2-1 A3 EFFICIENCY OF INFINITE ARRAYS WITH TRUNCATED EXCITATIONS

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In the analysis of phased-array antennas a large regular array of identical elements excited with uniform amplitude and linear progressive phase is frequently approximated by an array of infinite extent all elements of which are then excited with uniform amplitudes and linearly progressive phase. A question naturally arises as to how well the properties of a given finite array are approximated by the infinite array model which, evidently, cannot in any way account for edge effects. Insofar as we are aware no quantitative indication, possessing any degree of generality, has as yet been given in answer to this question. A quantitative but nevertheless simple approach to the problem may be found in consideration of the efficiency parameter associated with a finite excitation of an infinite array. A universal character is imparted to the quantitative result by considering arrays of antenna elements whose performance in the corresponding uniformly excited infinite array be ideal.

For a single excited element in an infinite array of identical antennas, the element efficiency is defined as the ratio of radiated to available power.[1,2,3] This efficiency may be expressed as an integral of the magnitude of the active reflection coefficient over the complete range of phasing angles. Thus, for a linear array, one has[1,2]

$$\eta = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} |\rho(\alpha)|^2 \, d\alpha \,, \quad (1)$$

where $\rho(\alpha)$ is the active reflection coefficient of the array having all of its elements excited with uniform amplitude and a linear progressive phase. When the array is excited with an arbitrary set of incident waves $\{a_m\}$, a physically meaningful generalization of efficiency is the ratio of total radiated to available power. It may be shown that in this case the efficiency may be written

$$\eta_T = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} |\rho(\xi)|^2 |A(\xi)|^2 d\xi, \quad (2)$$

where $A(\xi)$ is defined by the excitation coefficients:

$$A(\xi) = \sum_m a_m e^{-jm\xi}, \; \sum_m |a_m|^2 = 1 \,.$$

Of particular interest are truncated excitations of the form $\sqrt{2N+1}\, a_m = \exp\{jm\alpha\}$; $|m| \le N$, $a_m = 0$; $|m| > N$, for which

$$|A^{(N)}(\xi,\alpha)|^2 = \frac{\sin^2 [(N+1/2)\,(\alpha-\xi)]}{(2N+1)\sin^2(\frac{\alpha-\xi}{2})} \,. \quad (3)$$

The corresponding efficiency will be denoted by $\eta_T^{(N)}(\alpha)$. When a single element is excited, N=0, and (2) reduces to (1). On the other hand, when all the elements are excited, one has

$$\lim_{N\to\infty} \eta_T^{(N)}(\alpha) \equiv \eta_T = 1 - |\rho(\alpha)|^2 \,. \quad (4)$$

For any finite N the efficiency $\eta_T^{(N)}$ may be computed if only the magnitude of the active reflection coefficient for the infinite array is known. The

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deviation of $\eta_T^{(N)}$ from η_T provides a quantitative measure of the degree of approximation attainable by finite (i.e., truncated) excitations.

If one requires that the performance of an element in the infinite array be "ideal"[1] then (D is the element spacing)

$$|\rho(\alpha)| = \begin{cases} 0 \; ; & |\alpha| < kD < \pi \; , \\ 1 \; ; & kD < \alpha < \pi \; , \end{cases}$$

and the corresponding "ideal" or maximum efficiency is

$$\eta_T^{(N)}(\alpha) = \frac{1}{2\pi} \int_{-kD}^{kD} |A^{(N)}(\xi)|^2 \, d\xi \; . \quad (5)$$

The ideal efficiency constitutes an upper bound on $\eta_N^{(T)}$ for any element whatever, and provides a canonical result which may serve as a figure of merit in evaluating the (average) effects of "edge elements" in large phased arrays.

Figures 1, 2, and 3 show plots of the ideal efficiency for phasing angles $\alpha = 0$, $\pi/6$ and $\pi/2$, respectively. The rate of convergence with N is quite rapid and the efficiency begins to approach the step function ($N=\infty$) when only 27 element ($N=13$) are excited.

The above analysis is, of course, not limited to linear arrays but may be generalized to include infinite planar arrays.

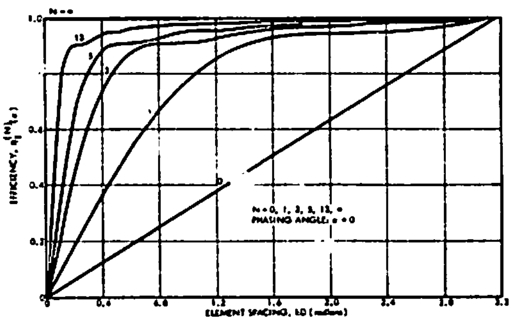

FIGURE 1. Efficiency of a Linear Array With 2N+1 Excited Elements

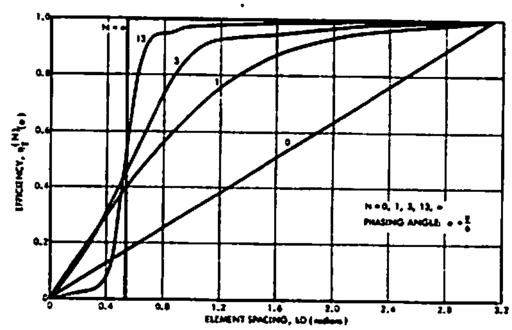

FIGURE 2. Efficiency of a Linear Array With 2N+1 Excited Elements

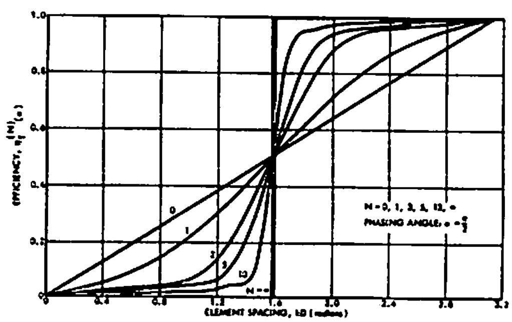

FIGURE 3. Efficiency of a Linear Array With 2N+1 Excited Elements

References

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