Fast Broad-band Angular Response Sweep Using FEM in Conjunction with Compressed Sensing Technique

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***Abstract-* The Compressed Sensing (CS) technique is applied to the finite element method (FEM) for fast radar cross section (RCS) calculation over a wide incident angular band. Rather than point-by-point solving multiple right-hand vectors of each incident angle, the right-hand sides are first compressed and then recovered to find each incident angle response in CS procedure. The numerical example shows the proposed approach leads a magnitude decrease of computation time without losing accuracy.**

I. Introduction

The finite-element method (FEM) is a full-wave numerical method that discretizes the variational formulation of a functional[1,2],has been widely used in electromagnetic analysis. For many practical applications such as radar imaging, our interests often a wide range of angles. In order to alleviate the computational burden and improve efficiency in this circumstance, some fast methods like interpolation [3] and extrapolation [4, 5] have been put forward to accelerate wide incident angular band calculation rather than point-by-point sweep. In this work, our renewed interest is to employ the novel compressed sensing technique for this scenario.

Compressed sensing was first developed in [6,7] by Candès et al in 2006,and applied in many fields recently, like CS radar[8], wireless sensor networks[9], magnetic resonance imaging[10]. It points out that if a sparse or compressible high-dimensional signal could be projected onto a low-dimensional space, with the sparse signal's priori conditions, the original signal could be recovered by a linear or non-linear reconstruction model.

Indeed, from the view of signal and system, the incident wave from different angler imposed on targets can be seen as a series of input signals, and the scatter fields can be viewed as the system response. So we can first compress these signals, subsequently solve the response of compressed signals and then recover to find the original response. If the computational burden of compression and recovery is minor, the angular response sweep calculation can be accelerated by using compressed sensing technique.

The rest of this paper is organized as follow. The total field formulations for scattering calculation using FEM are given in section II. A brief introduction of CS theory and some compression and recovery algorithms are presented in Section III. In section IV, the validity and efficiency of proposed method are demonstrated by numerical example. Some concluding remarks and observations are given in last section.

II. FEM Total Field Formulation for Scattering

For scattering applications, the FEM can be formulated in terms of either the total field or the scatter field. In the scatter field formulation, the incident field is localized to the scatters throughout the computation space and generates a dense right-hand side vector. While in the case of total field formulation, a function is typically defined that collocates an impressed source with an analytic (typically either first- or second-order) absorbing boundary condition on the boundary surface, thus the right-hand side vector is sparse owing to only minor unknowns on the boundary surface. For the sake of alleviating the computational burden of compressing and recovering in CS procedure, we choose the total field formulation.

The typical geometry of interest of FEM is shown in Fig.1, the surface S_{ab} defines the termination of the overall finite element region, and on this surface, the boundary value problem satisfies the Sommerfeld condition[2] given by (2.1)

$$\hat{\mathbf{r}} \times (\nabla \times \mathbf{E}^{sc}) = -jk_0 \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{E}^{sc}) \quad , \quad r \in S_{ab} \quad (2.1)$$

where $\mathbf{E^{sc}}$ is the scattered electric field. According to $\mathbf{E}^{sc} = \mathbf{E} - \mathbf{E}^{inc}$, the formula (2.1) can be written as

$$\begin{aligned} &\hat{\mathbf{r}} \times (\nabla \times \mathbf{E}) + jk_0 \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{E}) \\ &= \hat{\mathbf{r}} \times (\nabla \times \mathbf{E}^{inc}) + jk_0 \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{E}^{inc}) \end{aligned} \quad , \quad r \in S_{ab} \quad (2.2)$$

The total-field inside the FEM region satisfies the second-order wave equation derived from Maxwell equations as

$$\nabla \times \frac{1}{\mu_r} \nabla \times \mathbf{E} - k_0^2 \varepsilon_r \mathbf{E} = 0 \quad , \quad r \in V_t \quad (2.3)$$

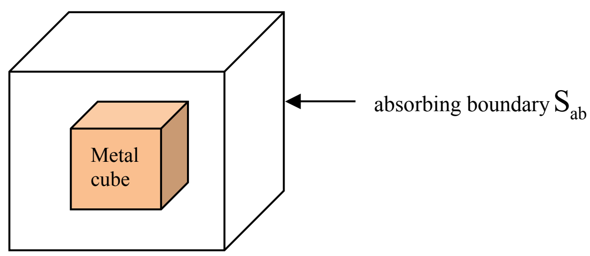

Figure 1. FEM domain

In a finite-element-based solution, the weak form of (2.3) can be obtained by Ritz procedure with edge-element base functions

$$\int_{V_t}\left\{\frac{1}{\mu_r}(\nabla\times\mathbf{N})\cdot(\nabla\times\mathbf{E})-k_0^2\varepsilon_r\mathbf{N}\cdot\mathbf{E}\right\}dV_t$$
$$-\int_{S_{ab}}\frac{jk_0}{\mu_r}\left[(\hat{\mathbf{n}}_{S_{ab}}\times\mathbf{N})\cdot(\hat{\mathbf{n}}_{S_{ab}}\times\mathbf{E})\right]dS_{ab}$$
$$=\frac{-1}{\mu_r}\int_{S_{ab}}\mathbf{N}\cdot\left[\hat{\mathbf{n}}_{S_{ab}}\times(\nabla\times\mathbf{E}^{inc})+jk_0\hat{\mathbf{n}}_{S_{ab}}\times(\hat{\mathbf{n}}_{S_{ab}}\times\mathbf{E}^{inc})\right]dS_{ab}$$

III. Compressed Sensing Theory

The core idea of CS theory contains three aspects, sparse representation, measurement matrix and reconstruction algorithms. Firstly, if the original signal $X\in\mathbb{R}^N$ is sparse or compressible based on the orthogonal basis Ψ like the Fourier or wavelet coefficients of smooth signal etc. [11], we need to work out transform coefficient Θ,

$$\Theta=\Psi^T X. \qquad (3.1)$$

Secondly, to ensure its convergence, a measurement matrix Φ should be given with dimension of $M\times N$ based on irrelevance of Ψ, which makes matrix $\mathbf{A^{CS}}$

$$A^{CS}=\Phi\Psi^T \qquad (3.2)$$

to satisfy Restricted Isomerty Property (RIP)[12].Common choices of measurement matrix Φ include Gaussian random matrix[13], Bernoulli matrix[13] etc.. Gaussian random matrix has a series of advantages that the entries independently subject to a distribution, and it is irrelevant to most sparse signals, the number of measurements for accurate recovery is smaller. Then the measurement set from the function

$$Y=\Phi\Theta=\Phi\Psi^T X \qquad (3.3)$$

can be obtained.

At last, l_0-regularization or l_1-regularization methods are used to acquire the approximation or the precise vector $\bar{X}$ of the original signal X, which is the sparsest vector based on Ψ, that is

$$\min\left\|\Psi^T X\right\|_{0or1} \qquad s.t. \qquad A^{CS}X=\Phi\Psi^T X=Y$$

Two major approaches of sparse recovery are greedy algorithm and convex programming. The greedy algorithm, like orthogonal matching pursuit (OMP)[14], regularized orthogonal matching pursuit(ROMP)[15], computes the support of X iteratively, finding one or more new elements and subtracting their contribution from the measurement vector Y at each step. Greedy methods are usually fast and easy to implement. Whereas, the convex programming, such as iterative threshold method [16], is to solve a convex program whose minimizer is known to approximate the target signal. OMP is mainly to pick a coordinate of Φ^*Y of the biggest magnitude. While ROMP, a variant of OMP, combines the speed and ease of implementation of the greedy methods with the strong guarantees of the convex programming methods. The flow chart as shown in Fig.2, illustrates the CS procedure.

IV. Numerical Example

To demonstrate the validity and efficiency of the proposed method, we calculate the monostatic RCS of a metal cube with 0.55 wavelength. The incident wave is vertical polarized, and angle θ ranges from 0 to 360.The measurement matrix we choose is a sparse column matrix based on Gaussian random matrix, and the compressed ratio is 0.2. OMP and ROMP are used as the reconstruction algorithms. The results are presented in Fig.3 and Fig.4 compared with point-by-point calculation. It can be seen that both two reconstruction algorithms have a good accuracy.

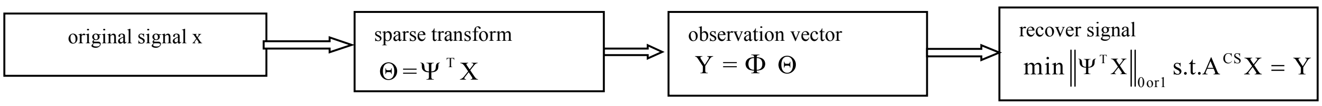

Figure 2. CS theory frame

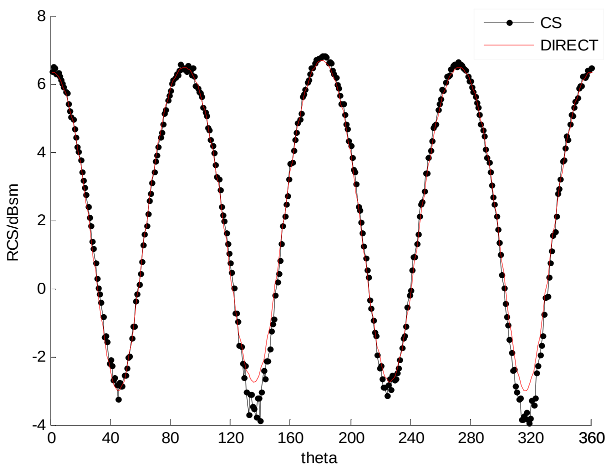

Figure 3. RCS via OMP

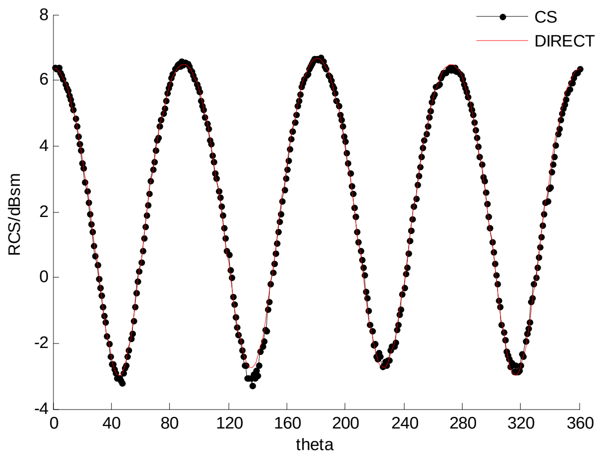

Figure 4. RCS via ROMP

The total CPU time of these two reconstruction algorithms and direct angular sweep method are given in Table I.

TABLE I
CPU TIME FOR DIFFERENT METHODS

Method	Time(s)	Speedup
DIRECT	1339.430	1
CS-OMP	468.826	2.86
CS-ROMP	318.486	4.21

Furthermore, the CPU time distribution of each part using CS_OMP and CS_ROMP are also given in Fig.5 and Fig. 6 respectively.

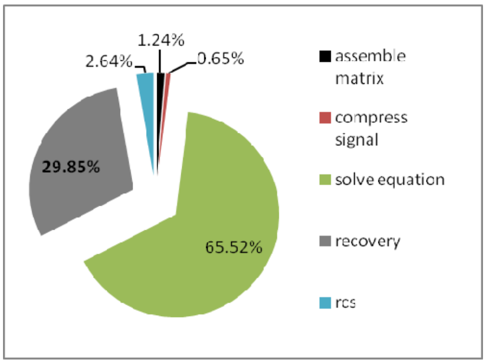

Figure 5. Time distribution chart by OMP

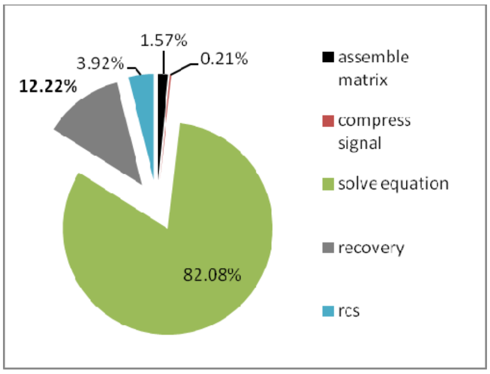

Figure 6. Time distribution chart by ROMP

In addition, relative errors for different algorithms are shown in Table II.

TABLE II
ERROR FOR DIFFERENT ALGORITHMS

Method	Error
CS-OMP	0.0746
CS-ROMP	0.0328

It can be concluded that ROMP shows good characteristics in recovering compared to OMP. In the time distribution chart, the CPU time for solving equation occupies a large proportion, while compression and recovery just take up much smaller part. So there is a foreseeable larger speedup when solving equation becomes a dominant part in whole FEM.

Besides, we also investigated the performance of different measurement matrices based on the same reconstruction algorithm-ROMP. In Fig.7, the recovered RCS by Band, Column, Gauss measurement matrices are presented. The relative error compared to direct method in Table III shows the sparse column matrix is more accurate than the other two.

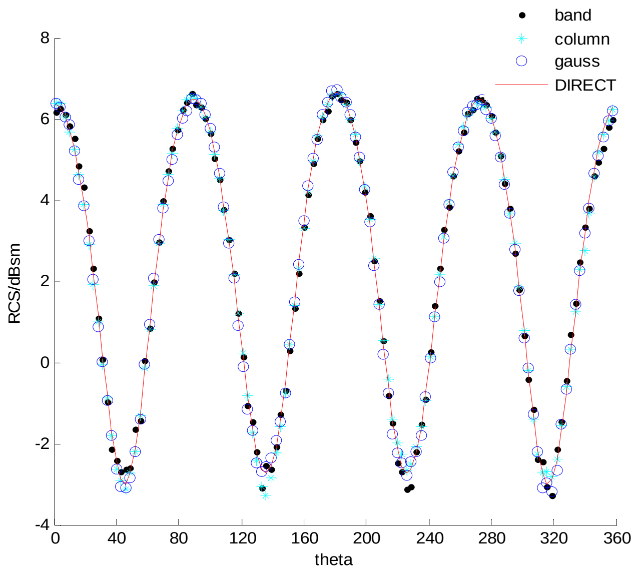

Figure 7. RCS for different measurement matrices by ROMP

TABLE III
ERROR FOR DIFFERENT MEASUREMENT MATRIX

Method	Error
Sparse Column Matrix	0.0328
Sparse Band Matrix	0.0458
Gaussian Random Matrix	0.0338

V. CONCLUSION

In this work, our innovative approach is to apply the novel compressed sensing technique to FEM to calculate RCS over a wide incident angular band. According to the numerical results, if the CPU time cost in solving equation occupies a large part of the total, this new approach could lead to a magnitude speedup. From a certain viewpoint, the proposed method improves the efficiency significantly under the premise of guaranteeing validity.

REFERENCES

[1] Xin-Qing Sheng, Wei Song, *Essentials of Computational Electromagnetics*, 1st ed., Singapore: Wiley, 2012.

[2] J.M.Jin, *The Finite Element Method in Electromagnetics*, 2nd ed., New York: Wiley, 2002.

[3] Ma J.F., Mittra R., Prakash V.V.S.,"Efficient RCS computation for incident angle sweep with the use of the FMM and MNM techniques", *Microwave and Optical Technology Letters*, v 34, n 4, p 273-276, August 20, 2002.

[4] Lu C.C.,"A simple extrapolation method based on current for rapid frequency and angle sweep in far-field calculation of an integral equation algorithm," *IEEE/ACES International Conf. on Wireless Communications and Applied Computational Electromagnetics*, p 333-6, 2005.

[5] Peng Zhen, Sheng XinQing,"Application of asymptotic waveform approximation technique to hybrid FE/BI method for 3D scattering," *Sci. China Ser. F-Inf. Sci*,vol.50,no1,124-134,2007.

[6] Candès, E.J., Romberg JK, Tao T.," Stable signal recovery from incomplete and inaccurate measurements," *Communications on pure and applied Mathematics*, 2006, 2107-1223.

[7] D L.Donoho,"Compressed sensing," *IEEE Trans. on Information Theory*,2006, 52(4):1289-1306.

[8] S Bhattacharya, T Blumensath, B Mulgrew,"Fast encoding of synthetic aperture radar raw data using compressed sensing," *IEEE Workshop on Statistical signal Processing, Madison,Wisconsin*,2007.448-452.

[9] W Bajwa, J Haupt, A Sayeed, "Compressive wireless sensing," *Proceedings of the fifth International Conference on Information Processing in Sensor Networks，IPSN'06*, *New York：Association for Computing Machinery*, 134-142, 2006.

[10] Haldar, J.P.,Hernando D. ,Zhi-Pei Liang, **"**Compressed-Sensing MRI with Random Encoding," *IEEE Transactions on Medical Imaging*, 2011.

[11] E J Candès, Tao, "Near optimal signal recovery from random projections Universal: encoding strategies," *IEEE Trans. on Information Theory*,2006, 52(12):5406-5425.

[12] Candès E J, "The Restricted Isometry Property and its implications for compressed sensing," *Academie des sciences*, 2006, 346(I):598-592.

[13] Emmanuel Candès, Justin Romberg, Terrence Tao,"Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. on Information Theory*,2006, 52(12):489-509.

[14] J A Tropp,A C Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," April, 2005, www-personal.umich.edu/_jtropp/papers/TG05-Signal-Recovery.pdf.

[15] Deanna Needell, Roman Vershynin, "Signal Recovery from Incomplete and Inaccurate Measurements via Regularized Orthogonal Matching Pursuit," *IEEE Journal of selected topics in signal processing*, vol. 4, no. 2, April 2010.

[16] I Daubechies, M Defrise, C De Mol," An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," *Comm. Pure. Appl. Math.*, 2004, 57(11):1413-1457.