An Efficient 3D DI-FDTD Method for Anisotropic Magnetized Plasma Medium

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***Abstract*—An efficient three-dimensional (3D) direct integration (DI) finite-difference time-domain (FDTD) method is introduced to solve electromagnetic wave propagation problems in anisotropic magnetized plasma medium using matrix method. Compared with previous 3D FDTD plasma method, our method is more flexible in adjusting medium parameters thus can deal with varied mediums. Through several simulations, the correctness of the method are proven and the advantages are clarified. Finally, potential applications of this method are discussed, such as the further study of the ionosphere.**

I. Introduction

Accurate simulation of electromagnetic (EM) wave propagation in plasma medium is often a difficult problem for its dispersive and anisotropic properties. However, an time domain approach proposed by Luebbers [1] presented a frequency-dependent formulation for transient propagation in plasma using the finite-difference time-domain (FDTD) method [2], which has become an efficient solution to such problems.

After that, the time domain numerical methods, instead of analytic ones, have become the major methods to solve these problems and many improved techniques were proposed. Some of the techniques [1, 3] are based on a difference approximation of Maxwells equations coupled to an iteration derived from the convolution integral form of the auxiliary differential equation, which are called recursive convolution (RC) methods. Some other techniques [4] are based on direct finite-difference approximations of the complete field equations of the medium, which are commonly referred to as direct integration (DI) methods. A systematic analysis of these techniques was reviewed by S. A. Cummer [5].

Among these techniques, the DI-FDTD method has much simpler forms and the accuracy is close to traditional RC-FDTD method, thus many two-dimensional (2D) and three-dimensional (3D) DI-FDTD methods were developed in the recent years.

Due to the existence of the geomagnetic field, the ionosphere surrounding the Earth becomes a gyrotropic plasma medium. Many plasma algorithms were applied to model the EM wave propagation in Earth-ionosphere system. An initial FDTD scheme that can deal with such an anisotropic medium was presented by Thèvenot [6], allowing the simulation of radiowave propagation in the Earth-ionosphere waveguide using a 2D spherical-coordinate FDTD method. Cummer [7] also reported a 2D FDTD method to simulate EM wave propagation in the Earth-ionosphere waveguide but using the cylindrical-coordinates. A 3D FDTD method was proposed by Lee [8] to study the transformation of an EM wave by a dynamic (time-varying) inhomogeneous magnetized plasma medium. The current density vector of such method is positioned at the center of the Yee cube to accommodate the anisotropy of the plasma medium. Nevertheless, it is only first-order accurate compared to another E-J collocated 3D FDTD method [9, 10] which is more accurate and has less memory-cost. However, the method is still not so perfect regarding the calculating time and computer memory.

In this paper, we have improved Yu and Simpson's 3D E-J collocated FDTD method [10] to a more flexible one, in which the parameter limits to maintain its accuracy are not as restrict as before. The robustness of parameter matrix is discussed to prove the method useful in a wide variety of fields. In Section II, the governing equations for the 3D magnetized cold plasma are described, as well as the the resulting FDTD time-stepping algorithm. Section III illustrate numerical examples of the 3D plasma FDTD method in which both unmagnetized and magnetized plasma cases are provided. Finally, Section IV concludes the paper and forecasts the future applications of the 3D FDTD method.

II. Approach

A. Governing Equations

In this paper, plasma mediums are assumed anisotropic and the method is based on a 3D Cartesian coordinate. Wave propagation effect introduced by electrons, positive ions and negative ions are included for generality. An extra magnetic flux density $\boldsymbol{B}$ is set here to simulate natural geomagnetic field. The governing equations of anisotropic magnetized cold plasma consist of three Lorentz current equations derived from Lorentz equation of motion which modeling the response of each charged particle species to the electric field $\boldsymbol{E}$ and the extra magnetic flux density $\boldsymbol{B}$, as well as two Maxwell's curl equations including total induced current density $\boldsymbol{J}_I$ and source current density $\boldsymbol{J}_S$. The whole governing equation is given by

$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J}_I + \boldsymbol{J}_S \tag{2}$$

$$\frac{\partial \boldsymbol{J}_e}{\partial t} + \upsilon_e \boldsymbol{J}_e = \varepsilon_0 \omega_{Pe}^2 \boldsymbol{E} + \boldsymbol{\omega}_{Ce} \times \boldsymbol{J}_e \tag{3}$$

$$\frac{\partial \boldsymbol{J}_p}{\partial t} + \upsilon_p \boldsymbol{J}_p = \varepsilon_0 \omega_{Pp}^2 \boldsymbol{E} - \boldsymbol{\omega}_{Cp} \times \boldsymbol{J}_p \tag{4}$$

$$\frac{\partial \boldsymbol{J}_n}{\partial t} + \upsilon_n \boldsymbol{J}_n = \varepsilon_0 \omega_{Pn}^2 \boldsymbol{E} + \boldsymbol{\omega}_{Cn} \times \boldsymbol{J}_n \tag{5}$$

$$\boldsymbol{J}_I = \sum_l \boldsymbol{J}_l = \boldsymbol{J}_e + \boldsymbol{J}_p + \boldsymbol{J}_n \tag{6}$$

Here the subscript l indicates the charged particle species in the plasma (e, p and n as electrons, positive ions and negative ions, respectively). $\boldsymbol{J}_e$, $\boldsymbol{J}_p$ and $\boldsymbol{J}_n$ are the current densities of each species. υ_e, υ_p and υ_n are the collision frequencies of each species. In addition, ω_{Pe}, ω_{Pp} and ω_{Pn} shows the plasma frequencies of each species with detailed structure in (7).

$$\omega_{Pl} = \sqrt{\frac{q_l^2 n_l}{\varepsilon_0 m_l}} \tag{7}$$

Further, $\boldsymbol{\omega}_{Ce}$, $\boldsymbol{\omega}_{Ce}$ and $\boldsymbol{\omega}_{Ce}$ are the cyclotron frequencies of each species given by (8).

$$\boldsymbol{\omega}_{Cl} = \frac{q_l \boldsymbol{B}}{m_l} \tag{8}$$

It is clear that the cyclotron frequency is a function of magnetic flux density $\boldsymbol{B}$ so that if the cross-product terms in (3)–(5) is set to zero, the whole governing equations will reduce to isotropic, in other words, the wave behavior is independent of its propagation direction.

The complete scalar equations derived from equations (1)–(6) are shown in [10].

B. FDTD Discretization Scheme

Take the scalar equations into the FDTD grids. J components locate at the same positions of E ones. Central differencing of the space derivatives is employed to transform them to the update equations which is easy for computing. On time derivatives, J and E components are supposed to be at the integer timesteps, indicated as n, while H components are at the semi-integer timesteps, indicated as $(n + 1/2)$.

Thus, the semi-implicit equations come with a new problem that each of E and J components must be iterated simultaneously. Three parameter matrices are then introduced to solve this problem perfectly. The approach used here is very similar to the matrix method described in [10] considering in detail the spatial averaging but without value scaling of $\tilde{H}_u = (\mu_0 \Delta u/\Delta t) H_u [u = x, y, z]$ and $\tilde{J} = (\Delta t/\varepsilon_0) J$. Matrix A and B represent the parameter coefficients of the E and J components at the present and previous timestep and matrix C represents the coefficients of H and J_S components at previous timestep. However, the matrix C used in Yu's method could be simplified as each two derivatives of H components in third column could be grouped so that the storage of parameter matrix is reduced. The last term of the equation array is described by

$$\begin{bmatrix} \frac{\Delta t}{\varepsilon_0} & & & -\frac{\Delta t}{\varepsilon_0} & & \\ & \frac{\Delta t}{\varepsilon_0} & & & -\frac{\Delta t}{\varepsilon_0} & \\ & & \frac{\Delta t}{\varepsilon_0} & & & -\frac{\Delta t}{\varepsilon_0} \\ \vdots & & & & & \vdots \\ 0 & & \cdots & \cdots & & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{\Delta H_z}{\Delta y} - \frac{\Delta H_y}{\Delta z} \\ \frac{\Delta H_x}{\Delta z} - \frac{\Delta H_z}{\Delta x} \\ \frac{\Delta H_y}{\Delta x} - \frac{\Delta H_x}{\Delta y} \\ J_{Sx} \\ J_{Sy} \\ J_{Sz} \end{bmatrix} \tag{9}$$

where the left side is the simplified matrix C.

C. Stability and Accuracy Analysis of the Scheme

Since the solutions of the semi-implicit differencing equations have a growth per timestep factor of $(1 - \upsilon\Delta t/2)/(1 + \upsilon\Delta t/2)$, most of the plasma FDTD methods require a strict criterion $\upsilon\Delta t \ll 1$ that limits the efficiency of the FDTD method [5]. However, the matrix method avoids such problem because the elements in diagonal of matrix A equals unit or $(1 + \upsilon_l \Delta t/2)$. Whether the criterion $\upsilon\Delta t \ll 1$ is fitted or not, the parameters $A^{-1}B$ and $A^{-1}C$ of iteration equation arrays are valid. Even for magnetized case, spatial grid-cell size is not necessary to be chosen carefully to satisfy the stability condition and accuracy requirements. For the same reason, the value scaling could be canceled to simplify the calculation process.

III. Demonstration

In this Section, the modified 3D FDTD plasma method is applied to simulate EM wave propagation in a space with a spheric plasma object inside. The uniaxial perfectly matched layer (UPML) boundary condition [11] is used here to absorb the wave outside the calculation region. All the E, H and J field components are discretized in x, y and z directions indicated i, j and k as their subscripts. $\Delta x = \Delta y = \Delta z = 2$ mm and $\Delta t = 3.3333$ ps. A pair of z-polarized differential Gaussian pulse dipole is generated as the wave source of our numerical demonstrations.

A homogeneous spheric plasma object is set inside the calculation space characterized by
electron plasma frequency:

$$\omega_{Pe} = 1.8 \times 10^{11} rad/s \tag{10}$$

and neutral-electron collision frequency:

$$\upsilon_e = 2 \times 10^{11} rad/s \tag{11}$$

Fig. 1 illustrates the snapshot of electric field E_z distribution after the pulsed wave propagating for 100 timesteps. The magnitude of all values is normalized. No extra magnetic flux density is applied here, in other words, $\boldsymbol{B} = 0$. In this case, $\upsilon\Delta t < 1$ is achieved to meet the fundamental requirement of traditional plasma DI-FDTD algorithm.

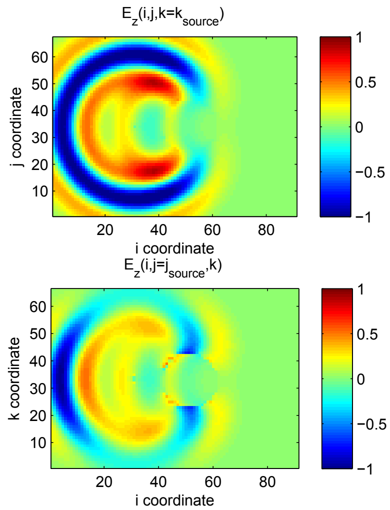

Fig. 1: Snapshot for pulsed wave propagation with a spheric region of unmagnetized plasma (a) upper panel: xOy-plane (b) lower panel: xOz-plane

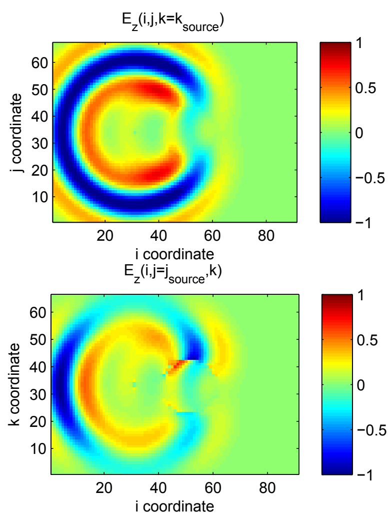

Fig. 2: Snapshot for pulsed wave propagation with a spheric region of magnetized plasma (a) upper panel: xOy-plane (b) lower panel: xOz-plane

We then apply an extra magnetic flux density $\boldsymbol{B}$ as $B_x = B_y = B_z = 2$ T in order to simulate the wave propagation through anisotropic magnetized plasma medium.

Fig. 2 shows the snapshot for the magnetized plasma medium case. The influence of the geomagnetic field is clear compared to Fig. 1 for its effect on wave rotation.

The final demonstration is shown in Fig. 3 as an example of particular case in which $v\Delta t$ is significantly greater than unit. We suppose that $\Delta = 60$ mm and $\Delta t = 100$ ps and other condition keeps the same as the first case so that $v\Delta t = 20$ which is not under the circumstance of $v\Delta t \ll 1$. The result shows that the improved method is still stable and accurate.

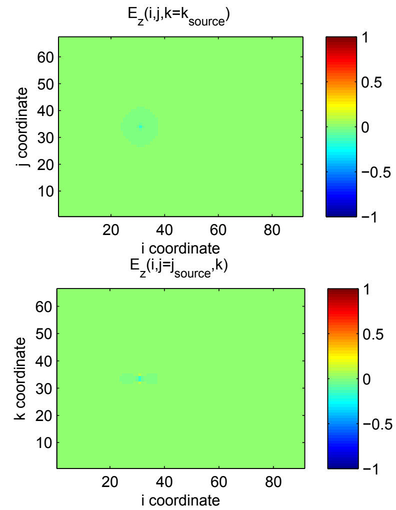

Fig. 3: Snapshot for pulsed wave propagation with a spheric region of unmagnetized plasma under the circumstance of $v\Delta t = 20$ after 10000 timesteps (a) upper panel: xOy-plane (b) lower panel: xOz-plane

IV. Summary and Conclusion

In this work, an efficient DI-FDTD method including anisotropic magnetized plasma medium is presented using the matrix method. The algorithm involves direct integration of current density term in semi-implicit equation arrays. Since the particular structure of the matrix, the parameter of the plasma is more flexible than that of traditional plasma FDTD methods.

Its validity is demonstrated by calculating the wave propagation of pulse generated from a pair of polarized dipole. The results of three simulations under different conditions, even over the limit of tradition algorithm, agree well with the expectation. Therefore, for its nice property in low frequency (Δt is large), the method is supposed to be efficient to a wide variety of applications such as the study of wave propagation in ionosphere.

Since ionosphere is known to be an anisotropic plasma medium to certain extent due to the geomagnetic field, the method presented in this paper may play a role in future study coupling to real models of the Earth-ionosphere system [12].

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