A-5-5 A Characterization of the Pulse Sequences Through Urban Radio Multipath Channel

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Outline

One method to describe urban radio multipath channel on a time domain is to measure the sequence of pulses (as shown in Fig. 1) reflected and diffracted by buildings after sending a single pulse[1]. A statistical model for the amplitude and the arrival times of each pulse was established on the base of the experimental data[2], where data fitting was a main object.

In this paper, we make clear the structure of the statistical model for the pulse arrival times using a Markov chain, and the procedure to calculate the pulse number distribution will be described by a state transition matrix. The pulse number distribution is important to characterize the pulse sequence reflected by buildings. Then, the relation between the real (empirical) pulse occupancy rate and the underlying pulse occupancy rate of the statistical model is also represented by a Markov chain and some numerical results of the convergence of the underlying pulse occupancy rate will be shown.

Representation of the Modified Poisson Model by a Markov Chain

Since the received pulses tend to arrive in groups reflecting the distribution form of buildings, it is reasonable to assume the following modified Poisson process to model the pulse arrival times. That is, the pulse occupancy rate is increased by factor K during Δ small time intervals(bins) after the pulse arrival time.

To represent this model by a Markov chain in a discrete time axis, let $S_{i,m}$ be the state whose pattern indicator in the last Δ bins is i and which has m pulses in the past. Numbering all $S_{i,m}$ in increasing order of m, we can construct a Markov chain with countable number of states. The state transitions for $\Delta = 1$ are shown in Fig. 2. This can be expressed by a state transition matrix P_n at the n-th bin as

$$P_n = \begin{pmatrix} 1-\lambda_n, & 0, & 0, & \lambda_n, & 0, & \cdots & & \\ 1-K\lambda_n, & 0, & 0, & K\lambda_n, & 0, & \cdots & & \\ 0, & 0, & 1-\lambda_n, & 0, & 0, & \lambda_n, & 0, & \cdots \\ \cdot, & 0, & 1-K\lambda_n, & 0, & 0, & K\lambda_n, & 0, & \cdots \\ \vdots & \vdots & 0, & 0, & 1-\lambda_n, & 0, & & \end{pmatrix} \quad \cdots(1)$$

Let $\tilde{\pi}_n = [\pi_{n1}, \pi_{n2}, \cdots]$ be the probability vector for each state at the n-th bin.

Then, $\tilde{\pi}_n = \tilde{\pi}_{n-1} P_n$ . . . . (2)

Assuming $\tilde{\pi}_0 = [1,0,0, \ . \ . \ . \ ]$ for the initial condition, $\tilde{\pi}_n$ can be obtained step by step, and the pulse number distribution at the n-th bin is calculated summing up all $\pi_{n,j}$ which corresponds to $S_{i,m}$ with respect to i. In the case of $\Delta = 1$, $\tilde{\pi}_2$, for example, is

$$\tilde{\pi}_2 = [(1-\lambda_1)(1-\lambda_2),\ 0\ ,\ \lambda_1(1-K\lambda_2), \lambda_2(1-\lambda_1),\ 0\ , K\lambda_1\lambda_2,\ 0,\ \ldots]$$

and the pulse number distribution $P_2[M=m]$ is given by

$$P_2[M=0]=(1-\lambda_1)(1-\lambda_2),\ P_2[M=1]=\lambda_1(1-K\lambda_2)+\lambda_2(1-\lambda_1)$$
$$P_2[M=2]=K\lambda_1\lambda_2\ ,\ P_2[M \geq 3]=0.$$

Since the numerical results of the pulse number distribution appear in [2], we show here the variances of the distributions in Fig.3 for the real pulse occupancy rate $r_n=0.3$.

As we expect, the variance increases as Δ increases for $K>1$, while it decreases as Δ increases for $K<1$. This means that the pulse sequence tends to arrive in groups for large values of Δ and $K(>1)$, and the pulse number distribution becomes flatter than that of the standard Poisson process.

The Relation between the Real Pulse Occupancy Rate and the Underlying Pulse Occupancy Rate

The analysis in the above used the underlying pulse occupancy rate λ_n. But the real occupancy rate r_n is usually given by experimental data. Therefore, it is essential to get the relation between them.

For $\Delta=1$, the relation is (see [2])

$$\lambda_n = r_n/[1+(K-1)r_{n-1}] \quad \ldots\ldots\ldots\ (3)$$

This relation can be drawn as Fig. 4 when r_n is constant. The impossible region arises from the restriction $K>0$.

When $\Delta \geq 2$, the relation is no longer simple as (3). We need to represent the structure by a Markov chain again. Let $\tilde{e}=(e_1,e_2,\ .\ .\ .\ ,e_\Delta)$ be the pattern in the last Δ bins where e_i takes 1 or 0 depending on whether a pulse exists in the bin or not, and let $\tilde{e}_\phi=(0,0,\ .\ .\ .\ ,0)$. Then by the similar idea to (3), there exists

$$\lambda_{n+1} = r_{n+1}/[(1-a_{n,\tilde{e}_\phi})K + a_{n,\tilde{e}_\phi}] \quad \ldots\ldots\ldots\ (4)$$

To obtain $a_{n,\tilde{e}_\phi}$, we define a state transition matrix Q_n with order 2^Δ as follows.

$$\mathrm{Prob}[(e_1,e_2,\ \ldots,\ e_\Delta) \longrightarrow (1,e_1,e_2,\ \ldots,\ e_{\Delta-1})] = K\lambda_n$$

$$\mathrm{Prob}[(e_1,e_2,\cdots,e_\Delta)\longrightarrow(0,e_1,e_2,\cdots,e_{\Delta-1})]=1-K\lambda_n \quad \cdots(5)$$

where not all e_i is zero. For $\tilde{e}_\phi$,

$$\mathrm{Prob}[(0,0,\cdots,0)\longrightarrow(1,0,\cdots,0)]=\lambda_n$$

$$\mathrm{Prob}[(0,0,\cdots,0)\longrightarrow(0,0,\cdots,0)]=1-\lambda_n \quad \cdots(6)$$

All other transition probability is zero. Though P_n is of infinite order, Q_n is of finite order as long as Δ is finite. For $\Delta = 2$,

$$Q_n = \begin{pmatrix} 1-\lambda_n, & 0, & \lambda_n, & 0 \\ 1-K\lambda_n, & 0, & K\lambda_n, & 0 \\ 0, & 1-K\lambda_n, & 0, & K\lambda_n \\ 0, & 1-K\lambda_n, & 0, & K\lambda_n \end{pmatrix} \quad \cdots\cdots(7)$$

Let $\tilde{a}_n$ be the probability vector whose component is the probability of each pattern $\tilde{e}$. Then,

$$\tilde{a}_n = \tilde{a}_{n-1}\,Q_n \quad \cdots\cdots\cdots\cdots\cdots(8)$$

Assuming the initial condition $a_0=(1,0,0,\cdots,0)$, $\tilde{a}_n$ is obtained recursively. Equations (4) and (8) represent the relation between r_n and λ_n.

When r_n is constant, the convergence of λ_n is pretty fast as shown in Fig. 5 for $\Delta = 2$ and $r_n=0.3$. The stationary value of λ_n can be obtained by

$$\tilde{a}_\infty = \tilde{a}_\infty\,Q_\infty \quad \cdots\cdots\cdots\cdots\cdots(9)$$

It is easily checked that the stationary values of λ_n in Fig. 5 satisfy (9).

Conclusion

The structure of the modified Poisson process which was used to model the pulse delay times through urban multipath channel,was characterized by a Markov chain and was investigated by calculating the variances of the pulse number distributions. Then, the relation between the real pulse occupancy rate and the underlying pulse occupancy rate,which is essential for the statistical model, was derived also using a Markov chain. It is noticed that this statistical model is applied not only to a pulse sequence with grouping property,but also to that with periodic tendency. Some simulation results of this model appear in [3].

Acknowledgment

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References

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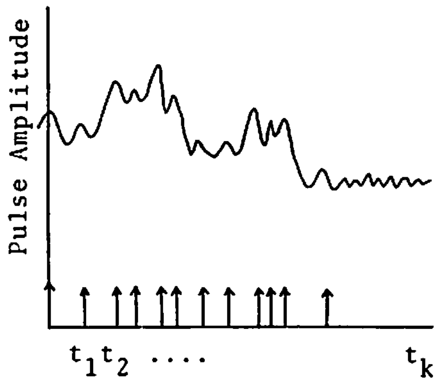

Fig. 1 A profile of a received pulse sequence

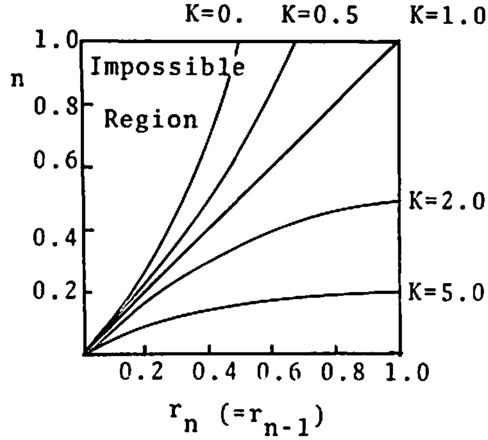

Fig. 4 The relation between r_n and λ_n

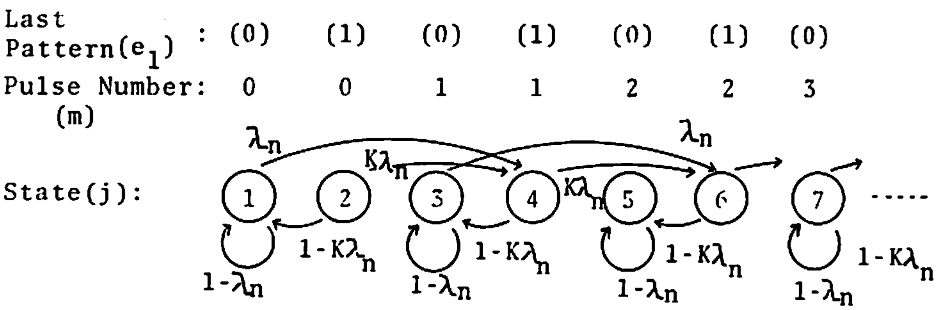

Fig. 2 The illustration of the state transitions

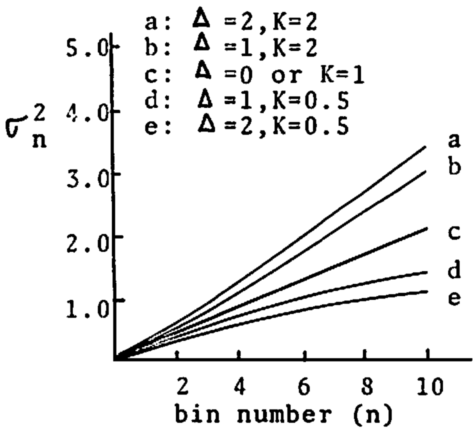

Fig. 3 Variances of the pulse number distributions

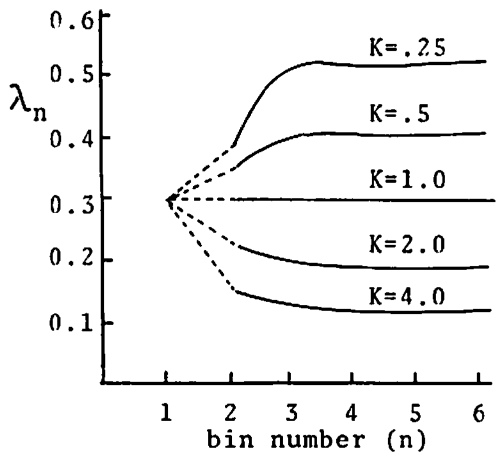

Fig. 5 Convergence of λ_n (r_n=0.3, Δ=2)