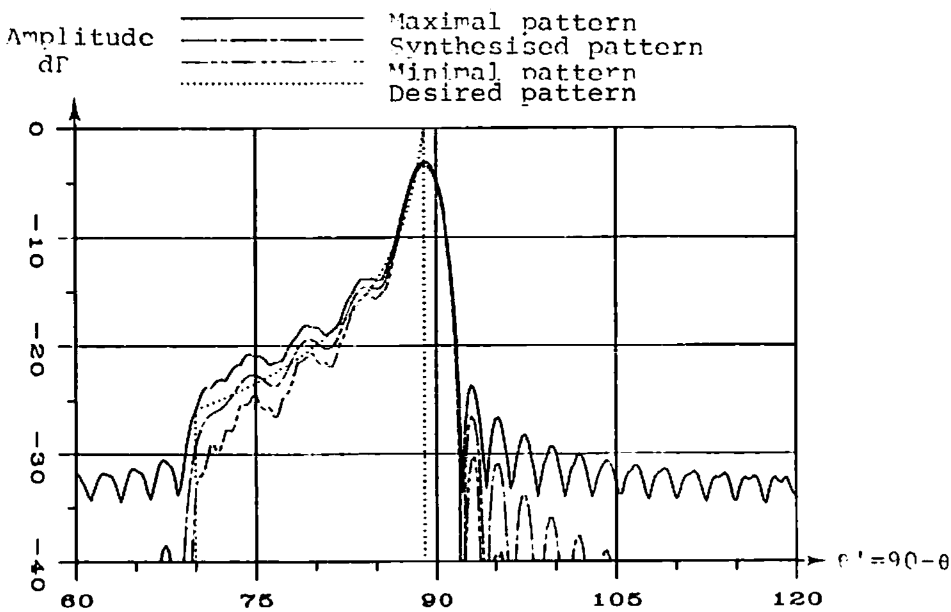

Fig. 4 : Computed error bounds for a csc^2 array