AN APPLICATION OF MUSIC METHOD TO THE GROUND-BASED DIRECTION FINDING OF MAGNETOSPHERIC VLF/ELF WAVES

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Abstract

A new ground-based direction finding method of magnetospheric VLF/ELF radio waves is proposed. The proposed method is based on the MUSIC procedure with use of the multiple electromagnetic fields. Simulations are performed on this problem, and it is found that the results indicate the effectiveness of the MUSIC method.

1. Introduction

Direction Finding of magnetospheric VLF/ELF waves (0.3 ~ 30kHz) is known to be a very important technique for the study of not only their generationand propagation mechanisms, but also the dynamics of magnetospheric plasma. Several methods for direction finding by using the simultaneous recording of multiple field components have been developed. The wave distribution function (WDF) method[1,2,3] is one of the most important. WDF means the spatial distribution of wave energy at the ionospheric base, and this method is based on numerical inversion from a spectral matrix composed of the auto- and cross-power spectra among observed field components as follows.

$$S_{ij}(\omega) = \frac{\pi}{2}\int_0^{2\pi}\int_0^1 a_{ij}(\omega,\theta,\phi,p_0)F(\omega,\theta,\phi)d(cos\theta)d\phi, \qquad (1)$$

where S and F indicate the spectral matrix and WDF, respectively, and ij corresponds the ijth element. ω, θ, and ϕ indicate the angular frequency, incident polar angle, and azimuthal angle. a_{ij} is known and the integration kernel. p_0 is the polarization. The maximum entropy concept or the linear regularization concept are adopted to recover the WDF image and the direction of wave arrival is determined by the peak of reconstructed WDF image. Therefore, this method is effective in the multi-wave-source case. However, there are problems on wave source separation in the case of closely distributed sources and the computation time. A method which overcomes these difficulties is required and the multiple signal classification (MUSIC) method[4] is examined in this paper for the ground-based direction finding.

2. Principle of the MUSIC method

Basic concept of MUSIC is the following. The waveform received at the M antenna elements are a linear combination of D incident waves and noise.

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} a(\varphi_1) & a(\varphi_2) & \cdots & a(\varphi_D) \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_D \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{bmatrix} \quad or \quad \mathbf{X} = \mathbf{AF} + \mathbf{W} \qquad (2)$$

The incident signals are represented in amplitude and phase by complex quantities $f_1, \cdots, f_D$. M $\times$ D matrix **A** is composed of $a(\varphi)$s which are known functions of the signal arrival angle and the antenna element location. The elements of **A** are complex in general. The jth column $a(\varphi_j)$ of **A** is a mode vector and φ_j indicates the direction of arrival of the jth signal. The elements of the measured **X** and the noise **W** are also complex. The M $\times$ M covariance matrix of the **X** vector is

$$\mathbf{S} \equiv \overline{\mathbf{XX}^*} = \mathbf{A}\overline{\mathbf{FF}^*}\mathbf{A}^* + \overline{\mathbf{WW}^*} = \mathbf{APA}^* + \sigma^2\mathbf{I}, \qquad (3)$$

under the assumption that the incident signals and the noise are uncorrelated and that the noise vector **W** has mean 0 and variance σ^2. The covarience matrix **S** is the same as the spectral matrix S mentioned in the WDF method. When the number of incident wave D is less than the number of antenna element M, $\mathbf{AP^*A}$ is singular. Using the eigenvalue decomposition of **S** matrix, eigenvalues $\lambda_1 > \cdots > \lambda_D \gg \lambda_{D+1} = \cdots = \lambda_M = \lambda_{min}$ and corresponding eigenvectors $e_1, \cdots, e_M$ are obtained. The rank of $\mathbf{AP^*A}$, which means the number of wave source, is determined directly from the eigenvalues of **S** and is D. The eigenvectors associated with λ_{min} are called noise eigenvectors which are spanned the noise subspace, and are orthogonal to the space spanned by columns of **A**. The number of noise vectors is M $-$ D. The direction findings is to determine the incident signal mode vector $a(\varphi)$. We assume that E_N is the M $\times$ (M $-$ D) matrix whose columns are the M $-$ D noise vectors. Then, we can estimate the dirrection of arrivals to search local maxima in function

$$P_{MU}(\varphi) = \frac{1}{a^*(\varphi)E_N E_N^* a(\varphi)}. \qquad (4)$$

3. Application to the ground-based direction finding

In the case of the ground-beased direction finding, three field components (two horizontal magnetic B_x and B_y, and one vertical electric E_z) can be observed by two orthogonal loop antennas and one vertical antenna because the ground is assumed to be a perfect conductor in the VLF/ELF range. The observed three field components are described based on the TM component $B_\perp$

$$\begin{bmatrix} B_x \\ B_y \\ E_z \end{bmatrix} = \begin{bmatrix} sin\phi - p_0 cos\theta cos\phi \\ -cos\phi - p_0 cos\theta sin\phi \\ sin\theta \end{bmatrix} B_\perp. \qquad (5)$$

The polarization p_0 is defined by $B_\parallel / B_\perp$. Fig.1 shows a coordinate sysytem of ground-based observation. θ and ϕ correspond the incident polar and azimuthal angles which we want to know. The coefficient vector of the right hand is corresponding to the mode vector $a(\varphi)$. Taken into consideration the additive internal noise, it is reasonable to apply the MUSIC method (M=3) for the present problem.

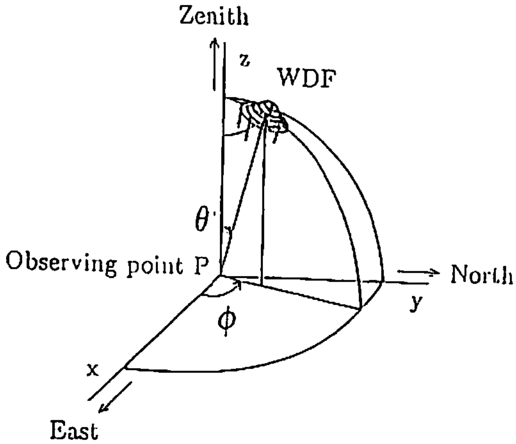

Fig.1 The coordinate system of the ground-based direction finding.

Computational experiments have been done on this problem. Experimental data (S matrix) are generated in the following procedure. We assume that the number of wave sources is $1 \sim 2$ and the incident waves have right-handed circular polarization ($p_0 = -i$). And each element of the S matrix is added the random noise. In a single wave source case, the direction of wave arrival is well-estimated within an error of a few degrees. Figs.2 and 3 show examples in the case of well-separated and close two wave sources, respectively, and SN ratio is assumed N/S=0.5. (a) indicates the assumed WDF and (b) shows the values of $P_{MU}(\theta, \phi)$. Fig.3 indicates there is a capacity of the separation of closely distributed sources. With the WDF method in this case, a single wave source would be estimated.

Through the simulations, it is found that the direction of arrival of incident waves is well-estimated with the high spatial resolution and with a small computation time compared with the WDF method.

4. Concluding remarks

A new ground-based direction finding method of magnetospheric VLF/ELF radio waves based on the MUSIC procedure is proposed. The results of computational experiments indicate that the good estimation of direction of wave arrival have been carried out with the high spatial resolution and the small computation time. The effectiveness of the new direction finding method is confirmed.

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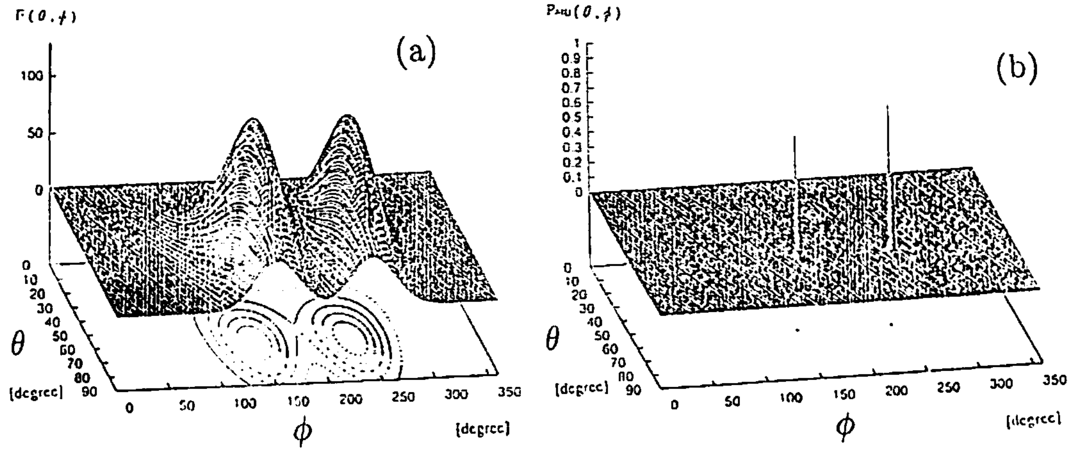

Fig.2 (a) WDF of the assumed two wave sources. Each source has a Gaussian-shaped energy distribution with $\sigma^2 = 20^o$. The location of the wave sources (θ, ϕ) are $(60^o, 150^o)$ and $(60^o, 240^o)$. (b) Estimated wave source location. $(\hat{\theta}, \hat{\phi}) = (52^o, 151^o)$ and $(54^o, 240^o)$.

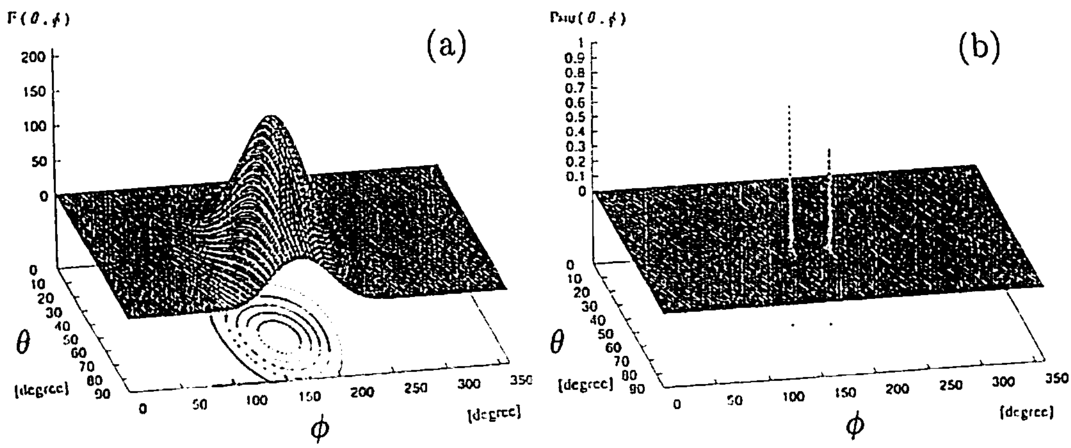

Fig.3 (a) The assumed WDF in the case of closely distributed two wave sources with $\sigma^2 = 20^o$. WDF seems a single wave source. $(\theta, \phi) = (60^o, 150^o)$, $(60^o, 180^o)$. (b) Estimated wave source location. $(\hat{\theta}, \hat{\phi}) = (56^o, 142^o)$ and $(55^o, 183^o)$.