Effects of Vibrating Lossless Dielectric on EM Fields: Numerical Simulation in One Dimension

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ABSTRACT
The effects of vibrating lossless dielectric medium on the electromagnetic fields were numerically simulated using the method of characteristics (MOC) in one dimension. Comparisons of the computational results of the transmitted and reflected electric fields were carried out and reported in this paper. In this problem there are two types of governing equations that MOC has to solve simultaneously: Maxwell's equations and Maxwell-Minkowski equations. The former governs when EM fields propagate in free space or in medium at rest whereas the latter rules in moving medium. The computational results reveal the effects of the vibrating medium on the EM fields, which are the direct proof that MOC has successfully solved the Maxwell-Minkowski equations. For the purpose of clear exhibitions the medium was set to vibrate in two different ways, sinusoidal and zigzag which have a maximum instantaneous velocity of 10 % of the speed of light.

1. INTRODUCTION
Dated back to as early as 1970s there have been many studies on the EM scattering problems from moving objects or moving boundary, on the moving medium, on the vibrating medium and objects. Most of them proposed theoretical solutions; while some provided numerical simulation results. The significance of computational Electromagnetics (CEM) aroused by the facts that most EM scattering problems are non-linear, complicated and almost impossible solved analytically and that the development of various useful methods for numerical solutions along with the rapid development of computer industry provides more and more powerful computing ability.

Two widely used numerical simulation approaches are the method of moments (MoM) and the finite-difference time-domain (FDTD) technique. The former provides information in the frequency domain and the latter records the evolution of EM fields as time advances. These two types of data are mutually available through proper Fourier transform. The method of characteristics (MOC), similar to FDTD, was proposed for numerically simulating EM scattering problems about fifteen years ago. It is reported that MOC had successfully simulated the reflected electric fields from boundary traveling at constant speed and/or vibrating with constant frequency and that MOC places all field variables in the cell centroid such that changes in the cell geometry due to the moving boundary can be resolved by accordingly adjusting the numerical time step of that cell.

2. GOVERNING EQUATIONS
The governing equations for the behavior of electromagnetic fields in free space and stationary uniform medium are the time-dependent Maxwell's equations. If EM fields are propagating inside medium that is moving, the Maxwell-Minkowski equations take over as the governing equations. Therefore, these are the two governing equations that in the present problem MOC has to solve simultaneously.

Since the present simulation is carried out in one dimension, and there are two types of governing equations, Maxwell's equations and Maxwell-Minkowski equations, have to be solved at the same time, the following arrangements have to be made. The excitation source is assumed to be plan Gaussian EM pulse, have only two components as $\vec{\mathbf{E}} = \hat{\mathbf{z}}\,\mathbf{E_z}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{y}}\,\mathbf{B_y}$. The medium may vibrate at an impractical high frequency of 0.9 or 1.5 GHz either in sinusoid or in zigzag with a maximum instantaneous velocity of one tenth of the speed of light (C = 3 × 10^8 m/s). Moreover, the instantaneous velocity is assumed to be $\vec{\mathbf{v}} = \mathbf{v}\,\hat{\mathbf{x}}$ with $\mathbf{v} = \pm\,\mathbf{0.1\,C}$.

With two conventional notations in mind: $\boldsymbol{\beta} = \frac{\mathbf{v}}{\mathbf{C}}$ and $\mathbf{n} = \sqrt{\boldsymbol{\mu}_\mathbf{r}\,\boldsymbol{\varepsilon}_\mathbf{r}}$, the Maxwell-Minkowski equations have variations from Maxwell's equation. The electric and the magnetic flux densities are modified as functions of both the electric and the magnetic field intensities

$$\vec{\mathbf{D}} = \boldsymbol{\varepsilon}\,\boldsymbol{\alpha}\cdot\vec{\mathbf{E}} + \boldsymbol{\Omega}\times\vec{\mathbf{H}} \tag{1}$$

$$\vec{\mathbf{B}} = \boldsymbol{\mu}\,\boldsymbol{\alpha}\cdot\vec{\mathbf{H}} - \boldsymbol{\Omega}\times\vec{\mathbf{E}}\,. \tag{2}$$

Symbols are defined as follows: $\mathbf{a} = \frac{1-\boldsymbol{\beta}^2}{1-\mathbf{n}^2\boldsymbol{\beta}^2}$, $\boldsymbol{\alpha} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{a} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{a} \end{pmatrix}$, $\boldsymbol{\Omega} = \frac{(\mathbf{n}^2-\mathbf{1})\boldsymbol{\beta}}{(\mathbf{1}-\mathbf{n}^2\boldsymbol{\beta}^2)\mathbf{C}}\hat{\mathbf{x}}$. To cast Maxwell's equations into the form of Euler equation, we first insert (1) and (2) into the Maxwell's equations, set $\mathbf{q} = [\,\mathbf{B}_\mathbf{x}\quad \mathbf{B}_\mathbf{y}\quad \mathbf{D}_\mathbf{z}\,]^\mathbf{T}$, $\mathbf{f} = [\,\mathbf{0}\quad -\mathbf{E}_\mathbf{z}\quad -\mathbf{H}_\mathbf{y}\,]^\mathbf{T}$, $\mathbf{g} = [\,\mathbf{E}_\mathbf{z}\quad \mathbf{0}\quad \mathbf{H}_\mathbf{x}\,]^\mathbf{T}$ and then transform the set of partial derivative equations from the Cartesian system (t, x, y) into curvilinear system (τ, ξ, η) and finally have the following

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{a} & \boldsymbol{\Omega}/\boldsymbol{\varepsilon} \\ \mathbf{0} & -\boldsymbol{\Omega}/\boldsymbol{\mu} & \mathbf{a} \end{bmatrix} \frac{\partial \mathbf{Q}}{\partial \boldsymbol{\tau}} + \frac{\partial \mathbf{F}}{\partial \boldsymbol{\xi}} + \frac{\partial \mathbf{G}}{\partial \boldsymbol{\eta}} = \mathbf{0} \tag{3}$$

where $\mathbf{Q} = \mathbf{J}\mathbf{q}$, $\mathbf{F} = \mathbf{J}(\xi_\mathbf{x}\mathbf{f} + \xi_\mathbf{y}\mathbf{g})$, $\mathbf{G} = \mathbf{J}(\eta_\mathbf{x}\mathbf{f} + \eta_\mathbf{y}\mathbf{g})$, and $\mathbf{J} = \begin{vmatrix} \mathbf{x}_\xi & \mathbf{x}_\eta \\ \mathbf{y}_\xi & \mathbf{y}_\eta \end{vmatrix}$ is the Jacobian of the inverse transformation. (3) is the Maxwell-Minkowski equations in the Euler equation form.

3. PROBLEM DEFINITION

The definition of the worked problem as schematically drawn in Figure 1 includes the electric field intensity and a dielectric medium characterized by its static relative permittivity. As shown the electric field is specified as follows: a peak value of 1 V/m, initially 1.1 meters from the boundary, a Gaussian profile in time with a width of 0.634 ns with a cut-off level of 120 dB. The dielectric medium is uniform and lossless and has its relative permittivity of 9 or 81 allowing the EM fields to propagate at a speed of one third or one ninth of the light speed, respectively. There are two sampling points to record the reflected and transmitted electric fields. They are located at 1.1 meters away from the boundary in the air and 1.0 meter in the medium. Note that the latter moves as the medium moves.

The grid system is a single array of square cells as in Figure 2. Because the problem involved with a vibrating medium, it is convenient to assume that the grids in the medium region are uniform and remain intact despite the fact that medium is vibrating. The only cell affected is the one in the air region immediately next to the boundary. Illustrated in Figure 2(b) explains how the N^{th} cell in the air zone is being eliminated little-by-little by the boundary moving to the left. Figure 2(c) shows that a (N+1) cell is being introduced bit-by-bit into the grid system when the boundary moves to the right. Note that the total number of cells eliminated or added may be multiple depending on the grid size, the maximum displacement, and the type of vibration. If the medium is vibrating sinusoidally at 0.9 GHz, the maximum peak-to-peak displacement is 10.61 mm. This number becomes 16.66 mm for the medium vibrates in zigzag at the same frequency. Provided that the grid size is one millimeter, they are corresponding to 10.61 cells and 16.66 cells. If the vibration frequency is 1.5 GHz, they become 6.37 and 10, respectively.

4. RESULTS

A series plots of the electric fields demonstrating the reflection and transmission of the propagation of EM fields onto a dielectric half-space (ε_r = 81, μ_r = 1) is depicted in Figure 3. Symbols t0 through t6 are used to indicate several time instances. First time interval (t1 – t0) is 0.8 m/C and the rest are 0.3 m/C. At each time point there are three electric fields corresponding to when the medium is at rest (solid) or vibrating sinusoidally (dotted) or in zigzag (dash-dotted). A close-up view at t4 is given showing the discrepancies are barely seen. There are six numbers used to indicate the locations of the peaks at t4 through t6. The reflected and the transmitted electric fields are measured -0.8 and 0.2 in strength. These observations agree with the expectation.

To closely inspect how the vibrating medium affects the reflected EM fields, two sets of plots bearing the discrepancies from that of the stationary medium were given in Figure 4. The differences have similar patterns, fluctuate accordingly with the vibration frequency, and increase in strength as the dielectric constant increases. Several observations are also revealed in the corresponding spectra as in Figures 5 and 6. The spectrum of the incident pulse was included as a reference. The reflected field spectra clearly demonstrate the signatures of vibration frequency despite the type of vibration. In Fig. 6, that the higher frequency components near the cutoff are omitted is noticeable, which is suspected as the results of insufficient number of grid since the refractive index of medium equals 9 and the EM pulse is narrower in width and smaller in strength. Also noticed is that the traces of vibration in zigzag are missing in the spectra of the transmitted electric fields inside medium. This may be because of the fact that the instantaneous velocity of the medium and the propagation speed of the EM fields are comparable such that EM fields are linearly accelerated more when they are moving in the same direction.

5. CONCLUSION

In this paper MOC has been shown successfully to simulate Maxwell's equations and Maxwell-Minkowski equations in one dimension and resolve the problem of the elimination and addition cells causing by the moving boundary. The effects of the vibrating medium on the EM fields were also demonstrated by exhibiting the differences in the reflected and transmitted electric fields. The computational results showed reasonable trends. This is made possible by MOC by placing all field variables in the cell centroid.

ACKNOWLEDGEMENT

The authors would like to thank Mr. Michael F. Chen of Cartell Chemical Co., Ltd., Chia-Yi, Taiwan, for making this work possible through the industry-academy cooperation project WFC-E-A2-9907-55.

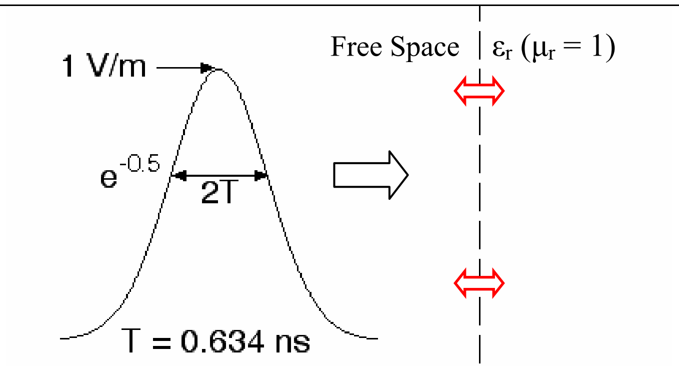
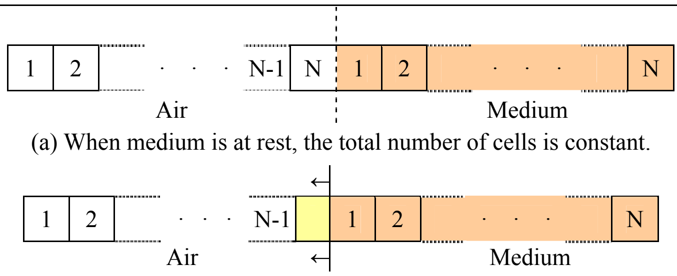
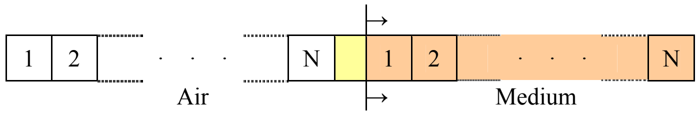


Figure 1: Gaussian EM pulse normally incident onto a vibrating medium.



(a) When medium is at rest, the total number of cells is constant.

(b) When medium moves to left, the cell N is gradually eliminated.



(c) When medium moves to right, the cell N+1 is gradually added.

Figure 2: How moving boundary affects the grid system.

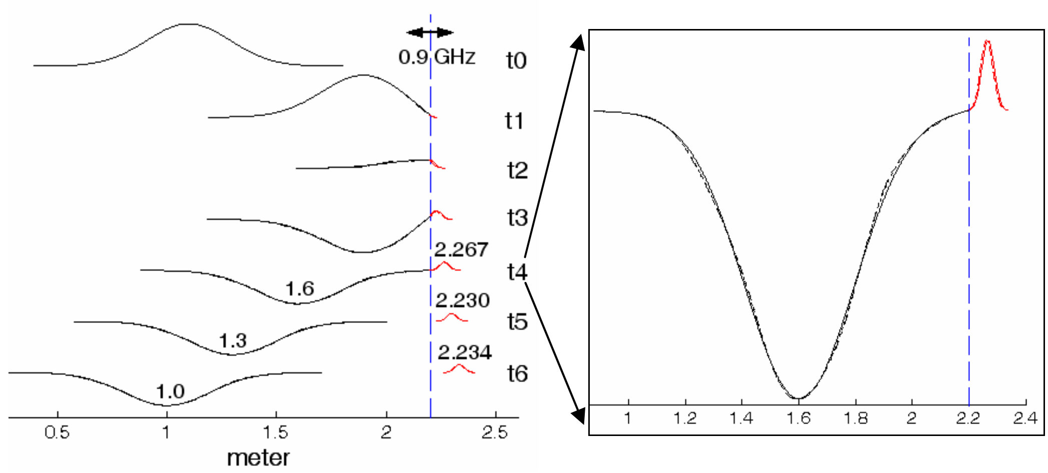
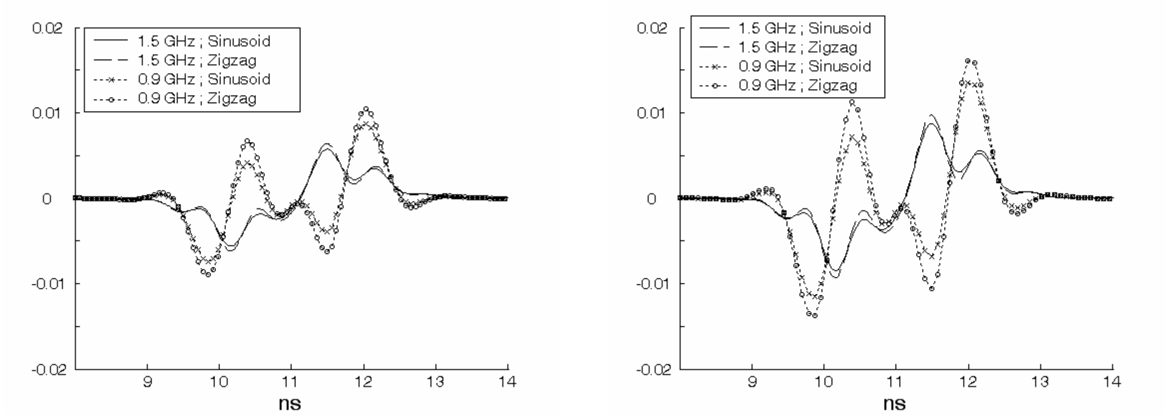


Figure 3: Propagation of EM pulse onto dielectric half-space ($\varepsilon_r = 81$, $\mu_r = 1$).



(a) $\varepsilon_r = 9$, $\mu_r = 1$

(b) $\varepsilon_r = 81$, $\mu_r = 1$

Figure 4: Reflected electric fields are affected by the vibrating medium.

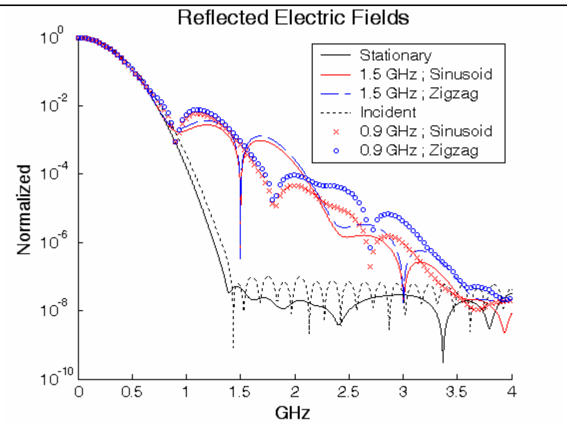


Figure 5: Spectra of the reflected electric fields from medium ($\varepsilon_r = 9$, $\mu_r = 1$).

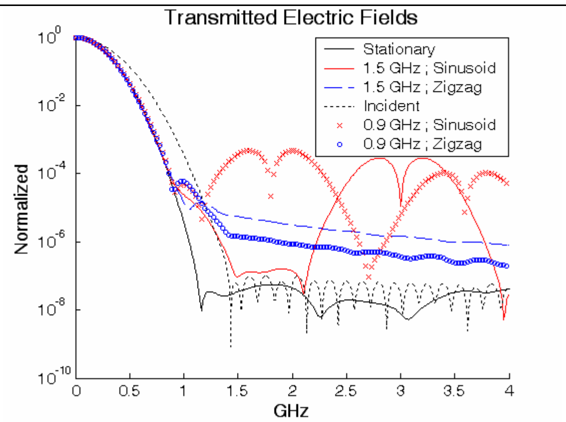


Figure 6: Spectra of the transmitted electric fields inside medium ($\varepsilon_r = 81$, $\mu_r = 1$).