

FUNDAMENTAL GAIN-BANDWIDTH-VOLUME LIMITATIONS OF ANTENNAS

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Overview

Historically, the analysis of gain-bandwidth relations for general antennas has been founded on the properties of a spherical mode expansion of the fields external to the antenna volume. In the classic works by Chu¹ and Harrington² the mode amplitudes have not been directly related to a (non-unique) charge-current distribution within the antenna volume, raising questions of physical realizability. Without more information, the bandwidth cannot be accurately calculated.

We have succeeded in deriving an expression for the fractional bandwidth of an antenna in terms of a rigorously defined time-average "reactive stored energy" at the center frequency. This energy is computed directly from any reasonable charge-current distribution within the antenna volume, with no reference to any spherical mode fields outside the volume. Since the antenna gain and time-average radiated power are also directly given by this distribution, we can address more realistically such questions as:

1) What bandwidth and gain-bandwidth product can one expect from an antenna with a specified charge-current distribution on the radiating elements?

2) For a given working volume, what is the maximum bandwidth for a specified directional gain or gain pattern?

3) For a prescribed working volume, what is the maximum directional gain-bandwidth product obtainable?

We could also answer a question raised by Harrington² about the minimum dissipation factor $D = P_{\text{diss}}/P_{\text{rad}}$ of an arbitrary antenna if we assume the time-average power dissipated, P_{diss} , is determined by the surface resistance on radiating elements.

Question 1) is answered directly by formulas below. To answer questions 2) and 3) realistically, we examine an ensemble of electric dipole distributions within the working volume. In each distribution, we compute the currents consistent with any constraints by a method described below. These yield the radiation quantities of interest and the bandwidth. That distribution with the maximal quantity of interest is taken to represent a realizable physical antenna, whose sources must support its current distribution.

Foundation

The analysis begins with the Panofsky-Phillips expressions³ for the spatial-temporal electric and magnetic fields at a space-time point, in terms of the retarded charge and current densities throughout the volume. The fields are separated into near* ($\sim R^{-2}$) and far or radiation ($\sim R^{-1}$)

* Neither near-field integrand has an R^{-3} -term in the space-time domain.

components. \vec{E}_{near} and \vec{E}_{far} satisfy the homogeneous Maxwell equation at each spatial point; \vec{H}_{near} and \vec{H}_{far} do not. There is an effective current $\vec{K}(\vec{r}, t)$ in the $\nabla \times \vec{H}_{\text{near}}$ -equation and $-\vec{K}$ in the $\nabla \times \vec{H}_{\text{far}}$ - equation, given by an integral over retarded sources $\partial \vec{J} / \partial t$. \vec{K} couples the near and far fields together. For this reason, the massless electrons flowing on the surfaces of the antenna elements convert near-field energy to radiated energy in a mathematically complicated way.

From these expressions, one may derive the transient energy later radiated across the radiation sphere during a time interval T of the excitation in the antenna volume. The formula for this energy is a single integral over time and a double integral over the charge-current distribution. The formula for time-average radiated power follows directly, but we shall not quote it because this power is more directly computed as a single integral of the Poynting flux on the radiation sphere.

One may also compute from Panofsky-Phillips sinusoidal-field expressions a "reactive stored energy" W_R defined as this integral over the surfaces of the antenna elements:

$$W_R = \frac{1}{4} \text{Imag} \int_{S_a} \partial \hat{E}_\omega / \partial \omega \times \hat{H}_\omega^* \cdot d\vec{S} \quad (1)$$

Here Imag denotes imaginary component, $\vec{E}(\vec{r}, t) = \text{Real}(\hat{E}(\vec{r})e^{j\omega t})$, and similarly for \vec{H} in terms of \hat{H}_ω , and $d\vec{S}$ points into the open volume. The fractional bandwidth B of a single driving circuit for all the generators which establish \hat{E}_ω on the antenna elements may be shown to be upperbounded** by

$$B = P_{\text{rad}} / (\omega W_R) = 1/Q \quad (2)$$

This is the quality factor Q relevant to bandwidth computation. We have tacitly assumed that the input reactance of the driving circuit varies linearly over the bandwidth, which usually implies $-0.5 \lesssim B \lesssim 0.5$. Any statements about maximal performance of a possible antenna within a given working volume will be less accurate for larger B within this interval.

Our expression for time-average W_R crucial for the computation of bandwidth at frequency ω , $k = \omega/c$, by Eq. (2) is

$$W_R = (16\pi\epsilon_0)^{-1} \text{Real} \left\{ \iint_{S_a} \hat{q}_\omega(\vec{r}') \frac{e^{-jkR}}{R} \hat{q}_\omega^*(\vec{r})(1 + jkR) dS dS' \right. \\ \left. + (1/c^2) \iint_{S_a} \hat{j}_\omega(\vec{r}') \frac{e^{-jkR}}{R} \hat{j}_\omega^*(\vec{r})(1 - jkR) dS dS' \right\} \quad (3)$$

** We have neglected the stored energy in the feed circuits; W_R is the pertinent stored energy in the space surrounding the antenna elements.

Here $R = |\vec{r} - \vec{r}'|$. The "1" in $(1 \pm jkR)$ represents "near-energy" terms important for small R ; the jkR -terms are the "far-energy" ones important in the larger antennas.

Q's of Low-Frequency Dipoles

Simple but significant examples of the application of Eq. (3) are the calculations of our Q for a low-frequency electric dipole of length $2l$, radius $a \ll L$, with a triangular current distribution, as well as for a magnetic dipole (loop) of radius r , wire radius $w \ll r$ and a uniform (sinusoidal) current. The results are:

$$Q_{el \text{ dip}} = 3 \ln \left(\frac{L}{ea} \right) / (kL)^3, \quad e = 2.71828 \dots \quad (4a)$$

$$Q_{mag \text{ dip}} = \frac{3}{\pi} \left[\ln \left(\frac{8r}{w} \right) - 2 \right] / (kr)^3 \quad (4b)$$

The former is more realistic (higher) than the value quoted by Hansen⁴ for Q as defined by Chu,

$$Q_{Chu} = 1 / (kR_{sphere})^3, \quad R_{sphere} = L \quad (5)$$

Computation of Maximal Antenna Properties from Continuous Current Distributions

To answer questions such as 2) and 3), we have examined the general problem for a spherical working volume of electrical radius ka at the center frequency. With directional gain defined in the x -direction, we assumed only $\hat{J}_z(\vec{r})$ current, expanded in spherical harmonics of the form:

$$a_{nm} \left(\int g(\alpha) j_n(\alpha r) \alpha^2 d\alpha \right) P_n^m(\cos\theta) \cos m\phi$$

for n, m both even or both odd. (The integral can be collapsed to $j_n(kr)$ if desired.) The expression for far-field $E_z(r, \pi/2, 0)$ is straightforward to evaluate with the aid of well-known integrals of the spherical harmonics, resulting in a sum $\sum a_{nm} f_{nm}(ka)$.

Computation of W_R by Eq. (3) written for a volume distribution is more formidable. The real quantity within the braces is:

$$\begin{aligned} \{ \} = & \omega^{-2} \iint_{V_a} [v' \cdot \hat{J}_z^R(\vec{r}') \nabla \cdot \hat{J}_z^R(\vec{r}) + v' \cdot \hat{J}_z^I(\vec{r}') \nabla \cdot \hat{J}_z^I(\vec{r})] \\ & \cdot (\cos kR/R + k \sin kR) dV dV' + c^{-2} \iint_{V_a} [\hat{J}_z^R(\vec{r}') \hat{J}_z^R(\vec{r}) + \hat{J}_z^I(\vec{r}') \hat{J}_z^I(\vec{r})] \\ & \cdot (\cos kR/R - k \sin kR) dV dV' \end{aligned} \quad (6)$$

Superscripts R, I denote real and imaginary components. If $\cos kR/R$ is expanded in spherical harmonics, the $\theta\theta'$ and $\phi\phi'$ integrals can be carried out explicitly but not the double integral over rr' . The result for the $\cos kR/R$ -portion of Eq. (6) is the triple sum,

$$\sum_{n, m \leq n} \sum_{n'} [a_{nm}^R a_{n'm}^R + a_{nm}^I a_{n'm}^I] [F_{nm} \delta(n, n') + G_{nm} \delta(n, n'+2) + H_{nm} \delta(n, n'-2)] \quad (7)$$

where $\delta(i,j) = 1$ for $i=j$, 0 for $i \neq j$. However, the far-energy or sinkR-portion of Eq. (6) cannot be similarly evaluated; the result is a full triple sum over n,m and n' . This means any maximal quantity will be evaluated as the extremal eigenvalue of a full matrix equation of rank $(n_{\max}+1)(n_{\max}+2)/2$.

Because of these complexities, we decided not to study maximal antenna performance from an optimized, continuous current-charge distribution within a specified spherical or other working volume. Instead, we will represent the source by various volume distributions of dipoles and solve for their amplitudes by a method described next.

Synthesis of Dipole Currents for Prescribed Radiation Field

The idea is to specify a geometrical distribution of dipoles with appropriate orientations within the working volume and solve for their current amplitudes to realize a constraint, such as maximum directional gain. Then we compute W_R by Eq. (3), including both self and mutual effects, and B by Eq. (2) in order to study gain and bandwidth as a function of dipole distribution.

The method of Sahalos⁵ based on orthonormalization of the dipole basis vectors is appropriate. The amplitudes of the new vectors are computed so as to represent a specified far field $\vec{E}(\theta,\phi)$ in a least-squares sense. These amplitudes yield the dipole current amplitudes and hence the gain and power radiated P_{rad} . Bandwidth B follows from Eqs. (2) and (3).

We are now studying the answers to questions 2) and 3) for several preliminary problems, such as a line of dipoles both disconnected and connected into a wire, a planar array of parallel lines of dipoles, and parallel planar arrays. These will serve as a guide in selecting dipole distributions to answer questions 2) and 3) for spherical and other working volumes.

References

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