SPACE-TIME COOPERATION DIVERSITY USING HIGH-RATE CODES

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1. Introduction

Recent developments on multiple-input-multiple-output (MIMO) systems have shown that, with the use of multiple transmit and receive antennas, reliable wireless links can be implemented with the system capacity exceeding the Shannon's channel capacity defined in a single-input-single-output (SISO) system. In many applications, however, it is often impractical for a transmitter to equip multiple antennas. Cooperative diversity exploiting cooperation among multiple terminals has been developed to achieve transmit diversity gain beyond the number of physical transmit antennas equipped at an terminal.

So far, the research on cooperative diversity have focused primarily on the protocol design and capacity analysis. Proper design of distributed space-time codes, although being equally important to make the ideas practically amendable, has so far received little attention. In this paper, we propose a cooperative diversity scheme that exploits the recently developed high-rate high-diversity space-time codes and demonstrate that the proposed scheme achieves higher data rate than existing cooperative diversity schemes. We then further consider the power optimization of the distributed space-time codes in the underlying cooperative systems.

2. System Model

Figure 1 illustrates the concept of cooperative wireless network. The users incorporate with each other in such a way that each acts as a relay terminal for the other. Therefore, each transmit user receives an attenuated and noisy version of the transmitted signals from other users, and relays them to the destination or other relays. The cooperation process can be divided into two phases, i.e., the broadcast phase and the relay phase (refer to Fig. 2).
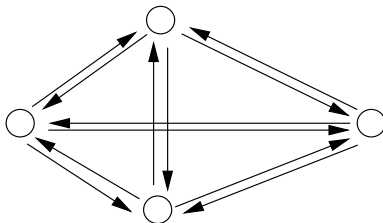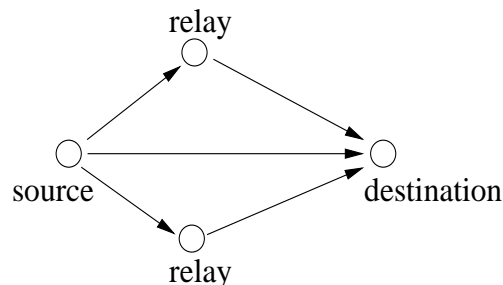

Figure 1. System model.

Figure 2. Data transmission through relays.

Depending on how the users relay other users' information, there are two major algorithms, namely, amplify-and-forward and decode-and-forward. At the expense of introducing additional complexity at the relay terminals, the decode-and-forward algorithm allows the removal of relay noise, and provides the flexibility of encoding the information at the relay phase so that higher spectral efficiency can be achieved [1]. On the other hand,

Frequency			
Ch. 1	1 transmits	2 repeats 1	$\cdots$	M repeats 1
Ch. 2	2 transmits	3 repeats 2	$\cdots$	1 repeats 2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Ch. M	M transmits	1 repeats M	$\cdots$	M−1 repeats M

(a) repetition-based diversity scheme

Frequency		
Ch. 1	1 transmits	1,2,...,M repeat 1
Ch. 2	2 transmits	1,2,...,M repeat 2
$\vdots$	$\vdots$	$\vdots$
Ch. M	M transmits	1,2,...,M repeat M

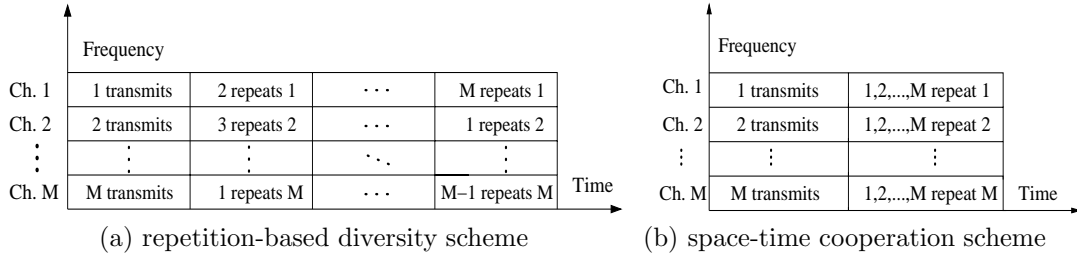(b) space-time cooperation scheme

Figure 3. Cooperative diversity schemes.

several cooperative diversity protocols have been proposed [1, 2]. The early protocols use repetition-based approaches, which is illustrated in Fig. 3(a). After a source broadcasts its information to the destination and the relays, all relays repeat this information in a sequential order. Therefore, a diversity gain of up to M is achieved at the expense of reducing the degrees of freedom (DOFs) by a factor of M. Recently, more effective protocols have been developed to take advantages of the development of MIMO space-time codes. For example, space-time cooperation protocols proposed in in [1, 2] are illustrated in Fig. 3(b). These protocols provide more effective bandwidth use, particularly when the number of cooperative users is large. These protocols allow the use of a fraction of up to 1/2 of the total DOFs in the channel.

3. High-Rate Space-Time Codes

Recently, non-orthogonal space-time codes have been introduced that achieve higher data rate than orthogonal ones, while a high diversity gain is maintained (e.g., [3, 4, 5, 6]). Among these codes, the high-dimensional lattice-based systematic cyclotomic space-time codes [6], summarized below for the two-antenna case, provide the full diversity gain (i.e., the data rate $R = N_t$ where N_t is the number of transmit antennas) and a high diversity product.

Denote $\phi(n)$ as the Euler number of n. We choose two positive integers m and n such that $N_t = \phi(N)/\phi(n)$ where $N = mn$. An $N_t \times N_t$ cyclotomic lattice generating matrix is defined as

$$\mathbf{G}(m,n) = \begin{bmatrix} \zeta_N & \zeta_N^2 & \cdots & \zeta_N^{N_t} \\ \zeta_N^{1+n_2 m} & \zeta_N^{2(1+n_2 m)} & \cdots & \zeta_N^{N_t(1+n_2 m)} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_N^{1+n_{N_t} m} & \zeta_N^{2(1+n_{N_t} m)} & \cdots & \zeta_N^{N_t(1+n_{N_t} m)} \end{bmatrix}, \tag{1}$$

where $\zeta_N = \exp(j2\pi/N)$ and $j = \sqrt{-1}$. A cyclotomic lattice $\Gamma_{N_t}(\mathbf{G}(m,n))$ is a set of $[x_1, x_2, \cdots, x_{N_t}]^T \subset \mathcal{C}^{N_t}$, where superscript T denotes transpose,

$$[x_1, x_2, \cdots, x_{N_t}]^T = \mathbf{G}(m,n)[u_1, u_2, \cdots, u_{N_t}]^T, \tag{2}$$

and

$$\begin{bmatrix} \mathrm{Re}(u_l) \\ \mathrm{Im}(u_l) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \cos(2\pi/m) & \sin(2\pi/m) \end{bmatrix} \begin{bmatrix} w_{l1} \\ w_{l2} \end{bmatrix}, \quad l = 1, 2, \cdots, N_t. \tag{3}$$

In(3), $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ denote real and imaginary part operators, respectively, and w_{l1} and w_{l2} are real integers which bear the source information.

For the single-layer cyclotomic space-time coding scheme, the following codeword is formed [6],

$$\mathbf{X} = \sqrt{g}\, \mathrm{diag}\,[x_1,\ x_2,\ \cdots,\ x_{N_t}], \tag{4}$$

where g is a gain normalization factor and diag[·] denotes the diagonal matrix operator. The cyclotomic space-time codes can be arranged to form a multi-layer structure which bear higher data rate [4, 5, 7].

4. Proposed Space-Time Cooperation Protocol

Figure 4 shows the proposed method for an M-terminal scenario. It transmits the high-rate cyclotomic space-time codes, $\mathbf{X}_i(t)$ instead of original data sequence and, as such, space-time cooperation is achieved in a more effective way. In the first $M-1$ time blocks, the ith row of the $M \times M$ codeword is broadcasted during the ith time block, where $i = 1, \cdots, M-1$. In particular, when the single-layer codes are used, there is only one element to be transmitted during each of the first $M-1$ time blocks. In the Mth time block, the full $M \times M$ codeword is transmitted from all the M virtual antennas.

The proposed method is amendable for the use of both amplify-and-forward and decode-and-forward schemes. When the former one is used, it does not requires decoding and encoding at the relays while it takes the advantage of high data rate and high diversity gain. It is evident that, in the proposed scheme, the DOF is always higher than $1/2$, compared to $1/M$ and $1/2$ in the aforementioned space-time cooperation schemes.
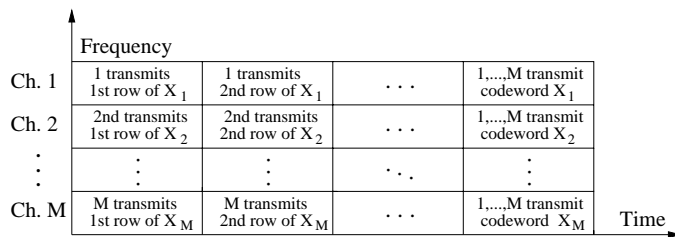
	Frequency			
Ch. 1	1 transmits 1st row of X$_1$	1 transmits 2nd row of X$_1$	$\cdots$	1,...,M transmit codeword X$_1$
Ch. 2	2nd transmits 1st row of X$_2$	2nd transmits 2nd row of X$_2$	$\cdots$	1,...,M transmit codeword X$_2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Ch. M	M transmits 1st row of X$_M$	M transmits 2nd row of X$_M$	$\cdots$	1,...,M transmit codeword X$_M$

Figure 4. The proposed cooperation protocol.

5. Power Optimization

Consider a two-user case that exploits the 2×2 single-layer space-time code defined in (4), the proposed protocols converts a codeword to an equivalent 2×3 distributed space-time codeword,

$$\mathbf{X} = \sqrt{g}\begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix} \quad \Rightarrow \quad \mathbf{X}' = \sqrt{g'}\begin{bmatrix} x_2 & \alpha x_1 & 0 \\ 0 & 0 & \hat{x}_2 \end{bmatrix} \tag{5}$$

where g' is a power normalization factor, α is introduced for power optimization, and $\hat{x}_i$ is a replica of x_i at the relay and its value depends on the applied cooperation protocol. For simplicity, the relay noise is ignored and $\hat{x}_i$ is assumed to be the exact replica of x_i.

To maintain a unit codeword energy, we have $g' = (2+\alpha^2)\sigma^2$, where σ^2 is the averaged energy of x_i, $i = 1, 2$, over all code constellations. Therefore, the determinant of the distributed codeword $\mathbf{X}'$ is

$$\det\left[\mathbf{X}'(\mathbf{X}')^H\right] = \frac{1}{(2+\alpha^2)^2}\left(|x_2|^4 + \alpha^2|x_1 x_2|^2\right). \tag{6}$$

It is known [6] that $\min|x_1 x_2| = 1$. Denote $\beta = \min|x_2|^2 < 1$, then the minimum value of the above equation becomes $(\beta^2 + \alpha^2)/(2+\alpha^2)^2$. Therefore, the optimum value of α corresponding to the maximum value of the above minimum determinant is $\alpha = \sqrt{2-\beta^2}$. When the modulation constellation is large, $\beta \ll 1$, and we have $\alpha \approx \sqrt{2}$.

6. Numerical Results

In the numerical simulations, we consider the codes depicted in (5). The prototype codeword has 256 constellations over two time slots. The relay noise is not considered in the simulations.

Figure 5(a) illustrates the minimum determinant of the codeword matrix versus the value of α. It is evident from this result that the minimum determinant achieves its
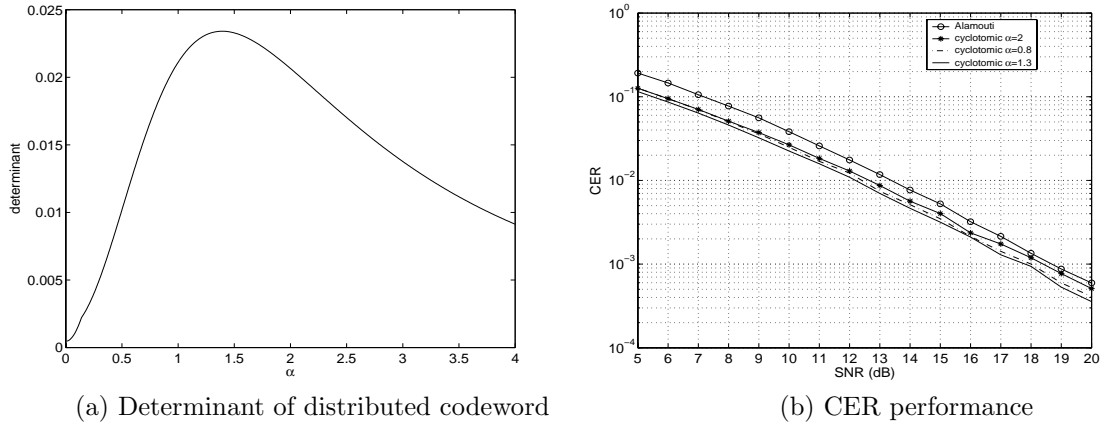
(a) Determinant of distributed codeword (b) CER performance

Figure 5. Simulation Results.

maximum value where α is slightly smaller than $\sqrt{2}$, and there is a 10% improvement in the peak value compared to that corresponding to $\alpha = 1$. Fig. 5(b) compares the CER performance of the proposed scheme for α=0.8, 1.3, and 2. For comparison, the results using the distributed space-time codes developed from Alamouti's space-time codes are also computed. It is seen that the results corresponding to α =1.3 outperforms the others. The proposed method outperforms the Alamouti's code-based scheme by about 1.5 dB.

7. Conclusion

We have proposed a space-time cooperation scheme that exploits full-rate full-diversity cyclotomic space-time codes in the cooperative diversity platforms. The proposed protocol is applicable to both the amplify-and-forward and decode-and-forward algorithms. Power optimization within a distributed codeword is also considered. The advantages of the proposed method were confirmed through numerical simulations.

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