AN APPROACH TO THE THINNING OF PHASE SHIFTERS IN PHASED ARRAYS

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Present-day radar requirements demand the use of phased arrays with many thousands of elements. For scanning purposes phase shifters are attached to the elements. The many phase shifters lead to high cost as well as to great complexity in control circuitry, and it would be highly desirable if their number could be reduced without impairing the array performance beyond tolerable limits. When the number of phase shifters is less than the element number, that is, when some common phase shifters are used to adjust the phase of more than one element, phase errors appear in array excitation. The phase errors depend on the direction of scan and cause a deterioration in the radiation pattern. One of the most objectionable results is an excessive rise in some of the sidelobes.

This paper deals with the problem of phase-shifter thinning by block phase excitation. In particular, it presents a technique for analyzing the extent of pattern deterioration when each phase shifter is used to control the phase of two elements, amounting to a 50 percent saving in the total number of phase shifters. Furthermore, a procedure for an optimum reduction in the high sidelobes by breaking the regularity of the phase errors is also presented. A steepest-descent process is used in the optimization procedure. The procedure is independent of the total number of array elements.

The radiation pattern of a linear array of 2K-1 elements excited symmetrically in amplitude and antisymmetrically in phase with respect to the center element can be written as

$$E(u) = \sum_{k=-(K-1)}^{K-1} I_k \exp(jku) \exp(-j\phi_k) \quad (1)$$

with

$$u = (\frac{2\pi d}{\lambda})(\sin \theta - \sin \theta_0 - \sin \theta_8),$$
 (2)

where θ_8 signifies an angle of scan away from the main-beam direction θ_0 , and ϕ_k is the phase error in the kth element as a consequence of exciting two adjacent elements through a common phase shifter. Denoting the designed pattern without phase error by

$$E_0(u) = \sum_k I_k \exp(jku),$$
 (3)

the error in the radiation pattern can be written as

$$\Delta E(u) = E(u) - E_{o}(u)$$

$$= -\sum_{k} I_{k} \exp(jku)f(k/K),$$
(4)

where

$$f(k/K) = 1 - \exp(-j\phi_k).$$
 (5)

In order to examine $\Delta E(u)$ in a systematic and general manner, a continuous function f(x) is defined for $-1 \le x \le +1$ such that it equals f(k/K) for integral values of k. Thus

$$f(x) = f_r(x) + jf_1(x),$$
 (6)

where

$$f_r(k/K) = 1 - \cos \phi_k$$
and
$$f_i(k/K) = \sin \phi_k.$$
(7)

For k = 0, we define

$$f_r(0) = 1 - \cos \phi_0 + \xi$$
 and $f_i(0) = 0$. (8)

f(x) in (6) can be represented by a
Fourier series with a period 2:

$$f(x) = \sum_{n=0}^{\infty} \{a_n \cos n\pi x + jb_n \sin n\pi x\}$$
 (9)

with

$$a_{n}=2\int_{0}^{1}f_{r}(x) \cos n\pi x \, dx$$
and
$$b_{n}=2\int_{0}^{1}f_{i}(x) \sin n\pi x \, dx.$$
(10)

Substitution of (7) and (9) in (4) yields an expression of $\Delta E(u)$ in terms of a_n , b_n , and $E_0(u)$. A similar consideration applies for an even number, 2K, of elements. The following is a general formula:

$$\Delta E(u) = \frac{1}{2} \sum_{n} \left[\pm (a_{2K+n} + a_{2K-n}) E_{o}(u - \frac{n\pi}{K}) + (b_{K+n} + b_{K-n}) E_{o}(u - \pi - \frac{n\pi}{K}) \right] + \xi I_{o}, \quad (11)$$

where the + and - signs are for 2K and-2K-1 elements respectively.

It is easy to extract from (11) information regarding the location and the level of sidelobes. The analysis of pattern deterioration due to phase-shifter thinning then is reduced to the determination of the coefficients a_n and b_n of the correct envelope function f(x).

High sidelobes can be reduced by disturbing the regularity of the phase errors. This is accomplished by exciting selected elements in the array at the correct phases by individual phase shifters. The problems here are:
(i) to define a proper envelope function f(x), (ii) to determine the Fourier coefficients an and bn, and (iii) to find the optimum locations, xm, of the elements which will have correct phases. An iterative, steepest-

descent process is used which minimizes a weighted sum of the magnitudes of the high sidelobes caused by phase errors. It was found for an 80-element array that the correct phase excitation of a single pair of elements would reduce the peak sidelobes which arose as a result of a 50 percent thinning in phase shifters by 4.6 dB. Other numerical data will also be presented.