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Antenna current optimization and optimal design

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***Abstract*—Antenna current optimization is used to determine optimal currents in the antenna design region. The currents provide understanding, physical bounds, and figures of merits for antenna designs. The problem is formulated as a convex optimization problem expressed in the currents on the antenna structure. These convex optimization problems are solved with a computational cost comparable to a Method of Moments (MoM) solution for an antenna in the same geometry. In this presentation, antenna current optimization and stored energy are reviewed. Numerical results for maximization of the gain to Q-factor quotient and minimization of the Q-factor for prescribed radiated fields are presented.**

***Index Terms*—stored energy, physical bounds, Q-factor**

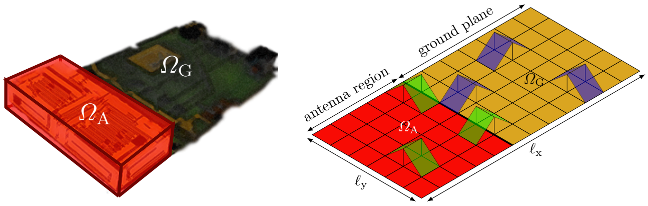

Fig. 1. Device geometry with a region Ω with current density $\boldsymbol{J}$, (top) with a device and (bottom) the numerical approximation with local basis functions. It is assumed that the currents $\boldsymbol{J}_{\rm A}$ can be controlled in the antenna region $\Omega_{\rm A}$. The currents $\boldsymbol{J}_{\rm G}$ in $\Omega_{\rm G} = \Omega \setminus \Omega_{\rm A}$ are induced by the currents $\boldsymbol{J}_{\rm A}$, see also [18].

1. Introduction

Design of small antennas is challenging because fundamental physics restricts the antenna performance [1–4]. Physical bounds express the trade-off between antenna performance and antenna size. The physical bounds on the Q-factor are determined from the stored energy around the antenna. The classical approach in [1, 5, 6] is based on calculations of the stored energy using mode expansions and gives expressions for the lower bound on the Q-factor for spherical geometries. Several generalizations of the bounds to arbitrary shaped geometries have been presented in the last years [7–17].

Optimization of the antenna currents is a general methodology to analyze and determine bounds on antennas [12, 13, 18]. The Q-factor is evaluated from the stored energy expressed in the current density [10]. The expressions are implemented using frequency differentiation of the method of moments (MoM) impedance matrix for the free space case [13, 19, 20]. Optimization of the current density on the antenna is formulated as a convex optimization problem and can hence be solved efficiently [13, 18]. Antenna parameters based on combinations of quadratic forms, such as the stored energy and radiated power, linear forms, such as near- and far fields and induced currents, and norms are used to formulate convex optimization problems relevant for specific antenna problems [13, 18].

2. Antenna current optimization

Antennas are often embedded into devices such as mobile phones and sensors, see Fig. 1a. The device structure Ω is divided into two regions; an antenna region $\Omega_{\rm A} \subset \Omega$ and the remaining part $\Omega_{\rm G} = \Omega \setminus \Omega_{\rm A}$. We assume that the antenna designer can specify the shape of the metal and dielectrics in the antenna region $\Omega_{\rm A}$. The electromagnetic properties of the remaining region $\Omega_{\rm G} = \Omega \setminus \Omega_{\rm A}$ are assumed to be fixed. The current density $\boldsymbol{J}_{\rm A}$ in $\Omega_{\rm A}$ is controllable and the current density $\boldsymbol{J}_{\rm G}$ in $\Omega_{\rm G}$ is induced by $\boldsymbol{J}_{\rm A}$. The optimal current distribution is determined from the solution of an optimization problem expressed in the currents [13, 18, 21].

For simplicity, the analysis is restricted to induced currents on a PEC ground plane. The induced currents depend linearly on the currents in the antenna region, and we use the electric field integral equation (EFIE) with impedance matrix $\mathbf{Z} = \mathbf{R} + {\rm j}\mathbf{X}$ to determine this relation [13, 18, 21]

$$\begin{pmatrix} \mathbf{Z}_{\rm AA} & \mathbf{Z}_{\rm AG} \\ \mathbf{Z}_{\rm GA} & \mathbf{Z}_{\rm GG} \end{pmatrix} \begin{pmatrix} \mathbf{I}_{\rm A} \\ \mathbf{I}_{\rm G} \end{pmatrix} = \begin{pmatrix} \mathbf{V}_{\rm A} \\ \mathbf{0} \end{pmatrix}. \tag{1}$$

The second row gives the equation

$$\mathbf{Z}_{\rm GA}\mathbf{I}_{\rm A} + \mathbf{Z}_{\rm GG}\mathbf{I}_{\rm G} = \mathbf{C}\mathbf{I} = \mathbf{0} \tag{2}$$

which is included as a constraint in the convex optimization problems. In the decomposition of the basis functions into its antenna, $\mathbf{I}_{\rm A}$, and ground plane, $\mathbf{I}_{\rm G}$, parts, we assign basis functions with support in both $\Omega_{\rm A}$ and $\Omega_{\rm G}$ to the antenna part $\mathbf{I}_{\rm A}$, see Fig. 1.

The MoM approximation of the stored energies [18] can be written as

$$W_{\rm e} \approx \frac{1}{8}\mathbf{I}^{\rm H}\left(\frac{\partial \mathbf{X}}{\partial \omega} - \frac{\mathbf{X}}{\omega}\right)\mathbf{I} = \frac{1}{4\omega}\mathbf{I}^{\rm H}\mathbf{X}_{\rm e}\mathbf{I} \tag{3}$$

for the stored electric energy and

$$W_{\rm m} \approx \frac{1}{8}\mathbf{I}^{\rm H}\left(\frac{\partial \mathbf{X}}{\partial \omega} + \frac{\mathbf{X}}{\omega}\right)\mathbf{I} = \frac{1}{4\omega}\mathbf{I}^{\rm H}\mathbf{X}_{\rm m}\mathbf{I} \tag{4}$$

for the stored magnetic energy, where the electric $\mathbf{X}_{\rm e}$, and magnetic $\mathbf{X}_{\rm m}$, reactance matrices are introduced and the superscript $^{\rm H}$ denotes the Hermitian transpose. The expressions (3) and (4) are identical to the stored energy expression (for surface current densities and free space) [10, 22], see also [20, 23]. It is noted that the computations of the reactance matrices only require minor modifications of existing MoM codes.

The constraint $\mathbf{CI} = \mathbf{0}$ in (2) is included in the G/Q optimization problem [13, 18] giving the convex optimization problem

$$\begin{aligned} &\text{minimize} && w \\ &\text{subject to} && \mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{e}}\mathbf{I} \le w, \\ &&& \mathbf{I}^{\mathrm{H}}\mathbf{X}_{\mathrm{m}}\mathbf{I} \le w, \\ &&& \mathbf{FI} = -\mathrm{j}, \\ &&& \mathbf{CI} = \mathbf{0} \end{aligned} \tag{5}$$

where $\mathbf{F}$ denotes the far-field matrix [13, 18]. This optimization problem is easily solved using *e.g.*, CVX [24] and provide upper bounds on G/Q for arbitrary shaped antenna and ground plane regions [13, 21].

There are many possible antenna current optimization formulations. The case with maximization of G/Q leads to minimization of the stored energy for a fixed radiated field in one direction (5). The generalization to antennas with directivity $D \ge D_0$ is obtained by addition of a constraint of the total radiated power. The stored energy can also be minimized for a desired radiated field or by projection of the radiated field on the desired far field [13]. The case with antennas embedded in a lossy background media is very different as there is no far field in the lossy case. It is however simple to instead include constraints on the near field [25]. It is also possible to impose constraints on the sidelobe level or radiation pattern in some directions similar to the cases in array synthesis [26].

3. Conclusions

Antenna current optimization can be used for arbitrary shaped antenna regions. In this paper, we focus on the case with antennas integrated with a PEC ground plane [13, 18, 21]. Generalization to antennas embedded in lossy media is considered in [25] and antennas above infinite ground planes in [27]. Geometries filled with arbitrary inhomogeneous materials are analyzed using stored energy based on state-space models. Combinations of electric and magnetic currents can also be analyzed [14–16]. Optimal antenna designs are *e.g.*, investigated in [8, 21, 28–30].

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References

[1] L. J. Chu, "Physical limitations of omnidirectional antennas," *J. Appl. Phys.*, vol. 19, pp. 1163–1175, 1948.

[2] J. Volakis, C. C. Chen, and K. Fujimoto, *Small Antennas: Miniaturization Techniques & Applications*. New York: McGraw-Hill, 2010.

[3] M. Gustafsson, D. Tayli, and M. Cismasu, *Physical bounds of antennas*. Springer-Verlag, 2015, pp. 1–32.

[4] A. K. Skrivervik, J.-F. Zürcher, O. Staub, and J. R. Mosig, "PCS antenna design: The challenge of miniaturization," *IEEE Antennas and Propagation Magazine*, vol. 43, no. 4, pp. 12–27, Aug. 2001.

[5] R. E. Collin and S. Rothschild, "Evaluation of antenna Q," *IEEE Trans. Antennas Propagat.*, vol. 12, pp. 23–27, Jan. 1964.

[6] H. L. Thal, "New radiation Q limits for spherical wire antennas," *IEEE Trans. Antennas Propagat.*, vol. 54, no. 10, pp. 2757–2763, Oct. 2006.

[7] M. Gustafsson, C. Sohl, and G. Kristensson, "Physical limitations on antennas of arbitrary shape," *Proc. R. Soc. A*, vol. 463, pp. 2589–2607, 2007.

[8] ——, "Illustrations of new physical bounds on linearly polarized antennas," *IEEE Trans. Antennas Propagat.*, vol. 57, no. 5, pp. 1319–1327, May 2009.

[9] A. D. Yaghjian and H. R. Stuart, "Lower bounds on the Q of electrically small dipole antennas," *IEEE Trans. Antennas Propagat.*, vol. 58, no. 10, pp. 3114–3121, 2010.

[10] G. A. E. Vandenbosch, "Reactive energies, impedance, and Q factor of radiating structures," *IEEE Trans. Antennas Propagat.*, vol. 58, no. 4, pp. 1112–1127, 2010.

[11] ——, "Simple procedure to derive lower bounds for radiation Q of electrically small devices of arbitrary topology," *IEEE Trans. Antennas Propagat.*, vol. 59, no. 6, pp. 2217–2225, 2011.

[12] M. Gustafsson, M. Cismasu, and B. L. G. Jonsson, "Physical bounds and optimal currents on antennas," *IEEE Trans. Antennas Propagat.*, vol. 60, no. 6, pp. 2672–2681, 2012.

[13] M. Gustafsson and S. Nordebo, "Optimal antenna currents for Q, superdirectivity, and radiation patterns using convex optimization," *IEEE Trans. Antennas Propagat.*, vol. 61, no. 3, pp. 1109–1118, 2013.

[14] A. D. Yaghjian, M. Gustafsson, and B. L. G. Jonsson, "Minimum Q for lossy and lossless electrically small dipole antennas," *Progress In Electromagnetics Research*, vol. 143, pp. 641–673, 2013.

[15] B. L. G. Jonsson and M. Gustafsson, "Stored energies in electric and magnetic current densities for small antennas," *Proc. R. Soc. A*, vol. 471, no. 2176, p. 20140897, 2015.

[16] O. S. Kim, "Lower bounds on Q for finite size antennas of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. 64, no. 1, pp. 146–154, 2016.

[17] L. Jelinek and M. Capek, "Optimal currents on arbitrarily shaped surfaces," *arXiv preprint arXiv:1602.05520*, 2016.

[18] M. Gustafsson, D. Tayli, C. Ehrenborg, M. Cismasu, and S. Nordebo, "Antenna current optimization using MATLAB and CVX," *FERMAT*, vol. 15, no. 5, pp. 1–29, 2016. [Online]. Available: http://www.e-fermat.org/articles/gustafsson-art-2016-vol15-may-jun-005/

[19] M. Gustafsson and B. L. G. Jonsson, "Antenna Q and stored energy expressed in the fields, currents, and input impedance," *IEEE Trans. Antennas Propagat.*, vol. 63, no. 1, pp. 240–249, 2015.

[20] R. F. Harrington and J. R. Mautz, "Control of radar scattering by reactive loading," *IEEE Trans. Antennas Propagat.*, vol. 20, no. 4, pp. 446–454, 1972.

[21] M. Cismasu and M. Gustafsson, "Antenna bandwidth optimization with single frequency simulation," *IEEE Trans. Antennas Propagat.*, vol. 62, no. 3, pp. 1304–1311, 2014.

[22] M. Gustafsson and B. L. G. Jonsson, "Stored electromagnetic energy and antenna Q," *Progress In Electromagnetics Research (PIER)*, vol. 150, pp. 13–27, 2015.

[23] W. Geyi, "A method for the evaluation of small antenna Q," *IEEE Trans. Antennas Propagat.*, vol. 51, no. 8, pp. 2124–2129, 2003.

[24] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 1.21," http://cvxr.com/cvx, Apr. 2011. [Online]. Available: http://cvxr.com/cvx/

[25] M. Gustafsson, J. Friden, and D. Colombi, "Antenna current optimization for lossy media with near field constraints," *Antennas and Wireless Propagation Letters, IEEE*, vol. 14, pp. 1538–1541, 2015.

[26] H. L. Van Trees, *Optimum Array Processing*. New York: John Wiley & Sons, Inc., 2002.

[27] D. Tayli and M. Gustafsson, "Physical bounds for antennas above a ground plane," *Antennas and Wireless Propagation Letters, IEEE*, vol. 15, pp. 1281–1284, 2016.

[28] R. W. Ziolkowski, M.-C. Tang, and N. Zhu, "An efficient, broad bandwidth, high directivity, electrically small antenna," *Microwave and Optical Technology Letters*, vol. 55, no. 6, pp. 1430–1434, 2013.

[29] S. R. Best, "Electrically small resonant planar antennas: Optimizing the quality factor and bandwidth." *IEEE Antennas and Propagation Magazine*, vol. 57, no. 3, pp. 38–47, June 2015.

[30] M. Shahpari, D. Thiel, and A. Lewis, "An investigation into the Gustafsson limit for small planar antennas using optimization," *IEEE Trans. Antennas Propagat.*, vol. 62, no. 2, pp. 950–955, Feb 2014.