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Linear and passive control method for the steady state amplitude in a parametrically excited hinged-hinged beam

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Abstract—Linear and passive control method for the steady state amplitude in a parametrically excited hinged-hinged beam is proposed theoretically. In general, the magnitude of steady state amplitude in parametrically excited system is determined by the effect of cubic nonlinearity in the system. By changing the boundary condition due to the attachment of a linear spring, we modify the magnitude of the cubic nonlinearity in the lateral direction and control the response amplitude of the parametrically excited hinged-hinged beam.

1. Introduction

There have been many researches on the parametric resonance because of the characteristic response behavior different from the resonance under the linear external excitation; for example, the steady state amplitude is determined by the inherently existing nonlinearity in the system, the trivial response is destabilized in parameter regions and at the boundaries, pitchfork bifurcations occur [1]. Recently, these features of the parametric resonance becomes attractive to enhance the performance of the mechanical signal processing devices, energy harvesters, and so on. As resonators in many microelectromechanical systems, the linear external resonance has been utilized to date. Recently, an application of the parametric resonance was proposed to MEMS oscillator [2]. Mechanical filters based on parametric excitation is theoretically and experimentally proposed [3]. The parametric resonance is produced only in some excitation frequency bands. In particular, the band-pass filter has the distinct advantage by using such a feature of parametric resonance. As one of difficulties, it is indicated that because the nontrivial responses can exist outside of the passband due to hysteresis and overhang, the detection of the boundaries of the passband is difficult. Furthermore, the parametric resonance has advantages in mass sensing and overcomes the limitation of sensitivity under the conventional linear external excitation in the measurement environments with the low Q factor [4]. The shift of the unstable region can be measured without effect of the viscosity by the modulation of the natural frequency in the case when the measurement mass is attached to the resonator. Also in this application of parametric resonance, the detection of the boundary of the unstable region determines the accuracy of the mass sensing. The hysteresis and overhang make it difficult to detect the boundary of the unstable region and to overcome this difficulty, some methods are proposed [5]. As one more application of the parametric resonance, the energy harvester can be mentioned [6]. In this paper, in order to carry out the above mentioned enhancement of filter, mass sensor, and energy harvester in the application of parametric resonance, we propose a amplitude control method in which the frequency response curve is suitably modified for the applications.

We consider a hinged-hinged beam under the periodic excitation in the axial direction. When the excitation frequency is in the neighborhood of twice the natural frequencies, parametric resonance occurs in the beam. The frequency response curve is affected by the nonlinearity in the system; in the case of hinged-hinged beam, the nonlinear curvature [7, 8, 9] determines the frequency response curve as soft-spring type. Therefore, the lower boundary of the unstable region is difficult to be detected because the discontinuous subcritical pitchfork bifurcation is produced and the overhang exists in the lower excitation frequency range [1]. We compensate the soft-spring nonlinearity by a linear spring attached to the supporting point in the axial direction and decrease the soft-spring characteristics to increase the response amplitude. As a result, the slope of the frequency response curve becomes large and the overhang in the frequency response curve is decreased.

2. Analytical Model and Equation of Motion

We consider a hinged-hinged beam subjected to periodic excitation in the axial direction as shown in Fig. 1(a). The excitation frequency is in the neighborhood of twice a natural frequency of the beam. We propose a method to change the frequency response curve to decrease the inherent softening characteristics due to the nonlinear curvature of the beam. As shown in Fig. 1(b), we attach a linear spring to the axial movable supporting point in the axial direction so that the linear spring is in the natural length in the state without lateral deflection of the beam. As a result, the linear spring affects the linear stiffness of the beam, but can modify only the nonlinear stiffness.

Following to the previous studies [8, 9], we introduce coordinate system as Fig. 2 to derive the governing equation

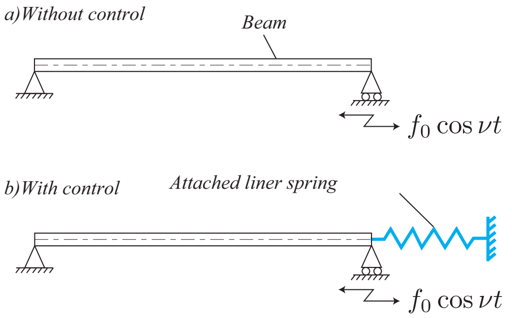

Figure 1: Parametrically excited hinged-hinged beam and amplitude control method by linear spring

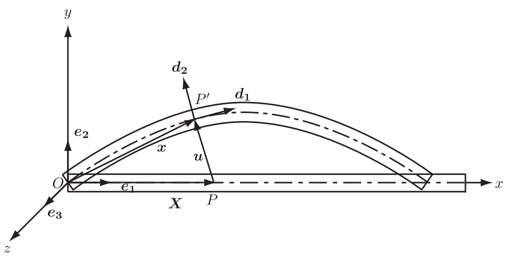

Figure 2: Co-ordinate system to derive the governing equation

of motion. $\boldsymbol{e_i}(i = 1, 2, 3)$ is Cartesian co-ordinate system, where $\boldsymbol{e_3} = \boldsymbol{e_1} \times \boldsymbol{e_2}$. The cross-section whose location P is expressed with vector $\boldsymbol{X}$ is moved to the point P' expressed with vector $\boldsymbol{u}$. Orthonormal basis or director is $\boldsymbol{d_i}(i = 1, 2, 3)$. The position vectors of $\boldsymbol{X}(x)$ and $\boldsymbol{x}(x, t)$ at the points, P and P', have the following relationship:

$$\boldsymbol{X}(x) = x\boldsymbol{e_1} \tag{1}$$

$$\boldsymbol{x}(x, t) = \boldsymbol{X}(x) + \boldsymbol{u}(x, t), \tag{2}$$

where

$$\boldsymbol{u}(x, t) = u(x, t)\boldsymbol{e_1} + v(x, t)\boldsymbol{e_2}. \tag{3}$$

Then, the strain is express as follows:

$$\begin{aligned}\epsilon &= \{(1 + u')\cos\theta + v'\sin\theta - 1\}\,\boldsymbol{e_1} \\ &+ \{v'\cos\theta - (1 + u')\sin\theta\}\,\boldsymbol{e_2},\end{aligned} \tag{4}$$

where θ is the rotation of the director vectors, $\boldsymbol{d_i}(i = 1, 2)$. Because the one end is freely moved in the axial direction, we have the constraint as follows:

$$v'\cos\theta - (1 + u')\sin\theta = 0 \tag{5}$$

or

$$\theta = \arctan\frac{v'}{1 + u'}. \tag{6}$$

The governing equation describing u and v are expressed as follows:

$$\begin{aligned}\rho A\ddot{u} &- \left(T' + \kappa\frac{M'}{1+\epsilon}\right)\cos\theta \\ &-\left(\frac{M''}{1+\epsilon} - \frac{\epsilon'}{(1+\epsilon)^2}M' - \kappa T\right)\sin\theta = 0\end{aligned} \tag{7}$$

$$\begin{aligned}\rho A\ddot{v} &- \left(T' + \kappa\frac{M'}{1+\epsilon}\right)\sin\theta \\ &+ \left(\frac{M''}{1+\epsilon} - \frac{\epsilon'}{(1+\epsilon)^2}M' - \kappa T\right)\cos\theta = 0,\end{aligned} \tag{8}$$

where

$$\begin{aligned}&M(x, t) = EI\kappa(x, t), N(x, t) = -M'(x, t), \\ &\kappa = \frac{v'' + u'v'' - u''v'}{(1+\epsilon)^2}.\end{aligned} \tag{9}$$

The tension T is expressed as follows:

$$\begin{aligned}T(x, t) = &-\textstyle\int_l^x \kappa M' dx - \int_l^x b_1 dx + N(l.t)\tan\theta(l, t) \\ &+\big(-m\ddot{u}(l, t) + f_0\cos\nu t - ku(l)\big)/\cos\theta(l, t),\end{aligned} \tag{10}$$

where k is the stiffness of the spring attached to the supporting point to change the nonlinear characteristics. Applying the inextensible condition $\epsilon = 0$, we have the governing equation of the lateral direction as follows:

$$\begin{aligned}&\rho A(1 - v'^2)\ddot{v} + \rho A v'\int_0^x(\dot{v}'^2 + v'\ddot{v}')dx \\ &+\rho A v''\int_l^x\int_0^x(\dot{v}'^2 + v'\ddot{v}')dxdx \\ &-\rho A v''\int_l^x v'\ddot{v}dx + (EIv'v''')|_{x=l}v'' \\ &-v'' f_0\cos\nu t|(1 + \frac{1}{2}v'^2)|_{x=l} \\ &+EI(v''^3 + 3v'v''v''' + v'''') + EIv''\int_l^x v''v'''dx \\ &-mv''\int_0^l(\dot{v}'^2 + v'\ddot{v}')dx \\ &-\frac{1}{2}kv''\int_0^l v'^2 dx = 0.\end{aligned} \tag{11}$$

The associated boundary conditions are

$$v(0) = v''(0) = v(l) = v''(l) = 0. \tag{12}$$

3. Change of Nonlinear Characteristics of Frequency Response Curve by Linear Spring

The dimensionless equation of motion in the dimensionless form is expressed by using the suitable representative values as follows:

$$\ddot{v}^* + \mu^*\dot{v}^* + v^{*\prime}\int_0^{x^*}(\dot{v}^{*\prime 2} + v^{*\prime}\ddot{v}^{*\prime})dx^* - \ddot{v}^* v^{*\prime 2}$$

$$+v^{*\prime\prime}\int_1^{x^*}\int_0^{x^*}(\dot{v}^{*\prime 2}+v^{\prime}\ddot{v}^{*\prime})dx^*dx^*$$
$$-v^{*\prime\prime}\int_1^{x^*}v^{*\prime}\ddot{v}^*dx^*$$
$$+v^{*\prime}(1)v^{*\prime\prime\prime}(1)v^{*\prime\prime}-v^{*\prime\prime}F^*\cos\nu^*t^*(1+\frac{1}{2}v^{*\prime 2}(1))$$
$$+v^{*\prime\prime 3}+3v^{*\prime}v^{*\prime\prime}v^{*\prime\prime\prime}+v^{*\prime\prime\prime\prime}$$
$$+v^{*\prime\prime}\int_1^{x^*}v^{*\prime\prime}v^{*\prime\prime\prime}dx^*-m^*v^{*\prime\prime}\int_0^1(\dot{v}^{*\prime 2}+v^{*\prime}\ddot{v}^{*\prime})dx^*$$
$$-\frac{1}{2}K^*v^{*\prime\prime}\int_0^1 v^{*\prime 2}dx^*=0, \quad (13)$$

$$v^*(0)=v^{*\prime\prime}(0)=v^*(1)=v^{*\prime\prime}(1)=0, \quad (14)$$

where * stands for non-dimensionalized values.

K^* is dimensionless spring constant of the linear spring for control, and m^* is dimensionless mass at the end where the axial motion is not fixed. For simplicity, * is omitted hereafter. It is seen that the effect of the attached linear springproduces the cubic nonlinearity in the lateral direction. As a result, the nonlinear characteristics of the frequency response curve can be modified by this linear spring.

4. Nonlinear Analysis

We assume a uniform expansion of the approximate solution as

$$v=\epsilon v_1+\epsilon^3 v_3. \quad (15)$$

The excitation frequency ν is expressed by using a detuning parameter σ to the deviation of the excitation frequency ν from twice the first natural frequency ω as follows:

$$\nu=2\omega+\sigma$$
$$\sigma=\epsilon^2\hat{\sigma} \quad (16)$$

Also, we introduce two time scales and the order estimation for the parameters as:

$$t_0=t, t_2=\epsilon^2 t, F=\epsilon^2\hat{F}, \mu=\epsilon^2\hat{\mu}. \quad (17)$$

By applying the method of multiple scales, we obtain the equations for each order as:

$$D_0^2 v_1+v_1^{\prime\prime\prime\prime}=0 \quad (18)$$
$$D_0^2 v_3+v_3^{\prime\prime\prime\prime}=-2D_0D_2v_1-\hat{\mu}D_0v_1$$
$$-v_1^{\prime}\int_0^x(\dot{v}_1^{\prime 2}+v_1^{\prime}\ddot{v}_1^{\prime})dx+D_0^2v_1v_1^{\prime 2}$$
$$-v_1^{\prime\prime}\int_1^x\int_0^x(\dot{v}_1^{\prime 2}+v_1^{\prime}\ddot{v}_1^{\prime})dxdx$$
$$+v_1^{\prime\prime}\int_1^x v_1^{\prime}\ddot{v}_1dx-v_1^{\prime}(1)v_1^{\prime\prime\prime}(1)v_1^{\prime\prime}$$
$$+v_1^{\prime\prime}\hat{F}\cos\nu t-v_1^{\prime\prime 3}-3v_1^{\prime}v_1^{\prime\prime}v_1^{\prime\prime\prime}-v_1^{\prime\prime}\int_1^x v_1^{\prime\prime}v_1^{\prime\prime\prime}dx$$
$$+mv_1^{\prime\prime}\int_0^1(\dot{v}_1^{\prime 2}+v_1^{\prime}\ddot{v}_1^{\prime})dx+\frac{1}{2}Kv_1^{\prime\prime}\int_0^1 v_1^{\prime 2}dx \quad (19)$$

The solution of Eq. (18) is

$$v_1=(A(t_2)e^{i\omega t_0}+\bar{A}(t_2)e^{-i\omega t_0})\sin\pi x, \quad (20)$$

where we consider only the first mode. We assume v_3 as

$$v_3=\Phi_3(t_2,x)e^{i\omega t_0}+C.C. \quad (21)$$

and substituting Eq. (21) into Eq. (19), the following solvability condition is enforced:

$$\frac{dB_r}{dt}-(\frac{\sigma}{2}+\frac{\alpha_3}{\omega}F)B_i+\alpha_1\mu B_r+\frac{\alpha_2}{\omega}(B_r^2+B_i^2)B_i=0 \quad (22)$$
$$\frac{dB_i}{dt}+(\frac{\sigma}{2}-\frac{\alpha_3}{\omega}F)B_r+\alpha_1\mu B_i-\frac{\alpha_2}{\omega}(B_r^2+B_i^2)B_r=0. \quad (23)$$

The dimensionless parameters, α_1, α_2, and α_3, are expressed as follows:

$$\alpha_1=\frac{c_2}{c_1}, \alpha_2=\frac{c_3}{c_1}, \alpha_3=\frac{c_4}{c_1}. \quad (24)$$

The parameters, c_1, c_2, and c_3, are expressed as follows:

$$c_1=\int_0^1-2\Phi_1^2dx, c_2=\int_0^1-\Phi_1^2dx$$
$$c_3=\int_0^1 c(x)\Phi_1dx, c_4=\int_0^1\frac{1}{2}\Phi_1^{\prime\prime}\Phi_1dx$$
$$c(x)=2\omega^2\Phi_1^{\prime}\int_0^x\Phi_1^{\prime 2}dx-3\omega^2\Phi_1\Phi_1^{\prime 2}$$
$$+2\omega^2\Phi_1^{\prime\prime}\int_1^x\int_0^x\Phi_1^{\prime 2}dxdx$$
$$-3\omega^2\Phi_1^{\prime\prime}\int_1^x\Phi_1\Phi_1^{\prime}dx-3\Phi_1^{\prime\prime 3}-9\Phi_1^{\prime}\Phi_1^{\prime\prime}\Phi_1^{\prime\prime\prime}+3\pi^4\Phi_1^{\prime\prime}$$
$$-3\Phi_1^{\prime\prime}\int_1^x\Phi_1^{\prime\prime}\Phi_1^{\prime\prime\prime}dx-2m\omega^2\Phi_1^{\prime\prime}\int_0^1\Phi_1^{\prime 2}dx$$
$$+\frac{3}{2}K\Phi_1^{\prime\prime}\int_0^1\Phi_1^{\prime 2}dx, \quad (25)$$

where $\Phi_1=\sin\pi x$. From the solvability condition, we can describe the steady state response and examine its stability. The approximate solution is expressed by B_r and B_i as follows:

$$v=\left(2B_r\cos\frac{\nu}{2}t-2B_i\sin\frac{\nu}{2}t\right)\sin\pi x \quad (26)$$

B_{rst} and B_{ist} in the steady state are given from Eqs. (22) and (23) with the condition $d/dt=0$. The steady state amplitude $a_{st}=\sqrt{B_{rst}^2+B_{ist}^2}$ are expressed as follows:

$$a_{st}=\sqrt{\frac{2\sigma\pm 4\sqrt{\alpha_3^2F^2-\alpha_1^2\mu^2}}{\alpha_2}}, \quad (27)$$

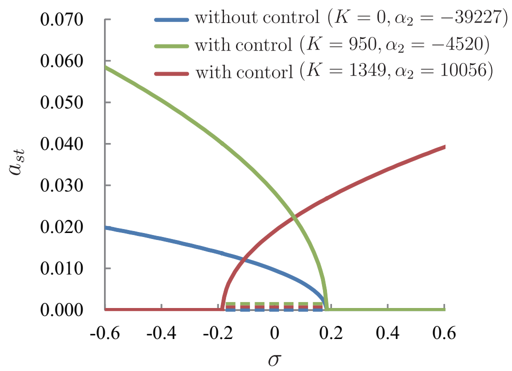

Figure 3: Change of nonlinear characteristics of frequency response curve depending on linear stiffness of attached spring: solid and dashed lines stand for stable and unstable steady state amplitudes, respectively.

where α_1, α_2, and α_3 are expressed as follows:

$$\alpha_1 = \tfrac{1}{2}, \alpha_3 = \tfrac{1}{4}\pi^2 \quad (28)$$

$$\alpha_2 = (-\tfrac{1}{6} - \tfrac{1}{2}m + \tfrac{15}{16\pi^2} + \tfrac{3K}{8\pi^4})\pi^8, \quad (29)$$

where K is the dimensionless stiffness of the attached linear spring.

Figure 3 shows the modification fo frequency response curve depending on the stiffness of the attached linear spring. In the case without spring, because of $\alpha_2 < 0$ from Eq. (29), the frequency response curve is softening type. It appears that attaching the spring makes the nonlinear characteristics hardening. In the case without spring, the subcritical and super critical pitchfork bifurcations are produced at lower and upper frequency boundary of the unstable region, respectively. Increasing the stiffness of the linear spring reverses their positions. If the spring is attached which is larger than the critical value:

$$K = \frac{1}{3}\left(\frac{\pi^4}{6} - \frac{\pi^4 m}{2} - \frac{15}{16}\right), \quad (30)$$

the subcritical and supercritical pitchfork bifurcations are produced at upper and lower frequency boundaries of the unstable region, respectively.

In the utilization as an energy harvester, K can be tuned to be small for the parametric amplification because smaller K causes larger response amplitude. Also, in the application of the parametric resonance to mechanical band-pass filters, the smaller K makes the bifurcation sensing easier.

5. Conclusions

We deal with amplitude control and amplification of the hinged-hinged beam under parametric excitation by attaching linear spring. By changing the longitudinal stiffness of the movable end in the axial direction by the spring, we can directly tune only the nonlinear characteristics of the beam in the lateral motion. Therefore, we can modify the frequency response of the parametric resonance by the attachment of the linear spring. Experimental result by a simple apparatus will be mentioned in the presentation.

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