2-I B3

THE BEAM POINTING ERROR OF ARRAYS

K. R. Carver and W. K. Cooper, New Mexico State University, Las Cruces, New Mexico, U.S.A.;
W. L. Stutzman, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, U.S.A.

A planar phased array is considered which has uniform amplitude isotropic elements spaced evenly on a square grid with distance d between adjacent elements. There are M+1 rows (each row parallel to the y axis) and N+1 columns (each column parallel to the x axis). Assuming negligible amplitude excitation errors and random phase excitation errors, the normalized radiated power is

$$P = \sum_{m}^{M} \sum_{n}^{N} \sum_{p}^{M} \sum_{q}^{N} \cos(\Gamma + \chi^{r}_{mn} - \chi^{r}_{pq}) \qquad (1)$$

where

$$\Gamma = kd\{ (m-p)(\cos\phi \sin\theta - \cos\phi_o \sin\theta_o) + (n-q)(\sin\phi \sin\theta - \sin\phi_o \sin\theta_o) \} \qquad (2)$$

with the coordinates as shown in the accompanying figure. If the slopes of the power pattern (1) are defined by $S(\theta,\phi) = \partial P(\theta,\phi)/\partial\theta$ and $T(\theta,\phi) = \partial P(\theta,\phi)/\partial\alpha$ where $\delta\alpha = \delta\phi \sin\theta_o$, we can use a first order Taylor series expansion of S and T to solve for the beam-pointing errors[1], i.e.

$$\Delta\theta = -\frac{S(\theta_o, \phi_o)}{\dfrac{\partial S(\theta_o, \phi_o)}{\partial\theta}} \qquad (3)$$

$$\Delta\alpha = -\frac{T(\theta_o, \phi_o)}{\dfrac{\partial T(\theta_o, \phi_o)}{\partial\alpha}} \qquad (4)$$

If we now assume that the probability density function of χ^{r}_{mn} is even and that the phase errors are uncorrelated between different elements, we find from (3) and (4) that

$$\overline{\Delta\theta} = \overline{\Delta\alpha} = 0 \qquad (5)$$

If either M or N is large, it follows that

$$\Delta\theta_{rms} = \frac{s}{(kd\cos\theta_o)\sqrt{F}} \qquad (6)$$

and

$$\Delta\alpha_{rms} = \Delta\theta_{rms} \cos\theta_o \qquad (7)$$

where

$$F = (M+1)(N+1)[M(M+2)\cos^2\phi_o + N(N+2)\sin^2\phi_o] \qquad (8)$$

and where s is a quantity depending on the statistical nature of the phase errors. If the phase errors are small and normally distributed with r.m.s. σ, then $s = 2\sqrt{3}\,\sigma$; if the phase errors are small and uniformly distributed between $\pm\chi_o$, then $s = 2\chi_o$. For a linear array with M+1 elements (M large) and setting $\phi_o = 0$,

$$\sqrt{F} = M^{3/2} \qquad (9)$$

For a square planar array with $(M+1)^2$ elements (M large),

$$\sqrt{F} = M^2 \qquad (10)$$

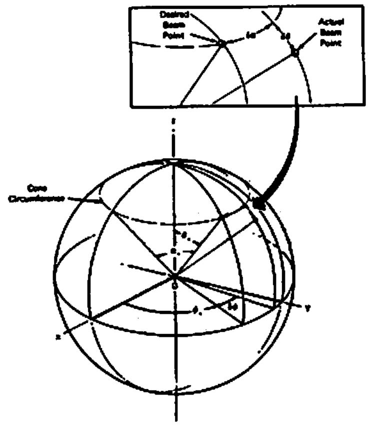
independently of ϕ_o.

Leichter[2] has considered a linear array with Gaussian phase errors and has obtained the result (6) and (9), but with $s = 2\sqrt{6}\,\sigma$;

the difference is due to the fact that he assumed a non-zero phase correlation factor between adjacent elements. Rondinelli[1] has obtained a formula for the beam-pointing error of a square planar array pointing at broadside; however, he assumed a Rayleigh distributed complex error current and thus his result cannot be compared to (6).

If the number of elements in the array is large, then we deduce from the Central Limit Theorem that both $\Delta\theta$ and $\Delta\alpha$ are approximately normally distributed, regardless of the statistical nature of the phase errors themselves.

These results have been checked for several test cases by using Monte Carlo simulation techniques on the IBM 360 Mod 50 digital computer[3]. The agreement obtained is good, typically 1%. Also, the Monte Carlo analysis confirms the approximate normal distribution of the beam-pointing error.



Illustrating the Beam-Pointing Angular Errors.

References

1. Rondinelli, L. A. "Effects of Random Errors on the Performance of Antenna Arrays of Many Elements," IRE 1959 National Convention Record, Part I, pp. 174-189.

2. Leichter, M., "Beam Pointing Errors of Long Line Sources," IRE Trans. Antennas and Propagation, AP-8, May 1960, pp. 268-275.

3. Carver, K. R., W. L. Stutzman and W. K. Cooper, "Planar Phased Array Beam-Pointing Errors," Proc. of Southwestern IEEE Conference, Houston, Texas, April 28-30, 1971.