Closed-Form Design Equations for Four-Port Crossover with Arbitrary Phase Delay

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***Abstract*- A novel procedure to analyse and design four-port crossover using admittance matrix is proposed. The closed-form design equations for crossover with arbitrary phase delay are obtained. For verification, both simulated and measured results of a fabricated crossover are given. The 15-dB return loss bandwidth is 35.4 %. And the return loss and isolation between adjacent ports are both below -24.7 dB with -0.7 dB insertion loss between the input and output.**

***Index Terms* —Four-port crossover, admittance matrix, arbitrary phase delay, closed-form design equations.**

I. INTRODUCTION

With the increasing complexity of microwave integrated circuits, designers are faced with the challenge of layout and routing. When two lines cross over each other, the traditional way to isolate signals on the same intersection area is to use via holes, air bridges, or bond wires. However, these structures have many shortcomings, one of which is that the characteristics of matching and phase delay are no good in higher frequency. Four-port planar crossover, the special case of couplers is a good candidate to solve these problems [1-7]. It has been widely used in Butler matrix for phased array systems [8, 9]. The even-odd-mode method has been used to analyse these structures [10]. But it is difficult to compute and simplify the *S*-parameter expressions manually if the crossover is complicated or cascaded. The admittance matrix has more direct and fundamental corresponding relationship with not only scattering properties but also topology structure. Since every admittance-parameter has a definite meaning, which is the input admittance or the transfer admittance when all other ports are short-circuited. Thus, for a given circuit model, its admittance matrix can be built more easily.

In this paper, a design method for a crossover with arbitrary phase delay based on the admittance matrix is introduced and closed-form design equations are obtained. To verify the design concept, a microstrip crossover worked at 6 GHz is designed. Simulations and measurements are presented to confirm that this approach is easy, simple and efficient.

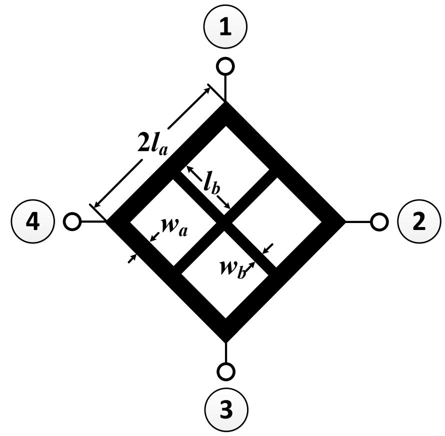

Figure 1. Layout of the crossover.

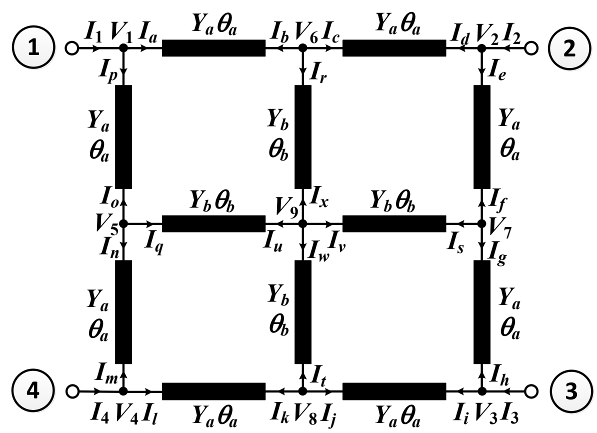

Figure 2. Equivalent circuit model of the crossover.

II. ADMITTANCE MATRIX APPROACH FOR CROSSOVER

The crossover is a special case of couplers, which allows a pair of signals to cross each other while maintaining isolation between the two signal paths. It also has the matching properties at all ports and a given phase delay θ between the ports in signal path. Thus, the scattering matrix of the crossover is as follows:

$$S = e^{-j\theta}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (1)$$

By adapting the standard scattering matrix to admittance matrix conversion operation with port admittance Y_0, its admittance matrix is obtained immediately:

$$Y = Y_0\left(U - S\right)\left(U + S\right)^{-1} = jY_0\begin{bmatrix} -\cot\theta & 0 & \csc\theta & 0 \\ 0 & -\cot\theta & 0 & \csc\theta \\ \csc\theta & 0 & -\cot\theta & 0 \\ 0 & \csc\theta & 0 & -\cot\theta \end{bmatrix} \quad (2)$$

where U is the unit matrix.

It is noted that all of the admittance parameters are zero or pure imaginary number, which means the network is lossless. It is also observed that the admittance matrix is rotational symmetric, which means the network is also rotational symmetric. And then, the crossover can be designed and fabricated using ordinary passive components, such as microstrip lines or lumped components.

Fig. 1 shows the layout of the microstrip crossover. It is composed of a square ring and inner crossed lines. The corresponding equivalent circuit model is depicted in Fig. 2, in which the characteristic admittance and the electric length of the square ring section are Y_a and θ_a. The counterparts of the inner crossed lines are Y_b and θ_b. In order to calculate its admittance matrix, the relationship between branch currents and node voltages is shown as follows:

$$\begin{cases} I_a = Y_{11}^aV_1 + Y_{12}^aV_6 \\ I_b = Y_{12}^aV_1 + Y_{11}^aV_6 \\ I_c = Y_{11}^aV_6 + Y_{12}^aV_2 \\ I_d = Y_{12}^aV_6 + Y_{11}^aV_2 \\ I_e = Y_{11}^aV_2 + Y_{12}^aV_7 \\ I_f = Y_{12}^aV_2 + Y_{11}^aV_7 \\ I_g = Y_{11}^aV_7 + Y_{12}^aV_3 \\ I_h = Y_{12}^aV_7 + Y_{11}^aV_3 \end{cases} \quad (3)$$

$$\begin{cases} I_i = Y_{11}^aV_3 + Y_{12}^aV_8 \\ I_j = Y_{12}^aV_3 + Y_{11}^aV_8 \\ I_k = Y_{11}^aV_8 + Y_{12}^aV_4 \\ I_l = Y_{12}^aV_8 + Y_{11}^aV_4 \\ I_m = Y_{11}^aV_4 + Y_{12}^aV_5 \\ I_n = Y_{12}^aV_4 + Y_{11}^aV_5 \\ I_o = Y_{11}^aV_5 + Y_{12}^aV_1 \\ I_p = Y_{12}^aV_5 + Y_{11}^aV_1 \end{cases} \quad (4)$$

$$\begin{cases} I_q = Y_{11}^bV_5 + Y_{12}^bV_9 \\ I_u = Y_{12}^bV_5 + Y_{11}^bV_9 \\ I_r = Y_{11}^bV_6 + Y_{12}^bV_9 \\ I_x = Y_{12}^bV_6 + Y_{11}^bV_9 \\ I_s = Y_{11}^bV_7 + Y_{12}^bV_9 \\ I_v = Y_{12}^bV_7 + Y_{11}^bV_9 \\ I_t = Y_{11}^bV_8 + Y_{12}^bV_9 \\ I_w = Y_{12}^bV_8 + Y_{11}^bV_9 \end{cases} \quad (5)$$

where

$$Y_{11}^a = -jY_a\cot\theta_a$$
$$Y_{12}^a = jY_a\csc\theta_a$$
$$Y_{11}^b = -jY_b\cot\theta_b$$
$$Y_{12}^b = jY_b\csc\theta_b$$

According to Kirchhoff's current law, the following equations can be obtained:

$$\begin{cases} I_b + I_c + I_r = 0 \\ I_f + I_g + I_s = 0 \\ I_k + I_j + I_t = 0 \\ I_o + I_n + I_q = 0 \\ I_x + I_v + I_w + I_u = 0 \end{cases} \quad (6)$$

$$\begin{cases} I_1 = I_a + I_p \\ I_2 = I_d + I_e \\ I_3 = I_i + I_h \\ I_4 = I_l + I_m \end{cases} \quad (7)$$

Based on the definition of admittance matrix, the admittance parameters corresponding to Fig. 2 can be calculated from equations (3-7):

$$Y' = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{12} \\ Y_{12} & Y_{11} & Y_{12} & Y_{13} \\ Y_{13} & Y_{12} & Y_{11} & Y_{12} \\ Y_{12} & Y_{13} & Y_{12} & Y_{11} \end{bmatrix} \tag{8}$$

where

$$Y_{11} = 2Y_{11}^{a} + 2Y_{12} - Y_{13}$$

$$Y_{12} = -\frac{(Y_{12}^{a})^2 Y_{11}^{b}}{2Y_{11}^{a}Y_{11}^{b} + (Y_{11}^{b})^2 - (Y_{12}^{b})^2}$$

$$Y_{13} = -\frac{(Y_{12}^{a})^2 (Y_{12}^{b})^2}{(2Y_{11}^{a} + Y_{11}^{b})(2Y_{11}^{a}Y_{11}^{b} + (Y_{11}^{b})^2 - (Y_{12}^{b})^2)}$$

Letting the admittance matrix expression (2) be equal to admittance matrix expression (8), three equations with four unknown parameters (Y_a, θ_a, Y_b and θ_b) are obtained. Subsequently, the parameters can be derived in term of the given phase delay θ:

$$\tan\theta_a = \pm\sqrt{\frac{3+\cos\theta}{1-\cos\theta}}$$

$$Y_a = \frac{Y_0}{2}\sqrt{\frac{3+\cos\theta}{1+\cos\theta}}$$

$$\theta_b = 90^o \tag{9}$$

while Y_b can be arbitrary value.

In this paper, we impose $\theta = 40^o$, Y_0=1/50=0.02 S, and Y_b =0.0088 S to give an example of designing microsrtip crossover. So, the parameters of the crossover can be calculated as θ_a=76º, Y_a=0.0146 S according to equation (9).

III. Results and Discussions

The designed crossover is fabricated on Rogers RO4003 substrates with dielectric constant 3.55 and thickness 0.813 mm. It has center frequency of 6GHz. Simulations are carried out with CST Microwave Studio and the measurements are performed using a vector network analyzer (Agilent 8722ES). Following the above approach, 0.3 mm is chosen for the width of inner crossed lines, with w_a = 1 mm, l_a = 7.5 mm and l_b = 9.25 mm.

Simulated and measured frequency responses of the crossover are shown in Fig. 3, together with phase delay between the diagonal ports (port 1 and 3) in Fig. 4. The matching (S_{11}) and isolation (S_{12} and S_{14}) characteristics are below -24.7 dB and -27 dB at 6 GHz. The insertion loss of transmission characteristics show S_{13} = -0.7 dB and the 15-dB return loss bandwidth is 35.4 %. The measured phase delay between the ports in signal path is 42.4º. Fig. 5 shows the photograph of the fabricated crossover.

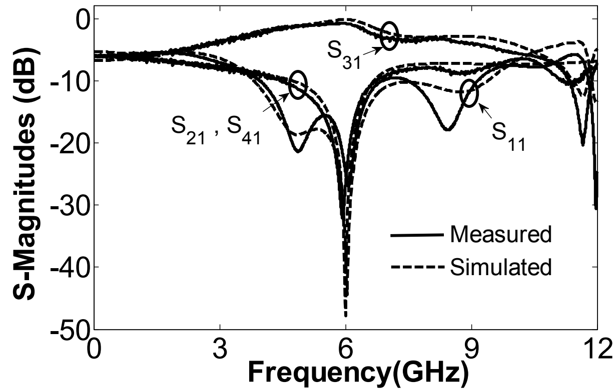

Figure 3. Simulated and measured responses of the crossover.

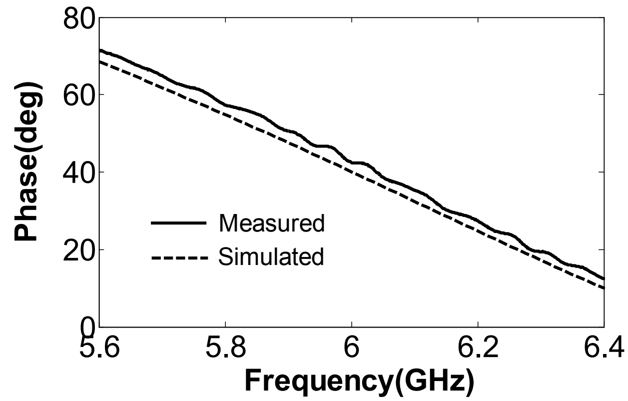

Figure 4. Phase delay between the ports in signal path.

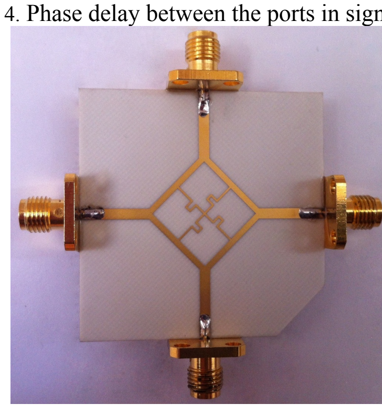

Figure 5. Photograph of the fabricated crossover.

IV. Conclusion

In this paper, a design method for crossover with arbitrary phase delay is proposed. The method derives closed-form design equations based on the admittance matrix. Simulations and measurements show that the procedure is easy, simple and efficient. One crossover is fabricated and measured for validity confirmation. It shows good insertion loss between diagonal ports and excellent isolation between adjacent ports. In addition, the crossover exhibits an accurate required phase delay between the ports in signal path.

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