Optimum Decision Rule for Fixed Relay Pairing Selection Application

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1. Introduction

Relay technologies have been recently studied and creatively incorporated in the next-generation mobile broadband communications systems (4G;WiMAX IEEE802.16j/m and LTE-Advanced standards) where the multi-hop radio link is to improve the transmission throughput and reduce the outage probability, especially the mobile station in the brink of cell-edge and shadowing area [1, 2, 3]. The relay selection presented in [4, 5, 6] is primarily based on the joint selection criteria of instantaneous signal-to-noise power ratio across the hops for the relay links. However, for fixed relay application, the radio links between relay nodes (R) and destination (D) of the second hop are assumed to be static fading channels which have deterministic channel statistics and same mean SNR value with amplify-and-forward (AF) protocol at relay nodes. In practice, this is true when R location is geographically fixed with a directional antenna to D (i.e. stationary sites; basestation) that also potentially reduces the transmission interference level. In the contrary, the links between source (S) and R in the first hop are usually treated as a stochastic fading channel (i.e. time-variant Rayleigh fading channel) due to its random walk mobility and unlimited extent rage from S (i.e. mobile unit). In our paper, a simple and effective relay selection method is explored for the two-hop fixed multi-relay wireless system using multiple-likelihood decision concepts. Fig.1 shows our proposed fixed M-relay network topology with Bayes optimum decision algorithm. We assume independent and identically distributed (i.i.d.) Rayleigh fadings are across all S-R and R-D relay hopes, as well as the channel gain form S-D direct link is weak enough to be negligible (no diversity gain). Meanwhile, perfect channel state information (CSI) is assumed for the channel statistics input to the optimum decision algorithm at the D. Hence, our relay selection is primarily dependent on the maximum channel power gain distribution of the first relay hop from the multiple relaying links, not on the comparative instantaneous signal-to-noise power from end-to-end link. Finally, the maximum channel power gain distribution and average Bit-Error-Rate (BER) performance are simulated against various Doppler effects on the first hop channels to validate our optimum selection criterion.

2. Optimal selection criterion

In previous selections based on the channel power gain [4, 5], the selection of the relay link is jointly performed as following;

$$L_m = \min\left(\chi_{\mathrm{L}_{12}\cap\mathrm{L}_{11}}, \chi_{\mathrm{L}_{22}\cap\mathrm{L}_{21}}, \;\; ...., \quad \chi_{\mathrm{L}_{M2}\cap\mathrm{L}_{M1}}\right) \qquad (1)$$

$$\Omega_m = \max\left(L_m\right) \;\; \text{for m=1,2,…, M} \qquad (2)$$

where $\chi_{L_{m2}\cap L_{m1}}$ is the instantaneous SNR across the first and the second hops (end-to-end link), less than threshold value χ_0; $\chi_{L_{m2}\cap L_{m1}} < \chi_0$ and χ_0 can be a QoS index defined in the medium access control (MAC) layer. The weaker links between the first hop and second hop, L_m, are ordered, and the one with the highest SNR, Ω_m, is selected. It assumed M relay nodes in which both S-R and R-D links experience independent and identically distributed (i.i.d.) Rayleigh fading so that the instantaneous channel gains, r_{m1} and r_{m2}, are random variables with Rayleigh probability distribution with mean values, $\bar{r}_{m1}$ and $\bar{r}_{m2}$, respectively, and corresponding $|r_{m1}|^2$ and $|r_{m2}|^2$ are the channel power gains of the first and second selected relay hops. The instantaneous SNR, $\chi_{L_{m1}} = SNR_1|r_{m1}|^2$ and $\chi_{L_{m2}} = SNR_2|r_{m2}|^2$, are measured at the first and the second hops for m = 1, 2, …, M respectively. These optimum links are then ordered in the basis of the weaker link L_m and the one with the highest instantaneous SNR, $\chi_m = \max_{m=1,2,..,M} \lfloor \chi_{L_{m2}\cap L_{m1}} \rfloor$, is selected for relay link. For the two-hop system, the end-to-end SNR of relay link is given as $\chi_{L_{m2}\cap L_{m1}} = f(\chi_{L_{m1}}, \chi_{L_{m2}})$ and $f(x,y) = \frac{xy}{x+y+1}$ is approached in [7]. Ω_m is selected as the candidate to relay the source information to the destination. Hence, the probability density function (pdf) of the link Ω_m is given by [5]

$$\mathrm{P}_{\Omega_m}(\chi_m) = \mathrm{M}\left[1-\exp\left(\frac{-\chi_m}{\chi_s/2}\right)\right]^{\mathrm{M}-1} \cdot \exp\left(\frac{-\chi_m}{\chi_s/2}\right) \Big/ \chi_s/2 \tag{3}$$

where $\chi_s = E[\chi_m]$ the average SNR value of the selected link. Using AF fixed relay, the relay amplifier gain is assumed known at destination and given by $G \cong \sqrt{1/(P_S|r_{m1}|^2)}$ with the transmitted source power P_S, to satisfy the power constraint at the relay node. Hence, the average channel gain threshold g_{m2} is assigned to each relay link where the second hops with the average channel gain with $\bar{r}_{m2} \geq g_{m2}$ are selected for candidate.

In our selection criterion, the joint pdf of the selected relay link at relay-*m*, L_m, can be expressed as joint probability $P_{rob}(L_m) = P_{r_{m2}}(\chi_{L_{m2}}) \cdot P_{r_{m1}}(\chi_{L_{m1}})$ for $m = 1,2..,M$. Note that $P_{r_{m1}}$ and $P_{r_{m2}}$ are characterized as Rayleigh pdf's corresponding to hops r_{m1} and r_{m2} respectively. For $\bar{r}_{m2} \geq g_{m2}$, the joint pdf of individual relaying link of L_m is ,therefore , given by

$$\mathrm{P}_{rob}(\mathrm{L}_m) = \left[1-\exp\left(-\chi_{\mathrm{L}_{m2}}/\bar{\chi}_{\mathrm{L}_{m2}}\right)\right] \cdot \mathrm{P}_{r_{m1}}(\chi_{\mathrm{L}_{m1}}), \quad \mathrm{m} = 1, 2, \ldots, \mathrm{M} \tag{4}$$

The average $\chi_{L_{m2}}$ of the individual R-D links has same mean channel power gain for all the relay nodes if R-D nodes are in stationary and pre-determined by AF relay. Hence, the selection relay link simply depends on the random variable r_{m1}^2 in which the maximum power gain distribution is achieved by using optimum decision algorithm. Recall (2), the selection criterion can be rewritten as

$$\tilde{\Omega}_m = \max\left\{P_{r_{m2}} \cap \int_{R_{m1}} P_{r_{m1}}(r)\,dr\right\},\ r \in R_{m1} \text{ and m=1, 2, .. ,M} \tag{5}$$

Noted that R_{m1} is the optimum range of the channel gain r_{m1}, and $P(r_{m1}) = \max_{r \in R_{m1}} \int_{R_{m1}} P_{r_{m1}}(r)\,dr$ has the maximum channel power gain distribution. Those optimum ranges, $R_{m1} = \{R_{11}, R_{21}, ..., R_{M1}\}$, are obtained using Bayes decision algorithm and performed every data block length (time window) β. In other words, the selection (5) is primarily determined from the M possible hypotheses in which one of the first hops *maximizes the channel power gain distribution*.

3. Optimal Bayes Decision Algorithm

For M relay stations to a specific source terminal, an extended Bayes decision rule [8] for a 1-by-M multiple links is observed over *M*-likelihood receiving relay node. The *average cost* for the Bayes decision is therefore selection of the optimal channel range, R_{m1}, for the channel gain, r_{m1}, such that the *average cost*, $\hat{C}$, is minimized [8],

$$\hat{C} = \sum_{k=1}^{M}\sum_{m=1}^{M} C_{k1,m1} P(deciding\ \ L_{k1}/L_{m1}\ ) P(L_{m1}) \tag{6}$$

where $P(deciding\, L_{k1}/L_{m1}\ )$ is the probability of deciding link L_{k1} given that L_{m1} is the correct selection link and can be written as

$$P(deciding\ L_{k1}/L_{m1}\ ) = \int ...\int_{R_{k1}} p(l / L_{m1}) dl \tag{7}$$

Substituting (7) into (6), there are M integrals can be defined as a new function [8]

$$y_{k1}(l) \cong \sum_{m=1}^{M} C_{k1,m1}\, p(l / L_{m1}) P(L_{m1}), \quad m = 1,2,...,M \tag{8}$$

The variance of channel gain, $\sigma^2_{r_{m1}}$, are calculated every data block length β to fulfil the input of $p(l / L_{m1})$ in (8). Equation (6) can be minimized if the optimum channel ranges R_{m1} are selected in the following way:

If $y_{m1}(l) < y_{k1}(l)$ then $l \in R_{m1}$ for all $k = 1, 2, \ldots, M$ and $k1 \neq m1$ (9).

Then, the channel variable l belongs to the optimum channel range R_{m1} (i.e., $l = r_{m1}$) via the relay link L_{m1}. By using a number of assumptions, $\hat{C}$ can be further simplified as follows;

1) A priori probabilities and conditional probability density functions; the statistical properties related to the M-hypotheses of relay stations can be categorized into the conditional probability density function, $P(l/L_{m1}\,)$, for channel gain variable l and a priori probability $P(L_{m1}\,) = {}^{1}\!/\!_{M}$.

2) Cost factors; associated with the decision of channel gain selected from the relay hop *k1* that it is from relay hop *m1*. A zero-one cost assignment is considered here that all costs for errors being 1 and all costs for correct decision being zero, as follows:

$$error\ \ decision: C_{k1,m1} = 1 \quad for \quad k1, m1 = 11,\ \ 21,\ \ ..., M1;\ \ k1 \neq m1$$
$$correct\ \ decision: C_{k1,m1} = 0\ \ for \quad k1 = m1$$

The optimum decision regions, $R_{11}, R_{21}, R_{31}, ..., R_{M1}$, corresponding to the first relay hops, are therefore given using (9). They are selected to be mutually exclusive and exhaustive, with a measured boundary range over three times the standard deviation of the channel gain $[0 \leq R_{11} \cup R_{21} \cup R_{31} \cup ..R_{M1} \leq 3r_0]$, corresponding to the probability of exceeding the Rayleigh envelope by one percentage (1%).

4. Simulations and Conclusion

Computer simulations were performed with following system parameters: number of the relay nodes M = 4, length of data block length β = 100, normalized Doppler frequencies $f_D T_S$ = 0.01, 0.02, 0.05 and 0.1 for characterizing the first hop channels. The parameter f_D is the maximum Doppler frequency, and T_S is the symbol duration (sampling data). The Jakes model [9] was employed to generate four i.i.d. complex Rayleigh fading channels with normalized Doppler frequencies for simulation. Fig.2 shows one example of simulation results, where the relay link r_{11} has maximum channel power gain distribution amongst relay nodes; R-1 is selected as the relay link. In Fig. 3, it shows the average BER performance is measured in terms of these channel parameters with QPSK modulating signals. For each relay link, the BER performances vs. SNR values were averaged over 1000 Monte-Carlo simulations. That shows, the maximum channel power gain distribution, $P(r_{11})$, occurs most likely at the first hop channel r_{11} which has less Doppler effect. It is noted that the relay node is selected and updated every 100 data block length (time window) and switch to the other relay node with better channel power gain if it is possible. This paper proposed a simple and effective relay selection method incorporated with an optimum decision rule for a two-hop fixed multi-relay network. This application is also emphasized on both relay nodes and destination in the stationary conditions.

Figures

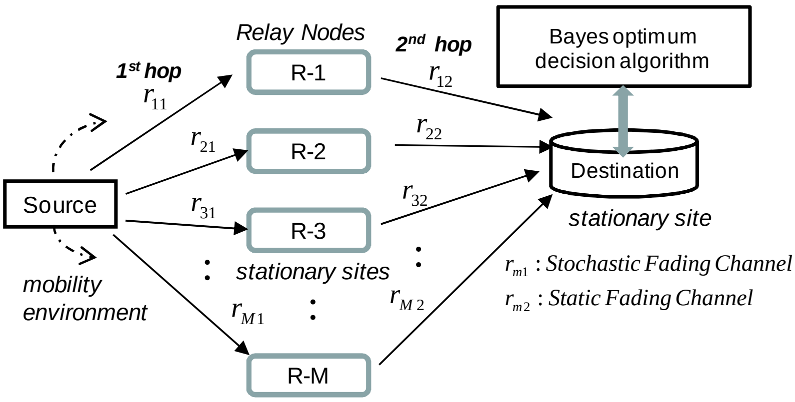

Fig.1 A two-hop fixed multi-relay wireless system incorporated with an optimum decision algorithm

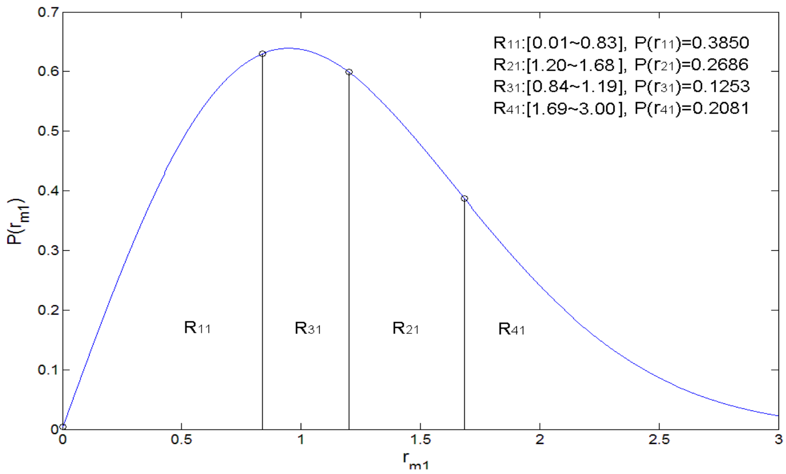

Fig.2 One measured channel power gain distribution vs. optimum decision regions of the first hop channels r_{11} , r_{21}, r_{31} and r_{41} and with normalized Doppler frequencies $f_D T_S$ =0.01, 0.02, 0.05 and 0.1, respectively. The relay hop r_{11} gives the maximum channel power gain over the Rayleigh pdf in accordance to an instantaneous channel conditions.

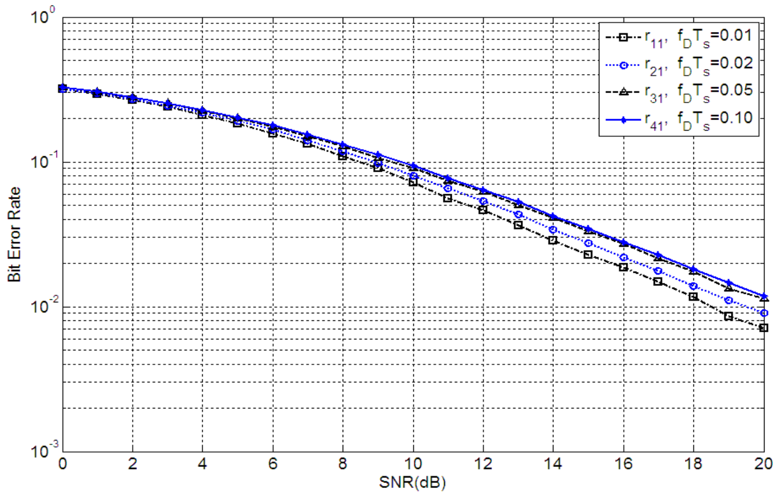

Fig.3 BER performance of the QPSK vs. SNR for the individual first relay hop (M = 4) with various Doppler frequencies in the Rayleigh fading channels.

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