

An Adaptive Evaluation Method of Material with Complex Permittivity in a Cylindrical Cavity

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1. Introduction

A cylindrical cavity resonator has been widely utilized to evaluate the complex permittivity of dielectric materials [1]-[4]. The most popularly used cavity modes for evaluation are the TE_{011} and TM_{010} modes. The TM_{010} mode has been chosen for the rod-like samples while the TE_{011} mode has been applied for the circular bulk or thin disk samples. The TE_{011} mode has a higher Q than other cavity modes, but it is not the dominant cavity mode [5]. When high permittivity and/or lossy samples are loaded in the cavity, the resonant frequency becomes lowered significantly and $TE_{01\delta}$ resonant mode for the loaded sample may be confused with other resonant modes.

This paper proposes an adaptive evaluation method of material with complex permittivity by using $TE_{01\delta}$ mode of a cylindrical cavity resonator. In the proposed method, the sample with various sizes and materials can be loaded at arbitrary height in the resonator. Therefore, the change in the resonant frequency can be kept in some limited variation. So that the $TE_{01\delta}$ resonant mode can stay in the uncrowded frequency region. The electromagnetic method used for the material evaluation is based on a hybrid EM method proposed by the authors [6].

2. Hybrid Electromagnetic Analytical Method : Basis for Measurement

Fig. 1 shows a cylindrical cavity resonator containing a circular-disc-shaped dielectric sample with dielectric supporting posts. The dielectric sample and supporting posts are assumed to be rotationally symmetric, and $TE_{01\delta}$ resonant mode with no ϕ variation is used for the measurement. The resonator with radius R and height H are designed using a mode chart [5] and considering that the TE_{011} resonant mode of the unloaded case should be located in the sparsely-distributed resonances and have higher Q value. The sample may be low-loss or lossy dielectric material, and the radius a and thickness t of the sample may be adjusted accordingly. Also the sample may be loaded at arbitrary height in the cavity.

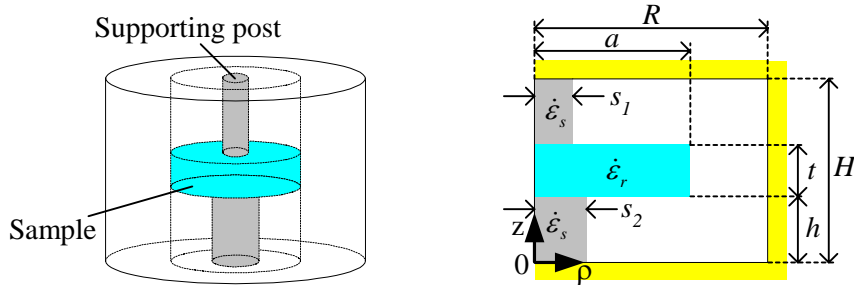
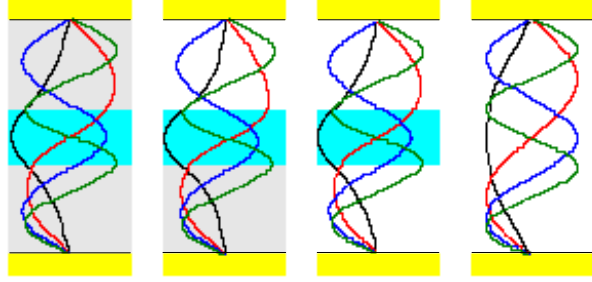
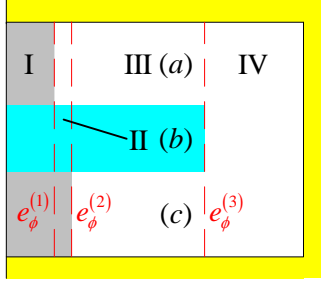


Figure 1: Cylindrical Cavity With Dielectric Sample

The electromagnetic field analysis, which is a basis of measurement method for the sample with various sizes and materials, should be versatile and free from errors due to approximations in the calculation process and it should enhance the efficiency for the iterative fitting process of parameters with the measured data. The electromagnetic analysis used in this evaluation is a hybrid EM method proposed by the authors, combining the extended spectral domain approach [7] with the mode matching method (ESDMM) [6]. The hybrid method was used to evaluate a sample material in a

rectangular waveguide [8]. Recently the method was applied to a cylindrical cavity containing a circular dielectric sample [4], where the aperture source fields were introduced at the top and bottom planes of the disc sample. In the present paper, the source fields are introduced at the circumferential surfaces, and the fields in each layer can be expanded in terms of eigenfunction consisting of simple sinusoidal functions, whereas eigenfunction in [4] were composed of cylindrical functions.

In the analysis, firstly, the electric fields are introduced at the circumferential surfaces and whole layer is partitioned into the concentric layers as shown in Fig. 2. Each layer can be treated independently quoting the equivalence principle [9]. When the layer is homogeneous and bounded by a perfect electric wall (layer IV in Fig. 3), the electromagnetic fields in the layers are expanded with series in terms of the simple sinusoidal functions which satisfy the boundary conditions at the upper and lower conductors of the cavity.



(i) Layer I (ii) Layer II (iii) Layer III (iv) Layer IV

Figure 2: Introduction of Aperture Fields Figure 3: Eigenfunctions in Each Layer

However, in the layers I, II and III, which contain a sample and/or support inhomogeneously, the fields cannot be expressed with simple series in terms of sinusoidal functions. The eigenfunctions in these inhomogeneous layers can be constructed by using a mode-matching procedure [6]. The inhomogeneous layer is divided further into homogeneous subregions (Fig. 2), and the eigenfunctions in homogeneous subregion (i) is expressed as

$$\Phi_m^{(i)}(z) = A_m^{(i)} \cos(\beta_m^{(i)} z) + B_m^{(i)} \sin(\beta_m^{(i)} z) \quad (1)$$

where $\beta_m^{(i)}$ is the eigenvalue in the subregion (i). The determinantal equation for the eigenvalue is derived by applying the continuity conditions at the interfaces between the homogeneous subregions. By proper choice of constants $A_m^{(i)}, B_m^{(i)}$ in Eq. (1), the eigenfunctions satisfy the following biorthogonal relation in the whole inhomogeneous subregions,

$$\int_{S_a} \Phi_m^{(a)}(z) \Phi_n^{(a)}(z) dz + \int_{S_b} \Phi_m^{(b)}(z) \Phi_n^{(b)}(z) dz + \int_{S_c} \Phi_m^{(c)}(z) \Phi_n^{(c)}(z) dz = \delta_{mn} \quad (2)$$

where S_i stand for the subregions. Fig. 3 shows the first few eigenfunctions obtained numerically.

By utilizing the biorthogonal relation, the electromagnetic fields in the inhomogeneous layers I-III can be transformed into spectral domain similarly in the homogeneous layer IV. The Green's functions in transformed domain are derived easily, and the EM fields are expressed with respect to the aperture fields. By applying the continuity conditions of the magnetic fields at the aperture surfaces in each layer, we can obtain the integral equations on the aperture fields. Applying the Galerkin's procedure to the integral equations, the determinantal equation for the resonant frequency can be derived. The complex resonant frequency and the aperture electric fields can be obtained numerically.

Once the aperture fields are determined, the field distribution in the resonator is determined and then the quality factor Q of the cavity itself is evaluated. That is, Q_c due to the conductor loss is calculated with the total stored energy W in the resonator and the power loss P_c at the conductor surface, as

$$Q_c = 2\pi f_0 \frac{W}{P_c} = 2\pi f_0 \frac{\frac{\epsilon}{2} \int_V |\mathbf{E}|^2 dv}{\frac{R_s}{2} \int_S |\mathbf{H}_{\tan}|^2 ds} \quad (3)$$

where \mathbf{H}_{tan} is the tangential component of the magnetic field at the conductor surfaces, and R_s is a surface resistance of the conductor.

Total Q of the resonator is calculated as follows.

$$Q = (1/Q_c + 1/Q_d)^{-1} \quad (4)$$

where Q_d is obtained by evaluating the power consumption in the sample [4][5].

3. Numerical Calculations and Verifications

Applying the Glerkin's method on the integral equations, the aperture fields are determined. The unknown aperture fields are expanded in terms of the appropriate basis functions $f_k^{(j)}(z)$ as

$$e_\phi^{(j)}(z) = \sum_{k=1}^N a_k^{(j)} f_k^{(j)}(z) \quad (5)$$

where $a_k^{(j)}$ are unknown coefficients to be determined. To calculate flexibly for the various type of lossless or lossy samples loaded at arbitrary loading height, the polynomial basis functions $f_k^{(j)}(z)$ are utilized to express the aperture fields. The matrix size in the determinantal equations is the order of the total number of basis function and far smaller than that of conventional mode matching method.

The unknown material parameters are estimated by solving the inverse problem of the resonance. A virtual experiment is performed to investigate the estimation accuracy of the material parameters. First, the permittivity of the sample is preassigned to be $\dot{\epsilon}_r^p = \epsilon_r^p (1 - j \tan \delta^p)$. The resonant frequency and quality factor with the sample loaded f_0^L , Q^L and unloaded f_0^U , Q^U are evaluated by the finite element method (FEM) using this preassigned permittivity $\dot{\epsilon}_r^p$. These resonant frequencies f_0 , and quality factor Q are treated as the virtual measured values, and the permittivity $\dot{\epsilon}_r^p$ is assumed to be unknown hereafter. The resonant frequency f_0^L , f_0^U and quality factor Q^L , Q^U are calculated by the present method (ESDMM) with first trial permittivity $\dot{\epsilon}_r$. The most probable $\dot{\epsilon}_r$ is determined by minimizing the difference in the resonant frequency f_0^L , f_0^U and quality factor Q^L , Q^U between virtual measured values and estimated values. Table 1 shows the estimated $\dot{\epsilon}_r$ and the preassigned $\dot{\epsilon}_r^p$ for the three materials. An accurate reproduction is observed.

Table 1: Virtual Experiments

Sample	Resonant Frequency	Pre-assigned		Estimation	
		ϵ_r^p	$\tan \delta^p (\times 10^{-4})$	ϵ_r	$\tan \delta (\times 10^{-4})$
A	9.4135GHz	2.080	4.000	2.080	3.999
B	8.2486GHz	5.600	41.00	5.599	41.00
C	7.1310GHz	9.500	3.000	9.498	3.000

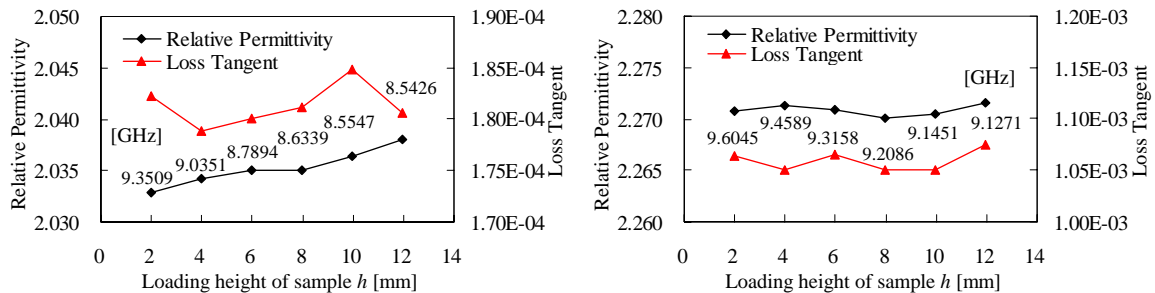
($R = 23.03$ mm, $H = 26.59$ mm, $\sigma = 6.173 \times 10^7$ S/m, $s_1 = 1.500$ mm, $s_2 = 2.500$ mm, $h = 12.30$ mm, $\dot{\epsilon}_s = 2.080 \times (1.000 - j 4.000 \times 10^{-4})$, $a = 11.52$ mm, $t = 2.000$ mm)

4. Measurement

The measurement system is set up in X-band for the practical evaluation of the material parameters. The radius and height of the cavity are $R = 23.03$ mm and $H = 26.59$ mm, respectively. The silver plated copper cavity is magnetically-coupled with small loops through the in and out holes in the cylindrical wall. The loops are connected to Agilent 8719ET Vector Network Analyzer through the coaxial cables. The radii of the upper and lower supporting posts are 1.440 and 2.496 mm, respectively, and the relative permittivity and loss tangent of the posts are $\epsilon_s = 2.030$, $\tan \delta = 1.800 \times 10^{-4}$, respectively. Measurement samples with different radii are prepared with Teflon and fluorine resin. The loading height of sample in the cavity is shifted by changing the lengths of the

supports.

The resonant frequency f_0 and Q are extracted from the measured scattering parameter S_{21} . Fig. 4 shows the relative permittivity and loss tangent with different loading height of samples. The resonant frequency changes with the loading height of sample as shown in the figures.



(a) Teflon ($a = 17.25$ mm, $t = 4.022$ mm) (b) Fluorine Resin ($a = 11.52$ mm, $t = 3.139$ mm)

Figure 4: Measurement of Complex Permittivity

5. Conclusion

We proposed an adaptive evaluation method of material with complex permittivity. The method can be applied to the sample with various sizes and materials and allows the resonant frequency to be adjusted. The measurement method is based on the accurate and efficient hybrid electromagnetic analysis method developed by the authors. The validity of the estimation method is verified by a virtual experiment using an FEM simulator and an experiment.

Acknowledgments

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