Improvements in the marching-on-in-degree method for time domain integral equations

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Abstract: In this paper, we introduced some techniques to speedup the computation of time domain integration equation (TDIE). Marching-on-in-degree (MOD) method is used in the solving procedure in order to acquire an absolute late-time stable result.

1 Introduction

Time domain integration equation (TDIE) is attracting more and more attentions in solving the transient responses of targets. Marching-on-in-degree (MOD) method is a method proposed in solving the TDIE problems.[1,2] Comparing to the original marching-on-in-time (MOT) method suffering from the late time unstable, the MOD method perfectly solves this problem by expanding the transient behaviors by the Laguerre polynomials, which are defined in the domain of [0,+∞). In the previous work, using the MOD method suffers for a low efficiency comparing to the MOT method and in this paper, some techniques are introduced to speedup this method

2 Combinations of Laguerre polynomials

On the boundary of PEC structures, the tangential electric field is zero and the TD-EFIE is shown as Eq. 1.

$$\left[\mathbf{E}\right]_{\tan} = \left[\mathbf{E}^i(\mathbf{r},t)\right]_{\tan} + \left[-\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t) - \nabla\Phi(\mathbf{r},t)\right]_{\tan} = \mathbf{0} \tag{1}$$

where $\mathbf{E}^i(\mathbf{r},t)$ is the incident wave, $\mathbf{A}(\mathbf{r},t)$ and $\Phi(\mathbf{r},t)$ are the magnetic vector and the electric scalar potentials, respectively Hertz potential is used in this paper to make the MOD method more simple. Hertz potential $\mathbf{c}(\mathbf{r},t)$ is defined in Eq. (2,3) and the EFIE can be expressed Eq. (4).

$$\mathbf{J}(\mathbf{r},t) = \frac{\partial}{\partial t}\mathbf{c}(\mathbf{r},t) \tag{2}$$

$$q(\mathbf{r},t) = -\nabla\cdot\mathbf{c}(\mathbf{r},t)\,. \tag{3}$$

$$\left[\frac{\mu}{4\pi}\frac{\partial^2}{\partial t^2}\int_l \frac{\mathbf{c}(\mathbf{r}',\tau)}{R}dl' - \frac{\nabla_r}{4\pi\varepsilon}\int_l \frac{\nabla_{r'}\cdot\mathbf{c}(\mathbf{r}',\tau)}{R}dl'\right]_{\tan} = \left[\mathbf{E}^i(\mathbf{r},t)\right]_{\tan} \tag{4}$$

Now we can expand the Hertz potential by spatial and temporal basis functions respectively.

$$\mathbf{c}(\mathbf{r},t) = \sum_{n=1}^{N} c_n(t)\mathbf{F}_n(\mathbf{r})\,, \tag{5}$$

$$c_n(t) = \sum_{j=0}^{\infty} c_{n,j} e^{-\frac{st}{2}}\left[L_j(st) - 2L_{j+1}(st) + L_{j+2}(st)\right], \tag{6}$$

where $L_j(x)$ is the Laguerre polynomials. In the previous work of MOD method, the Laguerre polynomials are used to expend the transient process. But the integral equation involves a second order directive and the directive of the Laguerre polynomial is a summation of its lower orders as shown by Eq. 7.

$$\frac{d}{dt}e^{-\frac{st}{2}}L_j(st)=-\frac{s}{2}e^{-\frac{st}{2}}L_j(st)-s\sum_{p=0}^{j-1}e^{-\frac{st}{2}}L_p(st) \quad (7)$$

This will result in a lot of summations accumulated in the final equations. Therefore, we proposed a new type of temporal basis functions, which is a combination of three terms of Laguerre polynomials as shown in Eq. 6. In this type of basis factions, the summation of the three terms will cancelled with each other and thus no summation will exist in the directives as shown in Eq. 8.

$$\frac{d^2}{dt^2}e^{-\frac{st}{2}}\left(L_j(st)-2L_{j+1}(st)+L_{j+2}(st)\right)=\frac{s^2}{4}e^{-\frac{st}{2}}\left(L_j(st)+2L_{j+1}(st)+L_{j+2}(st)\right) \quad (8)$$

By using this type of temporal basis functions, the TD-EFIE result can be simplified by reducing the summations.

3 Testing procedure

In the MOD method, there are spatial and temporal basis functions. In the previous work [1] points out that it is difficult to handle the restarted time and the central approximation is necessary if we apply the spatial testing first. Therefore, in this work, we apply the temporal testing first and then the spatial testing in order to avoid the central approximation which will results in significant errors. Eq. 9 is the result after the testing, which can be march-on-in-degree. Once we know the value of coefficients $c_{n,j}$ (n=1,2,…,N), we can solve the value of $c_{N+1,j}$ with this matrix equation.

$$\begin{aligned}&\sum_{n=1}^{N}c_{n,i}\left(\frac{s^2\mu}{4}a_{mn00}+\frac{1}{\varepsilon}b_{mn00}\right)=\Omega_{m,i}-\frac{s^2\mu}{4}\sum_{n=1}^{N}(c_{n,i-2}+2c_{n,i-1})a_{mn00}\\&-\frac{1}{\varepsilon}\sum_{n=1}^{N}(c_{n,i-2}-2c_{n,i-1})b_{mn00}-\frac{s^2\mu}{4}\sum_{n=1}^{N}\sum_{j=0}^{i-1}(c_{n,j-2}+2c_{n,j-1}+c_{n,j})a_{mnij}\\&-\frac{1}{\varepsilon}\sum_{n=1}^{N}\sum_{j=0}^{i-1}(c_{n,j-2}-2c_{n,j-1}+c_{n,j})b_{mnij}\end{aligned} \quad (9)$$

where

$$a_{mnij}=\int_l \mathbf{F}_m(\mathbf{r})\cdot\int_{l'}\frac{I_{ij}(s\frac{R}{c})\mathbf{F}_n(\mathbf{r}')}{4\pi R}dl'dl \quad (10)$$

$$b_{mnij}=\int_l \nabla_r\cdot\mathbf{F}_m(\mathbf{r})\int_{l'}\frac{I_{ij}(s\frac{R}{c})\nabla_{r'}\cdot\mathbf{F}_n(\mathbf{r}')}{4\pi R}dl'dl \quad (11)$$

$$\Omega_{m,i}=\int_l \mathbf{F}_m(\mathbf{r})\cdot\int_0^\infty e^{-\frac{st}{2}}L_i(st)\mathbf{E}^i(\mathbf{r},t)d(st)dl \quad (13)$$

$$I_{ij}(s\frac{R}{c})=\int_{s\frac{R}{c}}^{\infty}e^{-\frac{st}{2}}L_i(st)e^{-\frac{st}{2}}L_j(st-s\frac{R}{c})d(st)=\begin{cases}e^{-\frac{sR}{2c}}[L_{i-j}(s\frac{R}{c})-L_{i-j-1}(s\frac{R}{c})] & j<i\\ 0 & j>i\\ e^{-\frac{sR}{2c}} & j=i\end{cases} \quad (14)$$

4 Combination of the Green's function

The Laguerre polynomials of different degree are no longer orthogonal to its lower degree after a retarded time. And the influence of these lower degree terms have to be eliminate in the final equations and results in a lot of integration terms in the Eq. 9. We can combine the Green's function in a different mathematical format to reduce the calculation by introducing the follow terms.

$$k_{mni} = \int_S \mathbf{f}_m(\mathbf{r}) \bullet \int_S \mathbf{f}_n(\mathbf{r}') \frac{1}{4\pi R} \begin{pmatrix} \left(c_{n,i-2} + 2c_{n,i-1}\right) I_{ii}(sR/c) + \\ \sum_{j=0}^{i-1} \left(c_{n,j-2} + 2c_{n,j-1} + c_{n,j}\right) I_{ij}(sR/c) \end{pmatrix} dS' dS \tag{15}$$

and

$$h_{mni} = \int_S \nabla \bullet \mathbf{f}_m(\mathbf{r}) \int_S \nabla' \bullet \mathbf{f}_n(\mathbf{r}') \frac{1}{4\pi R} \begin{pmatrix} \left(u_{n,i-2} - 2u_{n,i-1}\right) I_{ii}(sR/c) + \\ \sum_{j=0}^{i-1} \left(u_{n,j-2} - 2u_{n,j-1} + u_{n,j}\right) I_{ij}(sR/c) \end{pmatrix} dS' dS \tag{16}$$

5 Improvement of the Computation Time

To quantize the improvement of the computation time for solving the TD-EFIE with the new MOD scheme, the computational efficiency can be approximated in terms of the order of magnitude $\mathcal{O}$ (called big O notation) of the total number of operations, which is mainly dependent upon the maximum number of spatial unknowns N and the maximum temporal degree I. From the TD-EFIE formulations given by [1,2] and Eq. (9), the number of operations needed for the floating-point divisions and the calculations of the Laguerre polynomials using the conventional MOD and the new MOD are counted and listed in Table 1.

	The conventional MOD	The new MOD
Floating point divisions	$\mathcal{O}(N^2I^2)$	$\mathcal{O}(N^2I)$
Laguerre polynomial computations	$\mathcal{O}(N^2I^3)$	$\mathcal{O}(N^2I^2)$

Table1. The number of operations for the conventional and the new MOD methods

6 Numerical examples

Numerical examples are designed to show the efficiency and accuracy of the proposed method. And with the improvement of the efficiency, it is possible to calculate a large problem such as a plane.

Example 1. Consider a 1-m long dipole placed along the *x*-axis that is illuminated by a *T*-pulse [3] of width 6 lm and starting at 8 lm with an approximate bandwidth of 200 MHz. It is incident from the *z*-axis and is polarized along the *x*-axis. Figure 1 plots the current at the central point of the dipole. The computation time needed for all time domain methods are listed in Table. 2. From Figure 1, it can be observed that both MOD methods agree well with the IDFT of the frequency domain solution. However, the computation time for the new MOD method over the conventional one is less by a factor of 18.

Methods	**Total calculation time (s)**
The conventional MOD	46.30
The new MOD	2.45
The MOT	2.04

Table 2. Comparison of the computation time for the analysis of the dipole using different time-domain methods

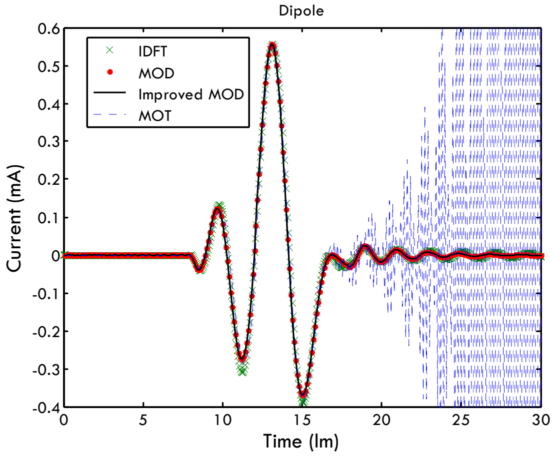

Figure 1. Transient response at the central point of the dipole caused by an incident *T*-pulse.

Example 2. A Boeing-737 aircraft with a size of 26 m × 26 m × 11 m is analyzed. The surface is discretized using 4721 triangular patches with 7327 edges. The structure is excited by a *T*-pulse coming from the head of the plane (θ= 90° and ϕ= 0°). The pulse has a pulse-width of 25 lm, a time delay of $ct_0 = 17.5$ lm, and a bandwidth of 50 MHz. The transient current distribution on the structure is computed using the new MOD method and is plotted in Figure 2.

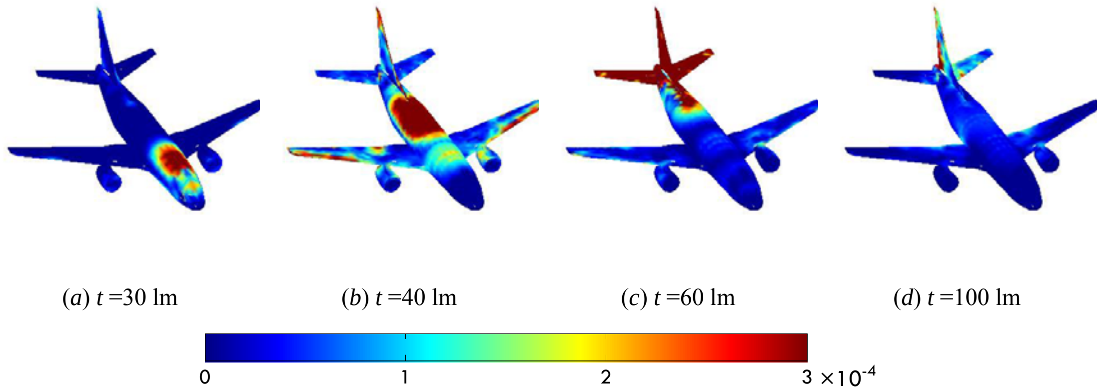

Figure 2. Transient current distributions on the Boeing-737 aircraft

7 Reference

[1]. Z. Ji, T. K. Sarkar, B. H. Jung, Y.-S. Chung, M. Salazar-Palma, and M. Yuan, “A Stable Solution of Time Domain Electric Field Integral Equation for Thin-Wire Antennas Using the Laguerre Polynomials,” *IEEE Trans. Antennas Propag.*, Vol. 52, No. 10, pp. 2641–2649, Oct. 2004.

[2]. Z. Ji, T. K. Sarkar, B. H. Jung, M. Yuan, and M. Salazar-Palma, “Solving Time Domain Electric Field Integral Equation Without the Time Variable,” *IEEE Trans. Antennas Propag.*, Vol. 54, No. 1, pp. 258–262, Jan. 2006.

[3]. Y. Hua and T. K. Sarkar, "Design of Optimum Discrete Finite Duration Orthogonal Nyguist Signals," *IEEE Trans. on Acoustics, Speech & Signal Processing*, Vol. 36, pp. 606-608, 1988.