Analytical Prediction of Shielding Effectiveness of Rectangular Enclosures with Rectangular Apertures under Normal Incidence

#Weiye Zhong [1], Zhongxiang Shen [2]
[1] Shanghai Astronomical Observatory
80 Nandan Road, Shanghai 200030, wyzhong@shao.ac.cn
[2] Nanyang Technological University
50 Nanyang Avenue, Singapore 639798

1. Introduction

As wireless communication becomes explosively popular, metallic enclosures are usually employed in electromagnetic compatibility (EMC) design to keep electronic equipment from being affected by electromagnetic interference (EMI). However, these enclosures usually consist of various apertures used for ventilation or cabling purposes. Shielding effectiveness (SE), which is measuring the shielding efficiency of a metallic enclosure, is defined as the ratio of field strength with enclosure present to field strength with enclosure absent.

In the past, a variety of analytical and numerical techniques were proposed to predict the SE of a rectangular metallic enclosure with different apertures [1]-[4]. In [1], the transmission line theory based on the dominant mode approximation was used to obtain an analytical solution to predict the SE. Various numerical methods [3, 4] can solve problems of complicated structure, but they normally require expensive computational resources.

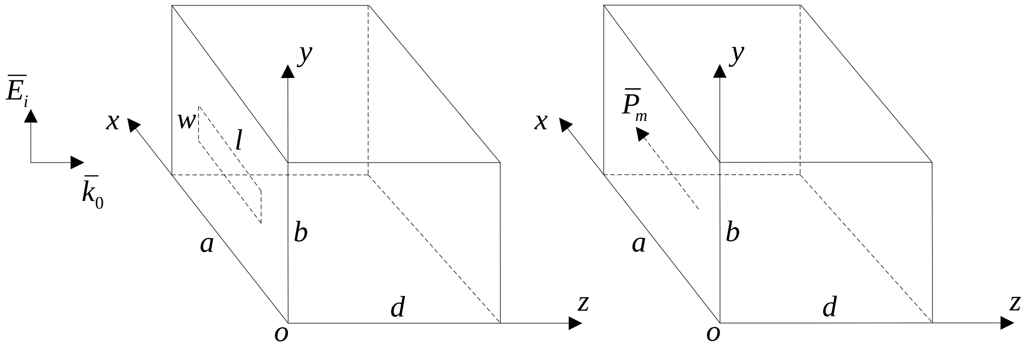
In this paper, the Bethe-Cohn aperture coupling theory is utilized to analytically calculate the electric fields inside the enclosure and to accurately predict the SE of rectangular enclosures with rectangular apertures. Bethe [5] investigated the coupling through a small circular aperture in a conducting wall of zero thickness based on equivalent electric and magnetic dipole moments. An extension of Bethe's work was developed by Cohn [6] enabling the theory to be applicable to large apertures. It is demonstrated that our computed results for SE based on the Bethe-Cohn aperture coupling theory are in good agreement with measured and calculated results available in the literature.

2. Formulation

Figure 1 shows a rectangular enclosure $(a \times b \times d)$ with a rectangular aperture $(l \times w)$ illuminated by a vertically polarized plane wave at normal incidence and its equivalent model based on Bethe-Cohn aperture coupling theory. Let $\overline{E}_i$, $\overline{H}_i$ be, respectively, the electric and magnetic fields for an incident plane wave, and $\overline{E}_a$, $\overline{H}_a$ be the fields at the aperture location when the aperture is absent. (x_0, y_0, z_0) are the coordinate variables of the slot center.

$$\begin{cases} \overline{E}_a \big|_{z=z_0} = \overline{E}_i + \overline{E}_r = 0 \\ \overline{H}_a \big|_{z=z_0} = \overline{H}_i + \overline{H}_r = \hat{x}(-2E_0/\eta_0) \end{cases} \tag{1}$$

where $\eta_0 = 120\pi\ \Omega$ is the wave impedance in free space, E_0 the magnitude of the incident electric field.



(a) (b)

Figure 1: (a) Enclosure with an aperture illuminated by one vertically polarized plane wave at normal incidence. (b) The equivalent model of (a) based on Bethe-Cohn theory.

Using the equations in [2], the electric and magnetic dipole moments are given by

$$\begin{cases} \bar{P}_e = -\varepsilon_0 p_e \bar{E}_a = 0 \\ \bar{P}_m = -p_m \bar{H}_a = \hat{x}(2p_m E_0/\eta_0)\delta(x-a/2)\delta(y-b/2)\delta(z) \end{cases} \tag{2}$$

where p_e and p_m are electric and magnetic polarizabilities over the small aperture, respectively.

Cohn [5] extended Bethe's small-aperture coupling theory to large-aperture by introducing a factor $(1-f^2/f_{apr}^2)$, where f is the operating frequency and $f_{apr}=1/(2l\sqrt{\mu_0\varepsilon_0})$ is the resonant frequency of the aperture. Therefore, the magnetic polarizability can be modified as follows [6, 7]

$$p_m = 0.132l^3/((\log_{10}(1+0.66l/w))(1-f^2/f_{ap}^2)) \tag{3}$$

In addition, (3) is valid up to the first resonance of the aperture as mentioned in [6]. This is essential to ensure the accuracy of our calculation.

The total magnetic current in the aperture in terms of Bethe-Cohn aperture coupling theory is

$$\bar{M}_x = j\omega\mu_0\bar{P}_m = \hat{x}(j2\omega\mu_0 p_m E_0/\eta_0)\delta(x-a/2)\delta(y-b/2)\delta(z) \tag{4}$$

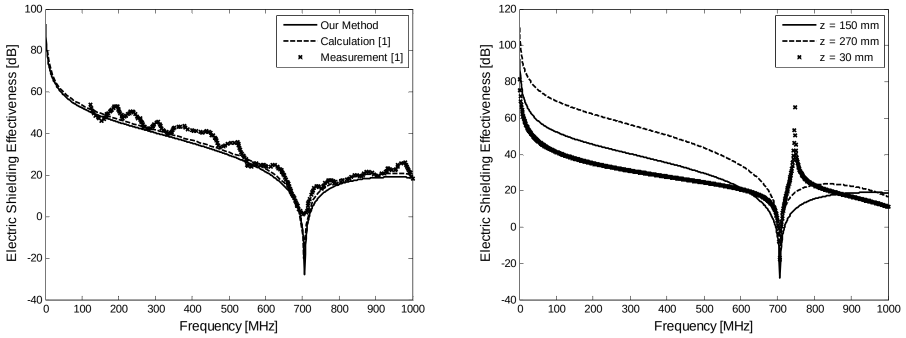
A simple mode-matching method is applied to calculate the transverse electric and magnetic fields inside the cavity due to the magnetic current along the cavity wall. Using the boundary condition $\bar{E}_t\big|_{z=d}=0$ and $\bar{M}_x=-\hat{z}\times\bar{E}_t\big|_{z=0}$ yields the following

$$\bar{E}_t = \sum_{i=1}^{N}[\sinh^2(\gamma_i d)-\cosh^2(\gamma_i d)]\frac{\sinh[\gamma_i(d-z)]}{\sinh(\gamma_i d)}\left[\int_s \bar{M}_x\cdot(\hat{z}\times\bar{e}_i)ds\right]\bar{e}_i \tag{5}$$

where i is the combined mode index for possible TE_{mn}, TM_{mn} modes, N the number of modes considered, $\bar{e}_i$ the modal function of the transverse electric field of the ith mode, γ_i the complex propagation constant.

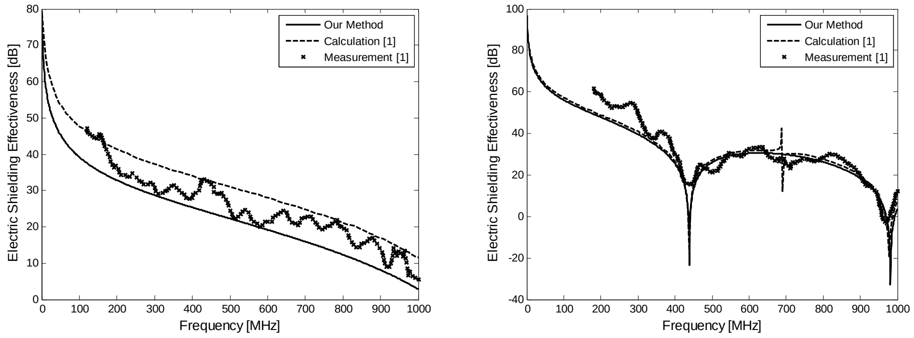
3. Numerical Results

In order to verify the formulation in the previous section, a rectangular enclosure of size $300\times120\times300$ mm^3 with an aperture of size 100×5 mm^2 is considered first. Figure 2(a) presents our calculated SE result and those obtained in [1] at the center of the enclosure $(z=150$ mm$)$. It is seen that our calculated SE values are in good agreement with those in [1] both below and above the cutoff frequency (500 MHz) of the rectangular waveguide. Figure 2(b) presents the calculated SE results at three different positions inside the same enclosure as Figure 2(a). From Figure 2, it is seen that the SE values around 700 MHz are negative due to the dominant (TE_{101}) mode resonance of the enclosure.



(a) (b)

Figure 2: (a) Calculated and measured SE results at the center of $300 \times 120 \times 300$ mm^3 enclosure with 100×5 mm^2 aperture. (b) Calculated SE results at three different positions inside the same enclosure with (a).



(a) (b)

Figure 3: (a) Calculated and measured SE results at the center of $222 \times 55 \times 146$ mm^3 enclosure with 100×5 mm^2 aperture. (b) Calculated and measured SE results at the center of $483 \times 120 \times 483$ mm^3 enclosure with 100×5 mm^2 aperture.

Figure 3 depicts calculated and measured SE results at the center of different enclosures with the same aperture. It is easily seen from Figure 3 that the larger enclosure resonates at 440 and 980 MHz, whereas the smaller enclosure does not have resonance over the whole frequency band of interest. In addition, comparing the curve obtained by our method with the calculated curve in [1], our result is closer to the measured curve in [1], especially at approximately 700 MHz where a false resonance is predicted in [1]. Finally, it is obvious that our results are slightly less than those in [1] over the entire frequency band probably due to the fact that the effect of enclosure wall thickness is not considered in our formulation.

4. Conclusion

In this paper, the Bethe-Cohn aperture coupling theory has been utilized to calculate the equivalent magnetic current on the aperture. The electric field inside a rectangular enclosure has then been evaluated from the equivalent magnetic current using the mode-matching method to analytically predict the SE of rectangular enclosures with rectangular apertures. It has been demonstrated that our calculated SE results are in good agreement with measured and numerical results available in the literature. The formulation is currently being extended to other geometries and other incident angles.

References

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