Estimation of the statistical uncertainty of measurements in a reverberation chamber

Christophe Lemoine, Philippe Besnier and M'hamed Drissi

INSA of Rennes, Institute of Electronics and Telecommunications of Rennes - IETR
Rennes, France, Email: {christophe.lemoine, philippe.besnier, mhamed.drissi}@insa-rennes.fr

Abstract—**Measurements in reverberation chambers are usually performed by changing the electromagnetic environment using either mechanical or frequency stirring. Independent stirrer positions or independent frequencies provide uncorrelated fields in the cavity and thereby uncorrelated power levels. The independence of samples is necessary to make a statistical analysis of data and provides a better accuracy of measured parameters. The autocorrelation function is generally used to estimate the independency of samples, supposing that only linear correlation can affect measurements. This paper focus on experimental results for estimating frequency and mechanical stirring efficiency by a recent approach that uses autocorrelation functions. Because uncorrelation does not necessarily implies independence, the use of autocorrelation function may lead to estimate an upper bound of the effective sample size.**

Keywords—**correlation, effective sample size, independence, measurement uncertainty, reverberation chamber**

I. INTRODUCTION

Reverberation chamber (RC) is an alternative tool for various EMC [1], [2] and non-EMC applications [3]. Above the lowest usable frequency (LUF), the electromagnetic field in the test volume of a reverberation chamber is homogeneous and isotropic. Due to the stochastic-like nature of the field, a measurement performed in a mode-stirred chamber is a random value. For instance, the immunity test of any electronic system in RC is a random experiment [4]. Therefore, the control of a test carried out in the cavity requires the quantification of the uncertainty over an estimation. Statistical methods are of a great interest for RC measurements analysis.

In RC, a mode stirrer is commonly used to change the boundary conditions for the electromagnetic field. Correctly selecting each location of the stirrer creates an independent field distribution. Similarly, independent electromagnetic environment can be provided by the electronic stirring process. The uncertainty level over an estimation in RC is directly related to the actual number of independent measurements, which are collected by mechanical and/or electronic stirring.

Received power measurements in RC can be viewed as a time series process. For a given correlated time series of length N, the information given by the N values is equal
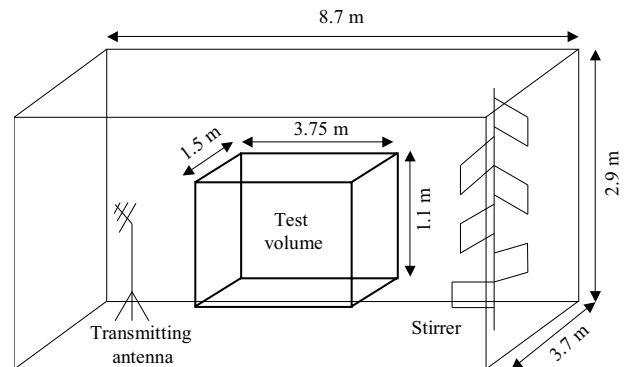

Figure 1. The IETR reverberation chamber.

to that given by an uncorrelated time series of length N', where $N' < N$. N' is the effective sample size (ESS) the RC user is looking for in order to evaluate correctly stirring efficiency.

Usually, the number of independent samples is evaluated using the autocorrelation function (ACF), which is strictly a measurement of the linear correlation. Therefore, the RC community does the common assumption that uncorrelated samples are also independent. However, we should keep in mind that uncorrelation does not necessarily imply independence. Let X be a random value which follows a zero mean normal distribution and $Y = X^2$. The correlation coefficient $\rho(X, Y)$ is null, although X and Y are highly dependent. As a consequence, ACF leads to estimate only a maximum number of independent samples, which are available among all measurements.

The normative part of the IEC 61000-4-21 [2] assumes statistically independent boundary conditions between two successive stirrer positions when the estimated first order ACF r is lower than $1/e \approx 0.37$. In (1), the series y is the same collection of data as x but shifted by one sample. The notations "covar" and "var" are for the covariance and the variance operators, respectively.

$$r = \frac{\text{covar}(x, y)}{\sqrt{\text{var}(x)}\sqrt{\text{var}(y)}} \tag{1}$$

Some authors have still pointed out that the criterion $\rho_0 = 0.37$ is generally not well adapted since the distribution of

the first order ACF is a function of the sample size N [5], [6]. Therefore, we will not base our analysis on this single criterion. In [6] we endeavoured to determine the correlation coefficient under more satisfactory statistical conditions: given $N = 1500$ samples, the estimated autocorrelation function of the N size sample is evaluated with a 5% uncertainty level.

This paper aims to highlight that using the ACF is a convenient way to estimate an upper bound of the actual number of independent samples. First, we briefly introduce the method that leads to an estimate of this upper bound. Second, experimental results are provided for both electronic and mechanical stirring.

II. ESTIMATION OF THE EFFECTIVE SAMPLE SIZE

The global formulation of an AR(k) model expresses the observation t of the dependent value y_t as a function of the former observations $y_{t-1}, y_{t-2}, \ldots, y_{t-k}$ and a residue ϵ_t. This last is supposed independently and identically distributed (IID).

$$y_t = \Phi_{k1}y_{t-1} + \Phi_{k2}y_{t-2} + \cdots + \Phi_{kk}y_{t-k} + \epsilon_t \quad (2)$$

The coefficients Φ_{kl} are easily calculated with a spreadsheet like Excel using the regression tool. In the case of an AR(1), we have the particular relationship $\Phi_{11} = r$. Increasing the order of an AR model is appropriate when there remains significant information in the residue that is not exploited yet. Therefore, the order k is deduced from a correlation analysis over the residues ϵ_t. For a series of $N = 1500$ samples, when the absolute value of the estimated first order ACF r is less than 0.10, we consider that the 1500 samples are uncorrelated, and then assume independence [6]. Thus, when the first order ACF $|r_\epsilon|$ of 1500 residues is lower than 0.10, then the residues are supposed independent.

The reader can find a detailed description of the calculation of the ESS of a N size sample in [6]. Only the useful relationship is given here. The ESS N' is estimated from a series of N dependent data, which are collected for instance using N correlated stirrer steps over a complete rotation. Let $y_1, \ldots, y_N$ be a sample of dependent data whose mean is μ_y and variance is σ_y^2. Among those N data, there are only N' values which are actually independent. Let $x_1, \ldots, x_{N'}$ be the sample of those N' independent data whose mean is μ_x and variance is σ_x^2. The effective sample size N' is calculated here using (3) for an AR(1) model.

$$N'_{AR(1)} = N \times \frac{1 - \Phi_{11}}{1 + \Phi_{11}} \times \left(\frac{\sigma_x}{\mu_x}\right)^2 \times \left(\frac{\mu_y}{\sigma_y}\right)^2 \quad (3)$$

This method is valid when no correlation does imply independence, since this analysis is based on autocorrelation functions which can only evaluate a linear dependency. In the general case for a N size sample, the relationship (3) leads to an upper bound of the actual effective sample size. When $\rho = 0$, then $N'_{AR(1)} = N' = N$; when $|\rho| > 0$,

then $N'_{AR(1)} > N'$. But, when we assume that only a linear dependency can affect measurements, then N' is well estimated from $N'_{AR(1)}$ (3).

The parameters μ_y, σ_y, Φ_{11}, Φ_{21}, and Φ_{22} are estimated experimentally from the series of N dependent data. The ratio σ_x/μ_x which is related to independent data is theoretically known [7]. For independent received power measurements, data follow an exponential distribution and therefore $\sigma_x/\mu_x = 1$. We previously checked that received power measurements with a large log-periodic antenna (ETS-Lindgren Model 3148: height 6.4 cm, width 85.6 cm, depth (length) 73.7 cm) fit correctly the exponential distribution, using appropriate goodness-of-fit tests [8].

III. EXPERIMENTAL RESULTS

This section aims to provide experimental results in order to evaluate the ability of the proposed method to estimate correctly the effective sample size in the case of RC measurements. The analysis is based on a comparison with the results given by the central limit theorem (CLT). All experiments are carried out in the IETR reverberation chamber (Fig. 1) whose LUF is established around 250 MHz.

A. Central limit theorem

In the case of independent samples, the central limit theorem is well known [9]. Let $x_1, \ldots, x_{N'}$ be an independent sample of the random variable X whose mean is μ_x and variance is σ_x^2. When the number N' of *independent* samples is sufficiently large (> 20), one can consider the estimated mean value $\hat{\mu}_x$ as normally distributed with the standard deviation

$$\sigma_{\mu_x} = \frac{\sigma_x}{\sqrt{N'}}. \quad (4)$$

We point out that the normal distribution of the estimation of a mean from N' observations enables to calculate a confidence interval associated with the estimation. The knowledge of this confidence interval leads to control measurements performed in RC. In Fig. 2, we indicate for instance the 95% confidence interval characterized by the 2.5% and 97.5% quantiles.

In order to assess if the correlation coefficient can be sufficient to claim that the data are independent, we propose to check the relationship (4), experimentally from received power measurements in RC. The purpose is to have access, with still a good precision, to the experimental standard deviation of the mean of N measurements, with N varying from 20 to 400 samples, generated either by frequency stirring (ESS denoted N'') or by mechanical stirring (ESS denoted N'). For small N, the CLT is checked replacing N' by N in (4), whereas when $N > N'$ for mechanical stirring or $N > N''$ for frequency stirring, the standard deviation should saturate at a level given by N' or N'', respectively.
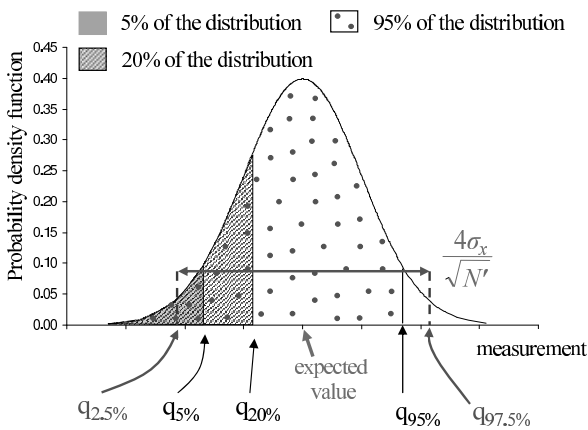
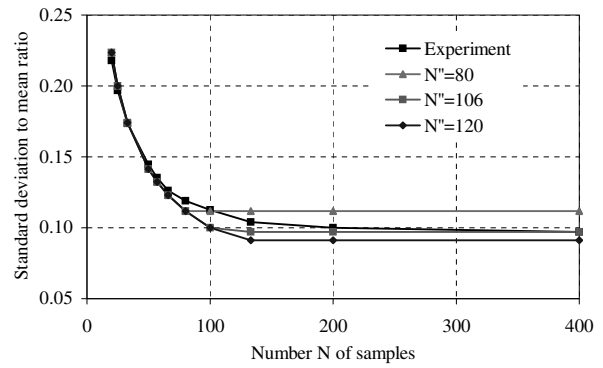Figure 2. Confidence interval associated with the estimation of a normal parameter from RC measurements.

Figure 3. Estimation of N'' using the central limit theorem in the frequency bandwidth [690 MHz - 710 MHz] (*Frequency* stirring).
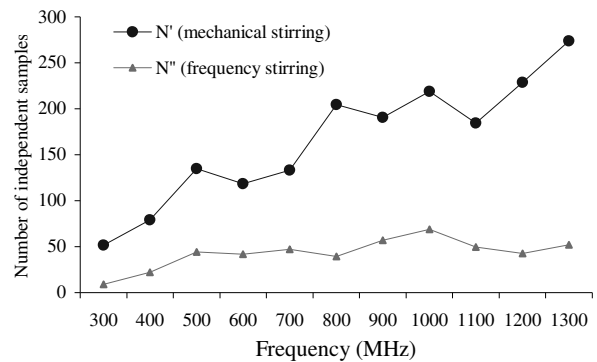

Figure 4. Evaluation of the numbers N' (mechanical sirring) and N'' (frequency stirring) of independent samples, using AR models. N' is the estimated number of independent stirrer locations available over a complete rotation. N'' is the estimated number of independent frequencies available in a 10 MHz frequency bandwidth [6].

Since 1500 estimated values of the mean of N samples are necessary to estimate correctly ($\pm 5\%$) the standard deviation of the experimental means, this experiment takes a long time. Either for frequency stirring, or for mechanical stirring, it takes approximately 180 hours. More details about the experimental setup and the results are given below.

B. Frequency stirring

From 690 MHz to 710 MHz, we select 400 samples with a constant frequency step, for one stirrer location and one receiving antenna position. We previously estimated (Fig. 4) that around 700 MHz, we have 50 uncorrelated samples in a 10 MHz frequency bandwidth. Therefore, we can assume that from 690 MHz to 710 MHz, there are approximately 100 uncorrelated samples using frequency stirring. Moreover, since we need 1500 estimations of the mean of N samples, we replicate this setup for 15 independent receiving antenna positions, and 100 stirrer locations over $360°$ per antenna position.

From each series of N data, we first estimate the mean μ_N of the N size sample. For instance, with $N = 20$, we select only one value out of twenty data; with $N = 50$, one value out of eight, and so on. Then, from 1500 series we are able to estimate precisely the expected standard deviation σ_{μ_N}. Here we call N'' the number of independent samples available in the $N = 400$ size sample, *i.e.* in the frequency range [690 MHz - 710 MHz].

Moreover, since in the case of received power measurements the standard deviation to mean ratio of data is simply 1 [7], using (4) the number N'' can be found solving the following equation:

$$\frac{\sigma_{\mu_N}}{\mu_N} = \frac{1}{\sqrt{N''}}. \quad (5)$$

When $N < N''$, the CLT must be statisfied using N instead of N'' on the right-hand side of (5). Then, when $N > N''$, the CLT cannot be used with N since there are

dependent samples in the N size sample. Consequently, the number N'' must be considered on the right-hand side of (5). The results for different values of N'' are compared with the experimental standard deviation to mean ratio of the estimated mean of the N size sample in Fig. 3.

As expected, the experimental standard deviation to mean ratio decreases when N increases, up to the critical value $N = N''$. When $N > N''$, the standard deviation cannot decrease since we only add dependent samples in series of measurements. The level of convergence of σ_{μ_N}/μ_N is obtained for large N and is 0.097. This level of convergence gives access to the number of independent samples available in the series of $N = 400$ measurements in the range [690 MHz - 710 MHz]. Using (5), we deduce $N'' = 106$ independent samples. In Fig. 3, one can clearly see, thanks to the precision of the estimation of the standard deviation to mean ratio, that higher values of N'' or smaller values do not let to fit correctly the experiment. The value $N'' = 106$ is consistent with the number of uncorrelated samples estimated from (3) using autoregressive models in [695 MHz - 705 MHz] (Fig. 4). Therefore, for frequency stirring, the first order ACF seems to be sufficient to test the independence of measurements.
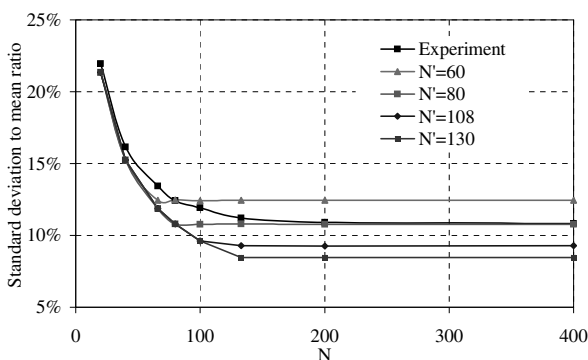
Figure 5. Estimation of N' using the central limit theorem, over a complete rotation of the mode stirrer and at 700 MHz (*Mechanical* stirring).

C. Mechanical stirring

Using the central limit theorem, we propose to determine the number N' of independent samples available over a complete rotation ($360°$) of the mode stirrer at 700 MHz. Consequently, we transpose the previous analysis for frequency stirring, in the case of mechanical stirring. We select $N = 400$ locations of the mode stirrer over $360°$. In order to get 1500 series of $N = 400$ samples, we also use 15 independent positions of the receiving antenna and 100 independent frequencies in the range [690 MHz - 710 MHz].

Here we call N' the number of independent samples given by the central limit theorem, among the $N = 400$ locations of the mode-stirrer. From (3), we estimated using autoregressive models that, at 700 MHz, there are 130 uncorrelated positions of the mode stirrer over $360°$ (Fig. 4). However, using the CLT we calculate that there are only 80 independent boundary conditions available with the mode stirrer over a complete rotation (Fig. 5). This leads to a significant shift in comparison with results for frequency stirring (Table I). Moreover, considering $N' = 130$ instead of $N' = 80$ corresponds with a significant shift of the standard deviation to mean ratio (Fig. 5). Therefore, for mechanical stirring, having uncorrelated samples may not necessarily imply that these samples are strictly independent.

TABLE I. ESTIMATION OF N' AND N'': AR MODEL[1] VS. CLT[2]

	N' over $360°$ at 700 MHz	N'' in [690 MHz-710 MHz]
AR model (3)	130	100
CLT (4)	80	106

[1] which estimates the number of *uncorrelated* samples.

[2] which estimates the number of *independent* samples.

A possible cause of such a behavior is here proposed. In the mechanical stirring case, when the stirrer is moving, we cannot predict that it is equivalent to reduce the electrical volume of the cavity or increase it in comparison with the wavelength λ. The variation of this electrical volume

is not monotonic, so dependency between subdivisions is possible. For frequency stirring, starting from a minimum to a maximum frequency is equivalent to reduce the electrical volume of the cavity with regards to λ. Therefore the evolution of the electrical volume is monotonic, and if frequencies are not correlated we may assume that there is also no dependency.

IV. CONCLUSION

The paper points out that having uncorrelated samples in a reverberation chamber does not necessarily imply that those samples are completely independent. For frequency stirring case, no linear correlation between samples enables to conclude independency. But, for mechanical stirring process, the evaluation of ESS leads only to an upper bound of the actual number of independent samples. The knowledge of $N'_{AR(1)}$ leads nonetheless to a maximum confidence interval of a parameter estimated in RC.

AKNOWLEDGEMENT

REFERENCES

[1] *DO-160D/E: "Environmental conditions and test procedures for airborne equipment"*, RTCA Std., 2000.

[2] *IEC 61000-4-21: "Reverberation chamber test methods"*, International Electrotechnical Commission Std., 2003.

[3] K. Rosengren, P.-S. Kildal, C. Carlsson, and J. Carlsson, "Characterization of antennas for mobile and wireless terminals in reverberation chambers: improved accuracy by platform stirring," *Microwave Opt. Technol. Lett.*, vol. 30, pp. 391–397, Sept. 2001.

[4] A. C. Marvin, T. Konefal, J. F. Dawson, and M. P. Robinson, "The use of reverberation chambers in the quantitative assessment of the risk associated with installation of electronic systems on vehicles," *EMC Workshop Europe 2007*, Paris, France, June 2007.

[5] O. Lundén and M. Bäckström, "Stirrer efficiency in FOA reverberation chambers. Evaluation of correlation coefficients and chi-squared tests," in *IEEE International Symposium on EMC*, vol. 1, Washington DC, USA, Aug. 2000, pp. 11–16.

[6] C. Lemoine, P. Besnier, and M. Drissi, "Estimating the effective sample size to select independent measurements in a reverberation chamber," *IEEE Trans. Electromagn. Compat.*, vol. 50, no. 2, pp. 227–236, May 2008.

[7] D. A. Hill, "Plane wave integral representation for fields in reverberation chambers," *IEEE Trans. Electromagn. Compat.*, vol. 40, no. 3, pp. 209–217, Aug. 1998.

[8] C. Lemoine, P. Besnier, and M. Drissi, "Investigation of reverberation chamber measurements through high power goodness of fit tests," *IEEE Trans. Electromagn. Compat.*, vol. 49, no. 4, pp. 745–755, Nov. 2007.

[9] J. D. Hamilton, *Time Series Analysis*. Princeton, NJ: Princeton University Press, 1994.