ARTIFICIAL NEURAL NETWORKS IN THE RECONSTRUCTION OF 2-D ENERGY DISTRIBUTIONS OF ELECTROMAGNETIC WAVES

Mehrez HIRARI and Masashi HAYAKAWA

Dept. of Electronic Engineering, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu-shi, Tokyo 182.

1 Introduction

Many problems in a broad class of engineering and scientific disciplines such as signal processing, control, and system identification, lead to the solution of systems of the following form,

$$y = Ax + \varepsilon \tag{1}$$

where y is the m-dimensional observation vector, $A = [a_{ij}]$ is the real coefficient matrix, x is the n-dimensional vector of unknowns, and ε is a corrupting noise. In many applications and under some assumptions it has been shown that the ordinary/penalized Least Squares Estimator (LSE) gives a good solution to the problem. But in some cases where these assumptions are no more realistic we seek more robust methods. The least squares estimator is a solution to the minimization problem

$$\min_x \{E(x) = \sum_{i=1}^{M} (y_i - y_i')^2\}, \tag{2}$$

where y_i and y_i' are respectively the observed and calculated values of the profile y. And it is well known that it is optimal if the data are Gaussian distributed. Moreover, if the number of unknowns to be determined is very large, the results might be unsatisfactory even under Gaussian conditions due to the excessive matrix operations. In this investigation we show the results of using unsupervised artificial neural networks to solve such inverse problems with particular application to the reconstruction of 2D distribution of the energy function of electromagnetic waves. The objective of the reconstruction is to estimate the direction of arrival (DOA) of an electromagnetic wave [1].

2 Neural Network Approach to Linear Inverse Problems

From a functionality point of view, artificial neural networks can be classified into supervised and unsupervised networks. The former require a learning set that consists of examples of the desired input-output mapping to set the connections weights. This process requires also a definite design; that is, the number of layers, the number of units in each layer, and a well defined architecture of to determine the disposition of the different layers and units. In the contrary, the second class known as unsupervised networks, do not need any examples of the desired input-output; indeed, only raw input data are given. And the network should look for irregularities in the data on the basis of which it can proceed towards the minimization of the energy function. The minimum of the latter corresponds to the optimal solution of the inverse problem. Hence, the adequate choice of this energy function is the key step into the design of an efficient network and this design widely known to be task dependent [2].

In the present application the number of parameters to be determined greatly exceeds the number of observations and the solution can only be estimated by constrained fitting to the data. The constraint consists in adding some additional information to restrict the domain of solutions. Explicitly, a solution that fits with observations and satisfies and additional constraint can be obtained by minimizing the following function

$$E(x) = \min_x \sum_{i=1}^{m} \rho(e_i) + \alpha\Phi(x). \qquad (3)$$

$\rho()$ is a suitable convex function and

$$e_i = y_i - y_i'$$

is the residual in which $y' = Ax$ and y is the observed quantity. The functional $\Phi(x)$ explicitly expresses the constraint, e.g, smoothness of the profile y and α is a tradeoff to balance between this smoothness and the fitness with the observations expressed by the first term of the right side in Eq.(3), often known as the error term. The neural network thus finds a solution by iteratively smoothing the above energy function.

Using the steepest gradient algorithm

$$x^+ = x + \mu\frac{\partial E(x)}{\partial x} \qquad (4)$$

and taking a particular form of ρ as

$$\rho(e) = \beta^2 \ln(\cosh(\frac{e}{\beta})), \qquad (5)$$

we find:

$$\begin{aligned} e_i(x) &= \sum_{j=1}^{n} a_{ij}x_j - y_i \quad i = 1, ..., m & (6)\\ \nabla E(x_j) &= \sum_{i=1}^{m} a_{ij}g(e_i) \quad j = 1, ..., n & (7)\\ \Delta x_j &= -\mu_j \nabla E(x_j) & (8)\\ x_j^+ &= x_j + \Delta x_j & (9) \end{aligned}$$

where

$$g(e_i) = \beta^2 \tanh(\frac{e_i}{\beta}) \qquad (10)$$

Equations (6)-(10) define the architecture of the network. This architecture can be implemented by a Hopfield-like three layers network. The connections' weights are fixed and are given by a_{ij}, the coefficients of the matrix A. μ_j's are the gain in the gradient algorithm and can be fixed to a relatively small positive constant, while α and β should be adequately chosen and their choice is investigated here.

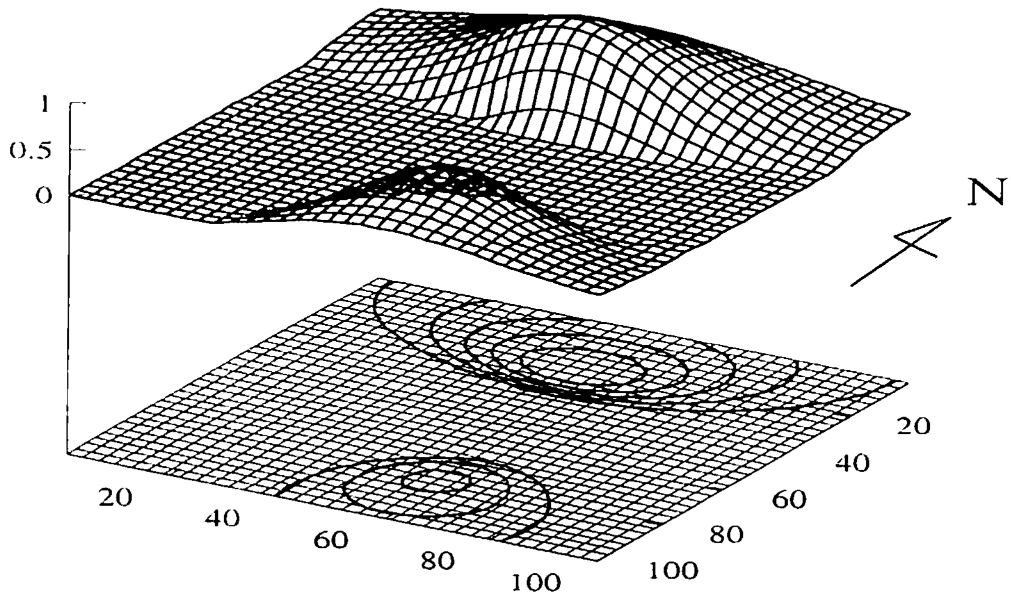

Figure 1: Reconstruction of two sources over an area of 300×300 km^2 divided into 100×100 pixels.

3 Results

We applied the proposed network to the ground-based direction finding for magnetospheric VLF/ELF radio waves such we try to estimate the energy distribution of those waves at the ionosphere from observation of the wave electromagnetic field on the ground. We performed the reconstruction for simulated and actually observed data for one source and two sources distributions. The DOA of the wave is determined by seeking the maximum/maxima of the distribution where each maximum correspond to an independent source at the the wave exit region. Figure 1 shows the reconstruction of two sources from actually observed data over a reconstruction area of 300×300 km^2, 80 km above the ground. As we can see, the whole distribution is not reconstructed because the sources are too broad. To this end, we should widen the reconstruction area and increase the number of pixels. Figure 2 shows 2D gray-scaled images of the the reconstruction from the same data over a reconstruction area 500×500 km^2.

4 Conclusion

In this research we have shown by means of computer simulation and real observed data that the unsupervised ANN can be used to solve ill-posed problems. The ANN offered the advantage to not impose any special assumption neither on the data nor on the parameters in one hand, and do not inquire excessive matrix operations in the other hand. These are two key advantages to handle large dimensioned and ill-determined systems, such as the inversion of electromagnetic data for the direction finding of magnetospheric VLF/ELF radio waves.

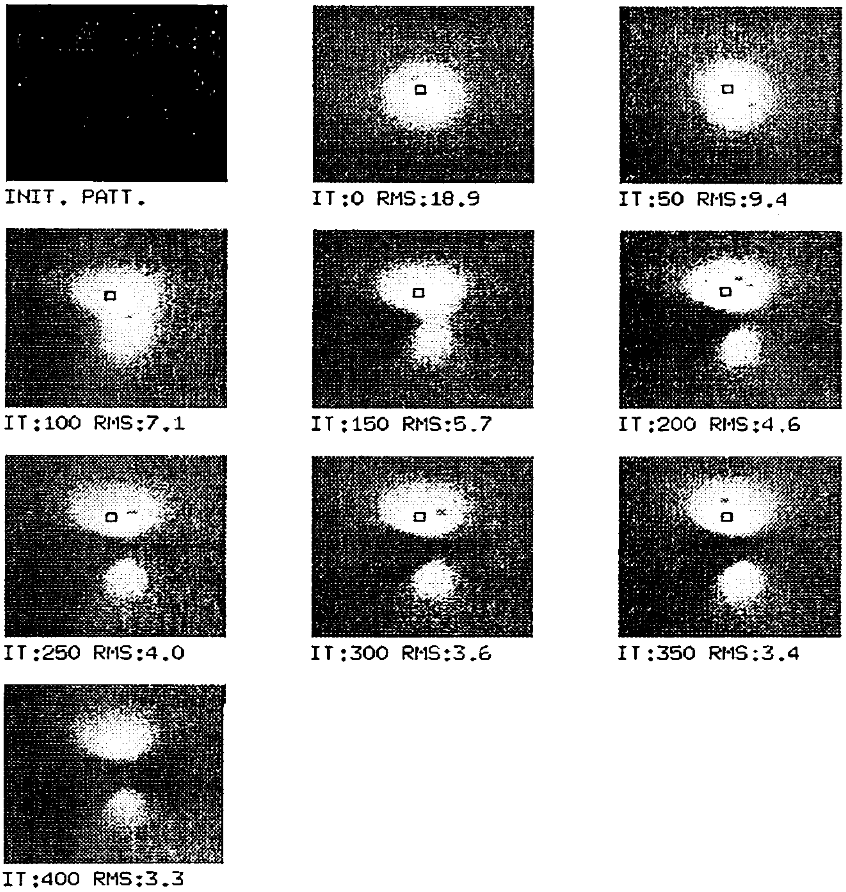

Figure 2: Reconstruction of two source distribution from observed data shown at different iterations (IT) over an area of $500 \times 500\text{km}^2$ divided into 120×120 pixels.

References

[1] Hirari, M. and Hayakawa, M., "Simulation Study on Ground-based Direction Finding of VLF/ELF Radio Waves by Wave Distribution Functions: a Bayesian Approach", IEICE Trans. Commun., Vol. E78-B, No. 6 June 1995.

[2] Cichocki, A., and Unbehauen, R., Neural Networks for Optimization and Signal Processing, John Wiley & Sons Ltd. & B. G. Teubner, Stuggart, 1993.