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Analysis on Information Diffusion of a Mathematical Model with Dynamical Sending Probability

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Abstract—Various kinds of information diffuse across networks in the real world. We have already proposed a mathematical model that realizes the effects of dynamical sending probability on the information diffusion including two specific diffusion dynamics. In this paper, we propose a new model based on the neural mechanism: when a node's interest for messages is higher than its intrinsic threshold value, the node sends messages to its neighbors. We found that the new model shows similar behavior to the former model, but the dynamics of the diffusion is much more closely related to that of the interest.

1. Introduction

In the real world, various complex phenomena occur by interactions between many components. If we regard the components as nodes and their interactions as links, we can describe many kinds of real systems as networks. For example, in a network of friendships, persons are nodes and their friendships are links. In the real networks, various kinds of information diffuse, such as diseases, electric pulses in neural networks, and so on. Therefore, it is very important to clarify how the information diffuses in the networks toward effective prevention of infectious diseases, understanding neural systems, and so on [1].

Using a simple mathematical model of infectious diseases, D. Watts and S. Strogatz showed that viruses easily diffuse across the random networks which have many shortcuts and few clusters [2]. On the other hand, D. Centola conducted an experiment through social networking service on the Internet and reported that social messages easily diffused across the lattice networks which have many clusters and few shortcuts [3]. We investigated the results obtained by Centola and proposed a mathematical model of information diffusion which can reproduce the results of Centola's experiments [4]. Our model includes two specific diffusion dynamics on the basis of Centola's experiments. First, if a node receives a second message, a probability with which the node sends messages to adjacent nodes will increase. We call this the sending probability. Second, if a node which has received messages from neighbors did not send messages during a certain period of time, the node becomes calm, or never sends messages subsequently. The results of numerical experiments showed that our model successfully reproduces the results of Centola's experiments.

In this paper, we extended the former model to treat two issues: (i) the sending probability increases depending on the number of received messages and (ii) the sending probability should gradually decrease with time. Then, we used a neural mechanism to construct a new model of information diffusion: when a neuron fires, it sends electrical pulses. We treated the internal state of a neuron as the interest for messages of a node. If the node receives messages, its interest gradually increases, depending on the number of messages. Then, when the interest is higher than its threshold value, the node sends messages to its neighbors. The interest changes dynamically during a process of diffusion. We investigated the temporal changes of the interest and how the information diffuses by using two updating methods. As a result, we found that the information diffuses widely even though the network is regular such as ring-lattice networks. These results of our new model agree with Centola's experiments [3]. The results also indicate that the diffusion performance depends on how we update the interest of nodes.

2. Dynamics of information diffusion

2.1. Former proposed model [4]

Our proposed model of the information diffusion mainly consists of two diffusion dynamics [4]: First, if the node receives a second message, the sending probability r increases by α where α (< 1) is a constant parameter. In this model [4], we assume that $r + \alpha < 1$. Second, if the node has received messages but does not send messages during a given time period of T, the node never sends messages after T because the node has no interest in the messages.

2.2. Interest for information

In the former model, we assume that r increases when the node receives the second message, but does not increase even if the node receives more than three messages for the sake of simplicity. We assume that the node suddenly stops sending messages after the temporal period of T in the former model. However, r should gradually decreases with time. To modify these aspects of the former model, we propose a new model of information diffusion which introduces a similar dynamics to a neural dynamics. Our proposed model is based on the mechanism that neurons receive and send electrical pulses in neural networks. The neural networks consist of neurons. Each neuron has the internal state. When the neuron receives electrical pulses, the internal state increases. If the internal state gets higher than its intrinsic threshold value, the neuron fires and sends pulses to its neighbors. The internal state decreases with time and converges to 0. We assume that this neural mechanism corresponds to a person's message sending mechanism. We treat the interest for messages as the internal state of neurons and the messages as the electrical pulses. Every person has his/her own interest for messages. When he/she receives a message, he/she becomes interested in the message. When he/she receives multiple messages, his/her interest for the messages gradually increases. It is considered that every person has his/her own criterion whether he/she sends messages or not. Then, if the interest becomes higher than his/her threshold value, he/she decides to send the messages to neighbors. Without receiving messages, his/her interest decreases with time, and he/she hardly sends messages.

2.3. Dynamics of Interest

We consider the relationship between the interest of the node i at time t, $x_i(t)$, and the interest $x_i(t+1)$ at time $t+1$. In general, the interest decreases gradually after the node receives messages. When the node receives another message at time t, $x_i(t)$ increases depending on the number of neighbors which send messages. In addition, it is natural that without receiving another message, the node increases the interest by itself and sends messages. This could happen if the node has a specific interest in received messages. Therefore, we can describe the relationship between $x_i(t)$ and $x_i(t+1)$ as follows:

$$x_i(t+1) = sx_i(t) + \sum_{j\in G_i} \alpha_j H(x_j(t) - \theta_j) + \beta Z_i(t), \quad (1)$$

$$H(y) = \begin{cases} 0 & (y < 0) \\ 1 & (y > 0), \end{cases} \quad (2)$$

where s (< 1) is a positive parameter which shows how much the interest decreases at each time step, α_j is a scaling parameter for the messages from its neighbors $j \in G_i$, G_i is a set of indices of nodes which are connected to the node i and which has not sent messages by the time t, $H(x_j(t)-\theta_j)$ takes unity when the node j sends messages at time t, otherwise zero, $Z_i(t)$ takes unity with the probability r_i, otherwise zero, and β denotes how much the node i increases the interest by itself. The interest always decreases by s exponentially, while receiving additional messages from adjacent nodes and increasing the interest by itself are the stochastic behavior.

Because Eq. (1) is a linear difference equation, we can solve it. The solution of Eq. (1) is

$$\begin{aligned}
x_i(t+1) &= sx_i(t) + \sum_{j\in G_i} \alpha_j H(c_j(t)) + \beta Z_i(t) \\
&= s\left\{ sx_i(t-1) + \sum_{j\in G_i} \alpha_j H(c_j(t-1)) + \beta Z_i(t-1) \right\} \\
&\quad + \sum_{j\in G_i} \alpha_j H(c_j(t)) + \beta Z_i(t) \\
&= s\left[s\left\{ sx_i(t-2) + \sum_{j\in G_i} \alpha_j H(c_j(t-2)) + \beta Z_i(t-2) \right\} \right. \\
&\quad \left. + \sum_{j\in G_i} \alpha_j H(c_j(t-1)) + \beta Z_i(t-1) \right] \\
&\quad + \sum_{j\in G_i} \alpha_j H(c_j(t)) + \beta Z_i(t),
\end{aligned}$$

where we define $x_j(t) - \theta_j$ as $c_j(t)$. Eventually, we obtain

$$x_i(t+1) = s^{t+1} x_i(0) + \sum_{n=0}^{t} s^n \left\{ \sum_{j\in G_i} \alpha_j H(c_j(t-n)) + \beta Z_i(t-n) \right\}. \quad (3)$$

Although we have several options to describe the interests, we defined s to $e^{-1/\tau}$ where τ controls how fast the interest decreases with time. Therefore, we rewrite Eq. (3) as follows:

$$x_i(t+1) = e^{-1/\tau} x_i(t) + \sum_{j\in G_i} \alpha_j H(c_j(t)) + \beta Z_i(t). \quad (4)$$

If $x_i(t+1) > \theta_i$ (θ_i is a threshold of the node i), the node i sends messages to its neighbors. The node i which has sent messages never send messages subsequently.

3. Numerical Experiments

3.1. Temporal change of Interest

In our experiments, first, we set $x_i(0)$ $(i = 1, \cdots, N)$ to 0. At $t = 0$, we randomly choose an initial spreader and the interest of its neighbors increases in α_j. We applied our proposed model to networks and investigated the temporal changes of the interest for each node. We used two networks, ring-lattice networks (RLN) [2] and random networks (RN) which are generated from RLN [2]. The nodes in RLN are arranged in a circular pattern, and sequentially numbered from 1 to N in the clockwise direction. The parameters are set to $\tau = 30, \alpha_j = 0.5$ ($j = 1, \cdots, N$), and $\beta = 0.1$. We assume that r_i obeys the exponential distribution whose average is 0.01 and the threshold θ_i obeys the uniform distribution whose range is $[0.4, 1.0]$.

We investigated the temporal changes of $x_i(t)$ ($i = 1, \cdots, N$). In the real world, people often receive messages and forward them to friends, and the friends might forward received messages again and again. This is an asynchronous update. Therefore, we adopted the asynchronous update for updating $x_i(t)$, but we adopted two kinds of updating methods. In the first method, we update $x_i(t)$ in the order of the index i. In the second method, we randomly choose a node i and update $x_i(t)$. Figure 1(a), (c), and (e) is the result for RLN ($N = 128, 512$, and 2048) when the interests are updated in the order of the index numbers of nodes. Even if N increases, $x_i(t)$ ($i = 1, \cdots, N$) rapidly increases and exceeds θ_i ($i = 1, \cdots, N$), so that all of them send messages only during a few steps. This is because RLN has a specific structure. For example, let us assume that the node i sends messages to the node $i-3$, $i-2$, $i-1$, $i+1$, $i+2$, and $i+3$ at t. At $t+1$, the interest of the node $i-3$ is updated. If $x_{i-3}(t+1) > \theta_{i-3}$, the node $i-3$ sends messages toward the node $i-6$, $i-5$, $i-4$, $i-2$, $i-1$, and i. Then, the interest of the node $i-2$ is updated. This node received messages from the node i at t and from the node $i-3$ at $t+1$. Therefore, $x_{i-2}(t+1)$ depends on the order of the updating, so that the sending probability of the node $i-2$ increases. Because the order of the indices of nodes and that of the updating are the same, the interests of all nodes in RLN increase rapidly only in a few step by repeating this procedure. However, when the interest is updated randomly, we cannot observe this tendency. From Fig. 1(a), (c), and (e), as N increases, it takes longer time for messages to diffuse across whole networks.

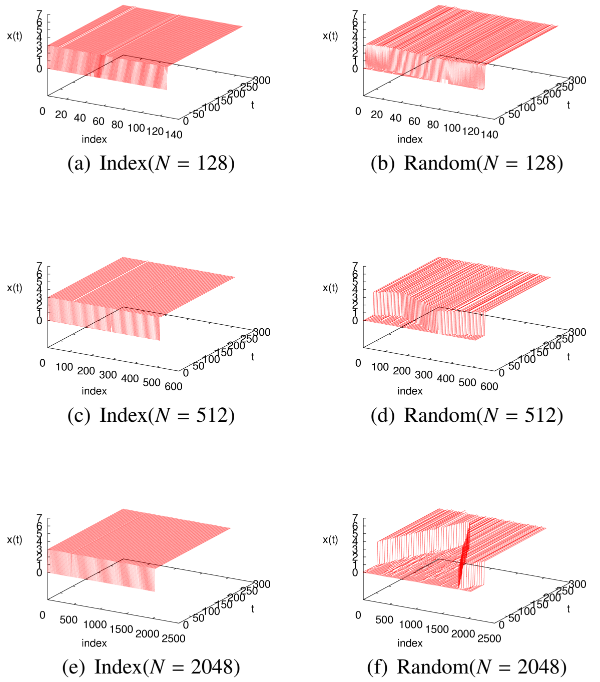
(a) Index($N = 128$) (b) Random($N = 128$) (c) Index($N = 512$) (d) Random($N = 512$) (e) Index($N = 2048$) (f) Random($N = 2048$)

Figure 1: Temporal changes of $x(t)$ for all nodes in RLN ($k = 6$) with two kinds of asynchronous update methods. To see the outline, data are plotted every five nodes in (c) and (d), and every twenty nodes in (e) and (f).

On the other hand, the results for RN are shown in Fig. 2. From Fig. 2, no difference is observed between the two updating methods. Because RN has no specific structure, that is, the nodes are connected randomly, no difference exists in the results between the updating methods. In addition, even if N increases, it takes shorter time for messages to diffuse in whole networks than the results in Fig. 1 for the random updating method.

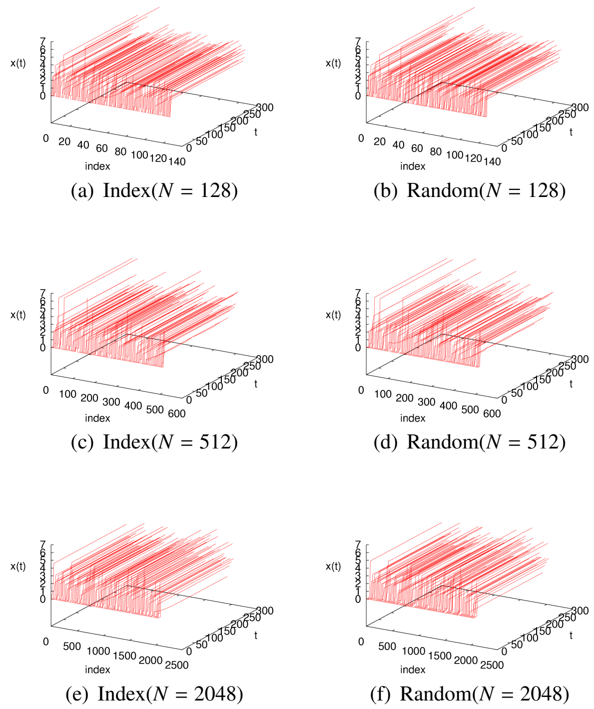
(a) Index($N = 128$) (b) Random($N = 128$) (c) Index($N = 512$) (d) Random($N = 512$) (e) Index($N = 2048$) (f) Random($N = 2048$)

Figure 2: The same as Fig. 1, but for RN.

3.2. Diffusion rate and convergence time

We investigated the diffusion rate and the convergence time. The diffusion rate quantifies the ratio of nodes that send the received messages at each time step and is described as follows:

$$R_t = \frac{1}{N} \sum_{i=1}^{N} a_i(t), \tag{5}$$

where if the node i has sent messages by the time t, $a_i(t) = 1$, otherwise $a_i(t) = 0$. If all of the nodes in the network have sent messages, the diffusion rate takes unity, and if no nodes have sent, the diffusion rate takes 0. The convergence time is the time when the diffusion stops. In the following, we evaluated the diffusion rate R for the proposed model in case of RLN and RN and then investigated the final diffusion rate and its convergence time.

We show the results in Fig. 3. The results are averaged over 100 trials. The difference between the updating methods does not affect the diffusion rate for both networks. Even if N increases, messages diffuse more widely in RLN than RN. These results are almost the same as the results obtained from our former proposed model [4]. These results indicate that to diffuse messages in whole networks, it is necessary to increase the interest through many clusters.

However, the convergence time depends on the updating methods. If we update the interest in the order of the index of the nodes, messages diffuse more quickly in RLN than RN (Fig. 3(b)). The reason for these results will be the same as the reason for Fig. 1(a), (c), and (e) as we mentioned in Sec. 3.1. In contrast, if we update the interest randomly, messages diffuse more quickly in RN than across RLN. As a result, the difference of the updating methods affects the information diffusion, and if the methods match with a network structure, the diffusion shows the specific characteristic such as Fig. 1(a), (c), and (e).

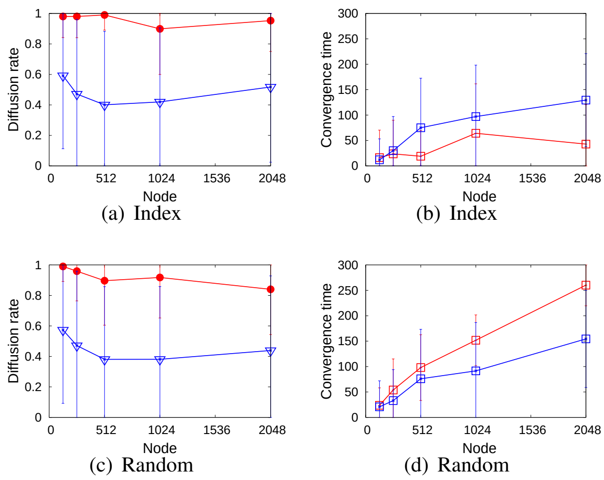

Figure 3: Diffusion rates and convergence times as a function of N with two kinds of asynchronous update methods. Red lines represent the results of RLN, and blue lines represent the results of RN. The results are averaged over 100 different trials. The vertical bars are standard deviations.

4. Conclusion

In this paper, we proposed a new mathematical model based on the neural mechanism. We considered the internal state of the neuron as the person's interest for messages. If the node receives messages, the interest increases, but if not, the interest decreases with time. When the interest exceeds the intrinsic threshold value, the node sends messages to its neighbors. We applied the new model to RLN and RN and investigated how the information diffuses. In the experiments, we used two updating methods for the interest. One is the update of the interest on the basis of indicied of nodes and the other is the random update.

As a result, messages diffused widely in RLN. This result agrees with the results obtained by the former model [4] and those of Centola's experiments [3]. It indicates that increase of the interest of nodes through many clusters is an important factor to diffuse messages in whole networks. In addition, if we update the interest in the order of index numbers, the interests of all nodes rapidly increase for only a few steps in RLN even if N increases. This is because the order of the indices of nodes and that of updating the interest are the same. This result implies that the network structures and the updating methods affect the behavior of the information diffusion.

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