Lightning-induced Voltages on an Overhead Line for Engineering Return-Stroke Models

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***Abstract*—This paper describes a calculation method of lightning-induced voltages on an overhead line to represent various engineering return stroke models. An analytical formula for a lightning stroke starting from an arbitrary altitude is used to the induced voltages. The lightning stroke in the proposed method is divided into many segments. This paper investigates the lightning-induced voltages for some return-stroke models.**

Key words: return stroke model, lightning-induced voltage, lightning protection, overhead line

I. Introduction

The authors derived analytical formulas of lightning-induced surges on an infinite length of overhead line caused by a lightning stroke starting from the ground [1]. The formulas are obtained on the basis of the Rusck model [2], which is one of famous coupling models for the estimation of the lightning-induced voltage based on the transmission line approximation [3]. The formulas are validated under the conditions that the ground and the line are perfectly conducting, and the return-stroke model is assumed to be the transmission line model (TLM) [4]. Many "Engineering" return-stroke models have been proposed [5]. The lightning-induced voltage depends on the return-stroke model. However, the Rusck model is consider only the TLM.

It is necessary to make effort in calculating the lightning-induced voltage associated with electromagnetic fields due to a complex return-stroke model. This paper proposes a calculation method of the lightning-induced voltage by dividing a lightning channel into some segments to extend the flexibility of the return-stroke model. The authors derive a modified formula of the lightning-induced surges generated by a lightning stroke starting from an arbitrary altitude to apply to the calculation method. This paper describes simulation results of the lightning-induced voltages for some engineering return-stroke models using the proposed method.

II. Analytical Formula of Lightning-Induced Surges on a Infinite Length of Line due to a Return Stroke Stating from Arbitrary Altitude

A. Assumptions of Analytical Formula

This paper adopts the following assumptions to derive an analytical formula of lightning-induced surges.

(1) The resistivity of the ground and a line is zero.
(2) The TLM [4] represents a return stroke. The return-stroke current develops with no wave deformation, no wave attenuation and a constant current velocity.
(3) The Rusck model is adopted to describe the electro-magnetic inducing effect.
(4) The lightning channel is located perpendicular to the ground plane.
(5) Time t=0 is defined as the time when the return-stroke current starts.

Fig. 1 illustrates a configuration of a lightning channel and an overhead line, where a rectangular coordinate system is adopted. The x- and z- axes are defined to be parallel to the overhead line and the lightning channel, respectively. The lightning channel is located at x=0. h and d are the line height and the distance between the line and the lightning channel, respectively. H is an altitude from which the lightning stroke starts to develop.

A return-stroke current $i_R(t)$ is approximated by piecewise linear characteristics given by:

$$i_R(t) = \sum_{k=1}^{N}(\alpha_{k+1} - \alpha_k)(t - T_k)u(t - T_k) \qquad (1)$$

$$\alpha_k = \frac{I_{k+1} - I_k}{T_{k+1} - T_k} \qquad (2)$$

where α_k is the tangent of the approximate return-stroke current on $[T_k, T_{k+1}]$, I_k is the instantaneous current at $t=T_k$, and $u(t)$ is the unit function of time.

B. Rusck Model

The Rusck model for a vertical return stroke is denoted by the following equations [2].

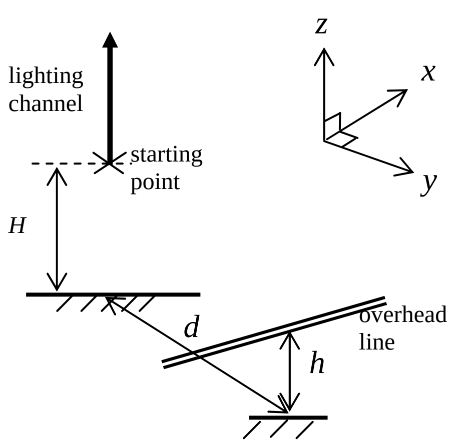


Fig. 1 A line configuration associated with a lightning channel

$$\frac{\partial V_s}{\partial x} = -L\frac{\partial I}{\partial t} \tag{3}$$

$$\frac{\partial I}{\partial x} = -C\frac{\partial (V_s - e_s)}{\partial t} \tag{4}$$

$$U = V_s + e_m \tag{5}$$

$$e_s = \int_0^h \frac{\partial V_i}{\partial z} dz \tag{6}$$

$$e_m = \int_0^h \frac{\partial A_i}{\partial t} dz \tag{7}$$

$$U = U_1 + U_2 \tag{8}$$

$$I = \frac{U_1 - U_2}{z_0} \tag{9}$$

$$U_n = V_{sn} + \frac{1}{2}e_m \tag{10}$$

where V_s is the induced scalar potential, I is the induced line current, U is the lightning-induced voltage, e_s is the inducing scalar potential, e_m is the inducing vector potential, V_i is the incident scalar potential, A_i is the incident vector potential, L is the line inductance per unit length, C is the line capacitance per unit length, z_o is the surge impedance of the line, and n=1: forward component, n=2: backward component.

C. Analytical Formula of Lightning-Induced Surges for Linearly-Rising Current

The lightning-induced voltage $U(x, t)$ and the induced current $I(x, t)$ caused by the return-stroke current αt are expressed by [5]

$$U_1,\ U_2 = \lambda\alpha\frac{30h}{v}\left[\ln\left\{\frac{\beta}{t_D \mp t_x}\cdot\frac{(t' \mp t_x)^2 + (t_y/\beta)^2}{\sqrt{t'^2 + \varsigma t_0^2} + \beta(t' \pm \varsigma t_x)}\right\} + \beta\ln\frac{t' + \sqrt{t'^2 + \varsigma t_0^2}}{t_a + t_a/\beta}\right] \tag{11}$$

$$U(x,t) = \lambda\alpha\frac{30h}{v}\left[\ln\frac{t_y^2}{t_y^2 + t_H^2}\left\{1 + \left(\frac{\beta}{t_y}\cdot\frac{t'^2 - t_0^2}{t' + \beta\sqrt{t'^2 + \varsigma t_0^2}}\right)^2\right\} + 2\beta\ln\frac{t' + \sqrt{t'^2 + \varsigma t_0^2}}{t_a + t_a/\beta}\right] \tag{12}$$

$$I(x,t) = \lambda\alpha\frac{30h}{v z_0}\ln\left\{\frac{t_D + t_x}{t_D - t_x}\cdot\frac{(t' - t_x)^2 + (t_y/\beta)^2}{(\sqrt{t'^2 + \varsigma t_0^2} + t_x/\beta)^2 + t_y^2}\right\} \tag{13}$$

where v_0 is the velocity of light in free space. v is the velocity of return stroke, λ is the direction cosine in the z direction of the return stroke (λ=+1: upward stroke, λ=-1: downward stroke), $\beta = v/v_0$, $t_x = x/v_0$, $t_y = d/v_0$, $t_H = H/v_0$, $t_a = t_D + \lambda t_H$, $t' = t + \lambda\frac{H}{v}$, $t_0 = \sqrt{t_x^2 + t_y^2}$, $t_D = \sqrt{t_x^2 + t_y^2 + t_H^2}$, $\varsigma = \frac{1 - \beta^2}{\beta^2}$.

III. Calculation Method of Lightning-Induced Voltages for Engineering Return-Stroke Models

A. Segmentation of Return-Stroke Model

Current and velocity in a return stroke are functions of altitude and so on. Thus, the return-stroke model is complicated in calculating electromagnetic field, and it is hard to estimate the lightning-induced voltage on the overhead line. This paper proposes a calculation method of the lightning-induced voltages considering various return-stroke models. The lightning channel is divided into a number of segments in the proposed method. The current and the velocity in the return stroke in a segment with short length can be regarded to be constant. As a result, the TLM is applicable to each segment, and it is not necessary to develop any special code for representing the return-stroke models. Fig. 2 illustrates the segments in the proposed method, where I_n is the current at the bottom of the n-th segment, $t_n = t - \tau_n$, τ_n is the time when the return stroke in the n-th segment starts, Δl is the length of the segment, and v_n is the return-stroke velocity in the n-th segment.

The analytical formulas of the lightning-induced surges is derived considering currents and charges along the whole lightning channel with semi-infinite length. Accordingly, the current with opposite polarity must be introduced to eliminate the current in the next segment for considering a boundary condition. The return-stroke current in the TLM has the following relation.

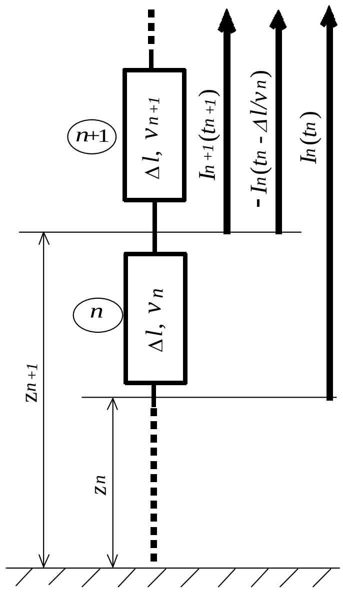


Fig. 2 Segmentation of return-stroke model

$$i(z+\Delta z,t)=i\left(z,t-\frac{\Delta l}{v}\right) \quad (14)$$

The current in the n-th segment is given by

$$i_n(t)=I_n(t_n)u(t_n)\delta(z_n)-I_n\left(t_n-\frac{\Delta l}{v_n}\right)u\left(t_n-\frac{\Delta l}{v_n}\right)\delta(z_{n+1}) \quad (15)$$

where $\delta(z)$ is the unit function of altitude.

Return-stroke parameters in segments can be independently determined because the current in a segment does not affect the currents in the other segments. The calculation method can represent TLM type return-stroke models by using the superposition theorem.

The lightning-induced voltage is estimated by a sum of the induced voltage due to the current in each segment.

$$U_t(t)=\sum_{n=1}^{N}U_n(t) \quad (16)$$

where $U_n(t)$ is the induced voltage caused by $i_n(t)$

It is very easy to calculate $U_n(t)$ caused by linearly approximated current using the analytical formula. Therefore, the proposed method enables engineers to calculate the lightning-induced voltages for various return-stroke models conveniently.

B. Return Stroke Models

This paper discusses the following "engineering" return-stroke models as a function of altitude H. The return-stroke current is given by

$$I(H,t)=P(H)I\left(0,t-\frac{H}{v}\right). \quad (17)$$

(a) TLM: v is constant, and $P(H)$=1.
(b) Modified TLM (MTLM) [6]: v is constant, and

$$P(H)=\exp\left(-\frac{H}{\phi}\right). \quad (18)$$

(c) MTLM with altitude-dependent velocity of return stroke (MTLVM):

$$v(H)=v(0)\exp\left(-\frac{H}{\theta}\right), \quad (19)$$

$$P(H)=\exp\left(-\frac{H}{\phi}\right) \quad (20)$$

where ϕ, and θ are the attenuation constants of current and velocity [m].

Typical velocities of a return stroke at the bottom and at the top of lightning channel are approximately 100 m/μs and 40 m/μs, respectively [7]. Thus, the return-stroke velocity is dependent on the altitude.

Fig. 3 shows a variation of current along a lightning channel for the return-stroke models, where $v(0)$=100m/μs, $I(0,t)$ is 1 A with shape of 2/40 μs, and the length of the lightning channel H_m is 2 km. The constant θ is chosen to satisfy the above observation result. It is clear from the figure that the distribution of the return-stroke current is dependent on the return-stroke model.

C. Waveform of Lightning-Induced Voltage for Return-stroke Models

Fig. 4 shows calculated waveforms of lightning-induced voltage on an infinite length of line with height of 10 m in cases of Δl=50 m and 100 m. Parameters of the return-stroke models are same as those used in the last section. The observation points are located at the closest to the lightning channel (x=0 m) and x=1 km for d=100 m.

It is confirmed from Fig. 4 that the lightning-induced voltage is dependent on the return-stroke model. The calculated result in case of Δl =50 m is almost equal to that in case of Δl =100 m.

D. Parameter Analysis of Lightning-Induced Voltage

This section investigates the influence of some parameters of the return stroke on the crest value of the induced voltage on an infinite length of overhead line. Fig. 9 shows an influence of the attenuation constants ϕ and θ of the return-stroke current on peak value of the lightning-induced voltage on an infinite length of line. The value in the figure is normalized by the crest return-stroke current at the bottom og the lightning channel. Observation point is x=0m. θ^{1}=0 is equivalent to the MTLM, and $\theta^{1}=\phi^{1}$=0 corresponds to the TLM.

From Fig. 9, the lightning-induced voltage decreases as ϕ^{1} or θ^{1} becomes larger. The reduction for ϕ^{1} is sufficient greater than that for θ^{1}. Therefore, it is necessary to consider the attenuation of the return-stroke current along the lightning channel for accurate lightning-induced effect analysis.

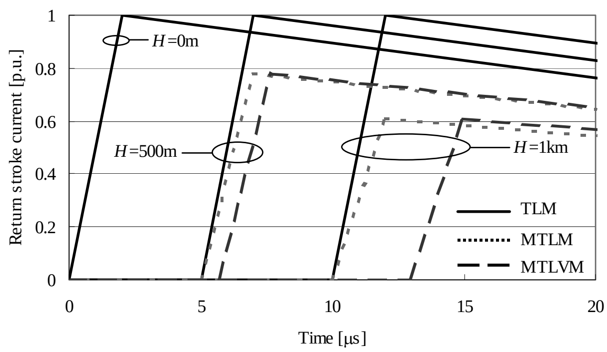
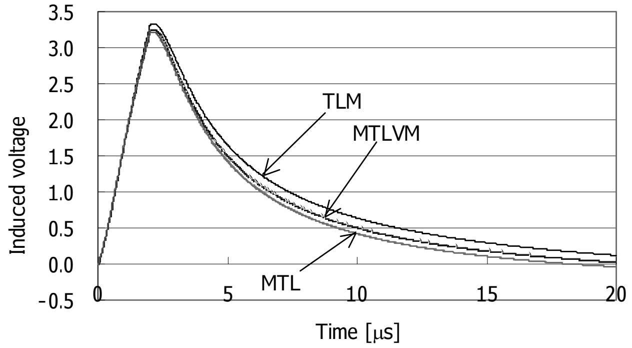
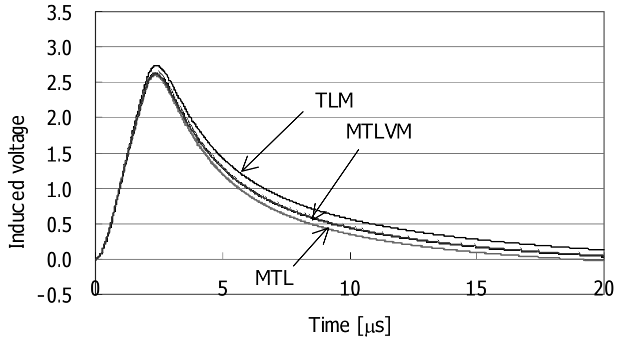


Fig. 3 Return-stroke current along a lightning channel for various return stroke models

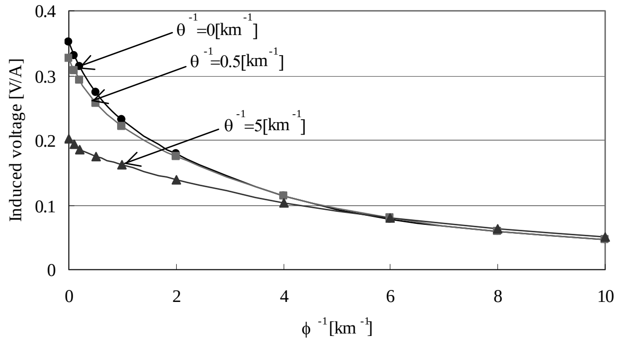


(a) x=0m

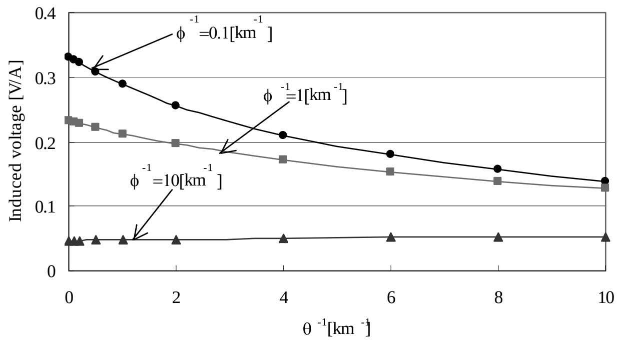


(b) x=1000m

Fig. 4 Calculated results of lightning-induced voltage for various return stroke models



(a) parameter ϕ



(b) parameter θ

Fig. 9 Influence of ϕ and θ on calculated results of lightning-induced voltage for various return stroke models (H_m=2 km, d=1 km, return stroke current waveform: 6/40 μs)

IV. CONCLUSION

This paper has described analytical formulas of lightning-induced surges on an infinite length of overhead line caused by a return stroke starting from an arbitrary altitude, and a calculation method of lightning-induced voltages in order to treat various "engineering" return-stroke models. A lightning channel is divided into a number of segments in the proposed model, and return-stroke parameters such as velocity and crest current in each segment are determined independently. The calculation method gives sufficient accuracy for representing transmission-line-type return-stroke models by choosing segment length properly.

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