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# A study on feedback control of intrinsic localized modes in a micro-mechanical cantilever array

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**Abstract**—Intrinsic localized mode (ILM), which is also called discrete breather (DB), is an energy localized vibration in nonlinear coupled oscillators. It is well known that the ILM can move in the system without decay of its energy concentration. This paper shows that the position of ILM can be controlled by proportional-derivative control. To create a force to the ILM, linear on-site coefficients are modulated linearly with respect to the lattice number. Namely, value of the linear on-site coefficients linearly increase/decrease as the lattice number increases. Magnitude of the tilt is adjusted with PD control scheme. As a result of numerical simulations, a standing ILM is successfully controlled toward a reference position with keeping its energy concentration.

## 1. Introduction

Spatially localized and temporary periodic vibrations often appear in nonlinear coupled oscillators [1]. The energy localized vibration in discrete media which is first discovered by A. J. Sievers and S. Takeno [2] is called intrinsic localized mode(ILM) or discrete breather(DB). Experimental observations of ILM have been reported for a variety of physical system in this decade as well as theoretical and numerical studies. In particular of them, the observation in micro-mechanical cantilever array allow us to expect the realization of applications using ILM in micro/nano-engineering [3], because it was also observed that ILM can move without decaying its energy concentration and can be manipulated by an extraneous stimulus [4].

For the realization of such application, the control scheme for the ILM should be established. In our previous research, it has been shown that a standing ILM loses its stability by parametric excitation [5]. This result implies that appropriately adjusting parameter can creates a force to standing ILM, because the parametric resonance is usually caused by changing a potential shape. Therefore, if the parameter of the system is changed appropriately based on the distance from the reference position, the position of ILM can be controlled by using an ordinary control method. In this paper, proportional-derivative control is applied to the control of the position of ILM. First, an approximate equation describing the motion of traveling ILM is derived. Second, behavior of moving ILM is shown when on-site linear coefficients are gradually changed with respect to the lattice number. Finally, the proportional-derivative control of ILM is demonstrated and discussed.

## 2. Coupled Cantilever Array and Standing ILM

Micro-cantilever array is one of nonlinear coupled oscillators having ILM [3]. By focusing on the first mode of beam vibrations, motions of each cantilever's tip are approximately described by the following ordinary differential equation [4, 6, 7],

$$\begin{aligned}\ddot{u}_n = &-\alpha_1 u_n - \alpha_2(2u_n - u_{n+1} - u_{n-1}) \\ &-\beta_1 u_n^3 - \beta_2(u_n - u_{n+1})^3 - \beta_2(u_n - u_{n-1})^3 \\ &(n = 1, \cdots 8),\end{aligned} \tag{1}$$

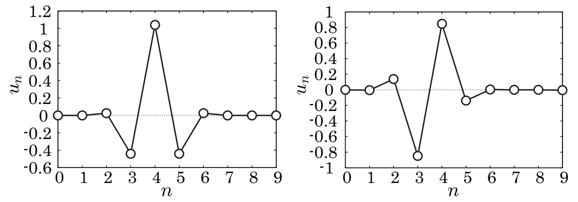
where $u_n$ denotes the displacement of $n$th cantilever from the equilibrium position. $\alpha_1$ and $\beta_1$ can be set at 1 by nondimensionalization. Coupling coefficients, $\alpha_2$ and $\beta_2$, are determined by the design of the array. In this paper, $\alpha_2$ is set at 0.1, $\beta_2$ at 0.5 referring the experiment by M. Sato [3]. The boundaries of Eq.(1) are set as $u_0 = u_8, u_9 = u_1$, namely the periodic boundary condition.

Amplitude distribution of typical standing ILMs at $\beta_2 = 0.5$ is shown with their Floquet multipliers in Fig.1. An ILM which has odd-symmetry in its amplitude distribution is shown in the left panel of Fig.1(a). This is called Sievers-Takeno mode(ST mode) [8]. On the other hand, even-symmetrical one is called Page mode(P mode) shown in the right panel of Fig.1(a). For the case that $\beta_2 = 0.5$, ST mode is stable whereas P mode is unstable as shown in Fig.1(b).

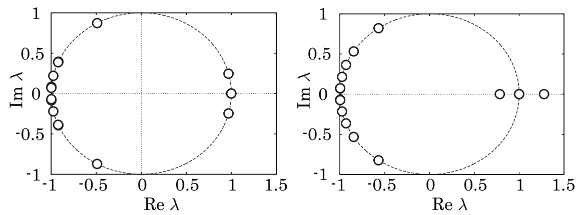
## 3. Traveling ILM

The trajectories of traveling ILMs show the structure which is quite similar to that of a pendulum [5]. In our previous research, it was shown that the position of traveling ILM can approximately be described by

$$\ddot{X} = -\frac{1}{2\pi}\Omega_0^2 \sin(2\pi X), \tag{2}$$



(a) Amplitude distribution



(b) Floquet multipliers

Figure 1: Sievers-Takeno mode and Page mode. All coexisting ST modes are stable whereas P modes are unstable for $\beta_2 = 0.5$.

where $X$ is the position of traveling ILM. $\Omega_0$ denotes the angular frequency of small fluctuation of traveling ILM around a stable ILM. The angular frequency depends on the parameters of Eq.(1). It is shown in Ref [5] that if a parameter is periodically changed in time, the parametric resonance occurs for a stable ILM. This implies that an effective potential for the position of ILM, which takes a sinusoidal-like shape, is deformed homogeneously by the parameter change. In the parametric excitation, equilibria of Eq.(2) are not changed, namely, standing ILM is not affected to its position by the parameter change. However, to control the position of ILM, a force which affects the position of ILM is necessary. In this paper, parameter values are varied with respect to the lattice number to create the force to ILM, which is called parameter tilt.

## 4. Parameter Tilt

The linear on-site coefficients $\alpha_1$ is gradually changed with respect to the lattice number to create the parameter tilt which is defined as,

$$\alpha_{1,n} = \alpha_{1,c} + m_{\alpha_1}(n - c), \tag{3}$$

where $n$ denotes the lattice number, $c$ is the center of the parameter tilt. In this paper, $c$ is fixed at 4. $m_{\alpha_1}$ is the gradient of the parameter tilt.

If there is no parameter tilt, namely $m_{\alpha_1} = 0$, standing ILMs locate at $n = \cdots, 4, 4.5, 5, \cdots$. However, as shown in Fig.2, the position of standing ILMs is changed

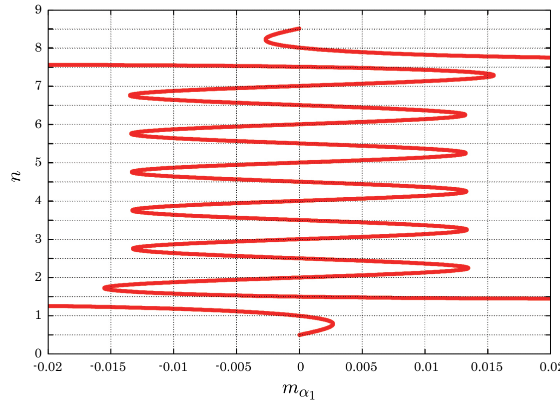


Figure 2: Position of standing ILMs vs. the parameter tilt.

with respect to the gradient of the parameter tilt $m_{\alpha_1}$. Stable ST modes and unstable P modes become close to each other as $m_{\alpha_1}$ decreases or increases. If $|m_{\alpha_1}|$ exceeds a certain value, ST and P modes disappear except ILMs located around $n = 1$ and $n = 8$ at which $\alpha_{1,n}$ has a large gap. The change of position of standing ILMs around the center of array corresponds to the case that a constant force term is added to Eq.(2), namely,

$$\ddot{X} = -\frac{1}{2\pi}\Omega_0^2 \sin(2\pi X) - F, \tag{4}$$

where $F$ is the term representing the force to the ILM position caused with the parameter tilt. If $F$ is adjusted based on the difference between the position of the traveling ILM and a reference position, the traveling ILM should be controlled. In the next section, several demonstrations of feedback control for traveling ILM will be shown.

## 5. Proportional-derivative Control

To control the position of ILM, the proportional-derivative(PD) control is applied in this paper. The parameter tilt is adjusted as,

$$\alpha_{1,n}(t) = \alpha_{1,c} + m_{\alpha_1}\{K_{\mathrm{p}}(N_{\mathrm{r}} - X_{\mathrm{ILM}}) + K_{\mathrm{v}}\dot{X}_{\mathrm{ILM}}\}(n - c) \tag{5}$$

where $N_{\mathrm{r}}$ is the reference position, $X_{\mathrm{ILM}}$ and $\dot{X}_{\mathrm{ILM}}$ are the position and the velocity of ILM, respectively. $K_{\mathrm{p}}$ and $K_{\mathrm{v}}$ denote the proportional gain and the derivative gain, respectively. In the initial state, an ILM stands at $n = 4$ and is stable as shown in Fig.1. The PD control is applied between $100 < t < 1000$ for all the simulations.

To detect the position of ILM, a complex projection $\mathcal{G} : \mathbb{R}^{2N} \to \mathbb{C}$ [9] is used, which is defined as

$$h = \mathcal{G}(\boldsymbol{u}, \dot{\boldsymbol{u}}) = \sum_{n=1}^{N}\left\{\left(\frac{1}{2}\dot{u}_n^2 + U_{\mathrm{On}}(u_n)\right)e^{i\frac{2\pi}{N}n} + U_{\mathrm{In}}(u_n - u_{n-1})e^{i\frac{2\pi}{N}(n+\frac{1}{2})}\right\} \tag{6}$$

where,

$$U_{\mathrm{O}n}(u_n) = \frac{\alpha_1}{2}u_n^2 + \frac{\beta_1}{4}u_n^4 \quad (7)$$

$$U_{\mathrm{I}n}(u_n - u_{n-1}) = \frac{\alpha_2}{2}(u_n - u_{n-1})^2 + \frac{\beta_2}{4}(u_n - u_{n-1})^4. \quad (8)$$

Here, $N$ denotes the number of oscillators and is set at 8 in this paper. The position of ILM can be estimated by

$$X_{\mathrm{ILM}} = \frac{\arg h}{2\pi}N + \frac{1}{2} \quad (9)$$

for the periodic boundary condition.

Figure3 shows the case that the reference value is set at $N_\mathrm{r} = 4.3$. The position of the ILM begins to sinusoidally oscillate when the control is turned on. In the case of $(K_\mathrm{p}, K_\mathrm{v}) = (0.1, 0)$, the oscillation does not decay. On the other hand, the oscillation decays and the position converges to $X_{\mathrm{ILM}} \simeq 4.15$ if $K_\mathrm{v}$ increases to 0.1. This means that the feedback of the velocity of ILM causes a damping effect into the motion of controlled ILM.

By increasing the proportional gain, the difference between the position after the controlled ILM converged and the reference position becomes small. In fact, the controlled ILM converges to $X_{\mathrm{ILM}} \simeq 4.25$ in the case that the proportional gain increases to $K_\mathrm{p} = 0.5$. However, the position after the controlled ILM converged does not coincide with the reference position even though the proportional gain increases further. On the basis of the control theory, an integral compensation will be needed to make the error zero.

By increasing the derivative gain, the speed to converge is enhanced as shown by the yellow curve in Fig.3. However, the error slowly increases after the ILM converged at $t \simeq 300$. For the corresponding control input shown in the lower panel of Fig.3, a small fluctuation is observed. As the time develops, the amplitude of the high frequency fluctuation becomes large. The fluctuation is caused by the vibration of each oscillator of Eq.(1) because ILM is a periodic solution of the system. The position detected by the complex projection fluctuates during each oscillator vibrates because the potential energy include nonlinear terms whereas the kinetic energy does not. As long as the complex projection Eq.(6) is used, detecting the fluctuation is unavoidable.

The cases that the reference position is set at $N_\mathrm{r} = 4.5$ or $N_\mathrm{r} = 5$ are shown in Fig.4. For $N_\mathrm{r} = 4.5$, the controlled ILM converges to $X_{\mathrm{ILM}} = 4.5$ and no error remains. After the control is turned off, the ILM loses its stability and begins to move because P mode is unstable for $\beta_2 = 0.5$. For $N_\mathrm{r} = 5$, the controlled ILM converges to $X_{\mathrm{ILM}} = 5$. If the control is turned off after the ILM is sufficiently close to $X_{\mathrm{ILM}} = 5$, the ILM keeps its position, because ST mode is originally stable. The control inputs tend to be zero as the time develops as shown in the lower panel of Fig.4. This implies that the error becomes zero without an integral compensation.

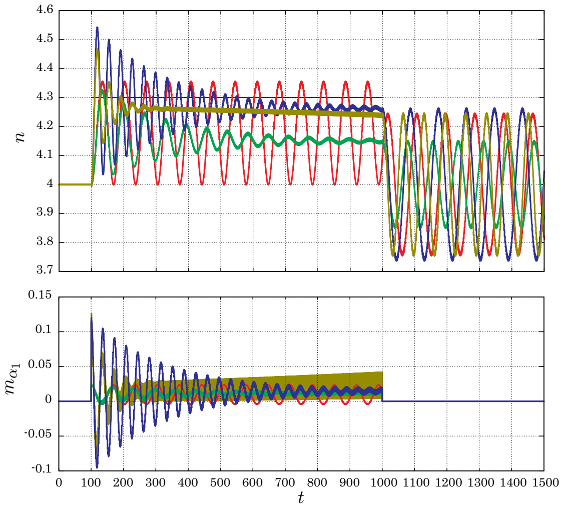



Figure 3: Trajectories of controlled ILMs and control input for $N_\mathrm{r} = 4.3$. Red: $(K_\mathrm{p}, K_\mathrm{v}) = (0.1, 0)$, green: $(K_\mathrm{p}, K_\mathrm{v}) = (0.1, 0.1)$, blue: $(K_\mathrm{p}, K_\mathrm{v}) = (0.5, 0.1)$, yellow: $(K_\mathrm{p}, K_\mathrm{v}) = (0.5, 0.5)$.

The time development of the energy distribution is shown in Fig.5. In the initial state, the energy is concentrated around $n = 4$. The energy concentration is kept during the control is applied and also after the control is turned off. The center of the energy concentration is finally located at $n = 5$. As a result, the ILM controlled from $n = 4$ to $n = 5$ without loss of the energy concentration. The feedback control is succeeded.

## 6. Conclusion

In this paper, a feedback control for the position of ILM was attempted. First, it was shown that the force to the position of ILM can be created by the parameter tilt of the system. Then, the magnitude of the parameter tilt was modulated by the difference between the position of ILM and the reference position by the proportional-derivative control scheme. Finally, it was demonstrated that ILM can be converged to anywhere near the original position.

However, if the reference position is not coincide with the place where ILM originally exists, the small and fast fluctuation was observed in the control input. To eliminate the fluctuation, the method to detect the position of ILM should be improved. In addition, the residual error was also observed in this case. The control scheme should be considered to include an integral compensation, namely, the proportional-integral-derivative(PID) control.

The area in which parameter values are adjusted should be reduced if the control scheme is applied to a real system because it is almost impossible to create the parameter tilt in the whole array if it consists of many oscillators. Since ILM usually concentrates in a few sites, the area, fortunately, seems to be able to narrow. In fact, the manipulation

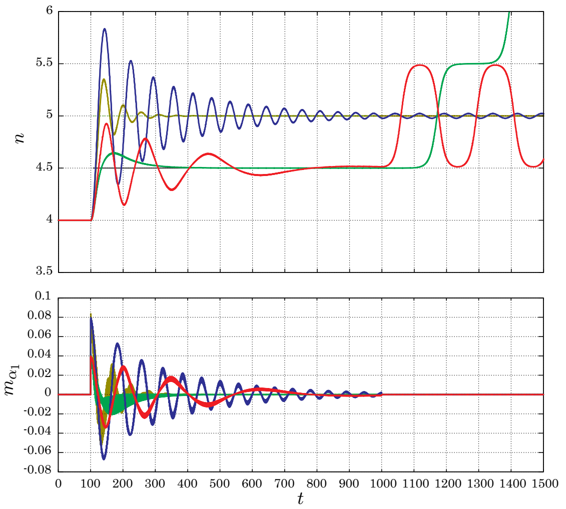



Figure 4: Trajectories of controlled ILMs and control input for $N_r = 4.5$(red and green) and 5(blue and yellow). Gains are set at $(K_p, K_v) = (0.1, 0.1)$ for the red and blue curves, $(K_p, K_v) = (0.1, 0.5)$ for the green and yellow curves.

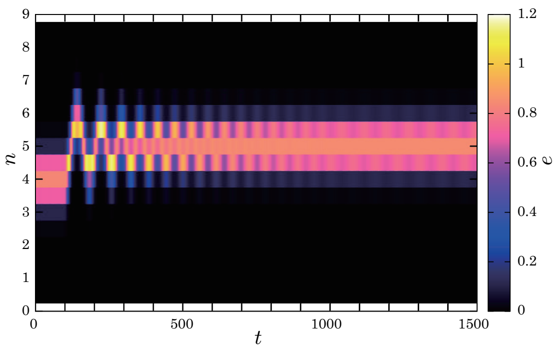



Figure 5: Time development of the energy distribution of the controlled ILM shown by the blue curve in Fig.4.

using a local defect created by an IR laser is already demonstrated for ILM in a micro-cantilever array by M.Sato and his coworkers [10]. If the position of the local defect is adjusted based on the difference between the position of ILM and the reference position, the feedback control will be able to achieve experimentally in micro-cantilever array. We will take into account such realistic restrictions for realization of the control of ILM in the future works.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### References


[1] S. Flach and A. V. Gorbach, "Discrete breathers – advances in theory and applications," *Phys. Rep.*, vol.467, p.1–116, 2008.

[2] S. Takeno and A. J. Sievers, "Anharmonic resonant modes in perfect crystals," *Solid State Commun.*, vol.67, p.1023, 1988.

[3] M. Sato, B. E. Hubbard, A. J. Sievers, B. Ilic, D. A. Czaplewski, and H. G. Craighead, "Observation of locked intrinsic localized vibrational modes in a micromechanical oscillator array," *Phys. Rev. Lett.*, vol.90, p.044102, 2003.

[4] M. Sato, B. E. Hubbard, and A. J. Sievers, "*Colloquium*: Nonlinear energy localization and its manipulation in micromechanical oscillator arrays," *Rev. Mod. Phys.*, vol.78, p.137, 2006.

[5] Masayuki Kimura, Yasuo Matsushita, and Takashi Hikihara. "A numerical study on parametric resonance of intrinsic localized modes in coupled cantilever arrays". In *Proceedings of NOLTA2012, Majorca, Spain*, 2012.

[6] M. Kimura and T. Hikihara, "Stability change of intrinsic localized mode in finite nonlinear coupled oscillators," *Phys. Lett. A*, vol.372, p.4592, 2008.

[7] M. Kimura and T. Hikihara, "Capture and release of traveling intrinsic localized mode in coupled cantilever array," *Chaos*, vol.19, (1) p.013138, 2009.

[8] S. Flach and A. Gorbach, "Discrete breathers in Fermi-Pasta-Ulam lattices," *Chaos*, vol.15, p.15112, 2005.

[9] Paule A. Houle, "Phase plane of moving discrete breathers," *Phys. Rev. E*, vol.56, p.3657, 1997.

[10] M. Sato, B. E. Hubbard, A. J. Sievers, B. Ilic, and H. G. Craighead, "Optical manipulation of intrinsic localized vibrational energy in cantilever arrays," *Europhys. Lett.*, vol.66, (3) p.318, 2004.