Accounting for Uncertainty in EMC Studies

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***Abstract*— The paper addresses the use of statistical techniques in the assessment of the impact of parameter uncertainties on important EMC parameters such as shielding effectiveness**

Key words: Statistical techniques, EMC, Shielding, Unscented Transform.

I. Introduction

Uncertainty is one of the most challenging aspects of EMC analysis. Frequently, the EMC compliance of a complex system is required when the layout and many component parameters are only known to certain accuracy or may vary in a random fashion. Therefore, many EMC parameters such as coupling, radiation and immunity can only be defined within statistical limits. Calculating these statistical parameters for EMC in complex systems directly is very time consuming and often impossible. Solving complex systems using a Monte Carlo approach is not feasible as it uses several hundred thousand simulations to obtain the statistics of the final result. For complex systems where each simulation may require hours this is not a practical method. Unscented Transforms (UT) offer a method of greatly reducing the computational burden needed during statistical analysis. The paper describes the basis of the UT approach and gives examples of its application in shielding effectiveness problems.

II. Theory

A. The Unscented Transform (UT)

The UT was developed by Julier and Uhlman in 1997 [1] and it is similar to the Moment Design Technique (MDT) [2]. Both techniques use the moments of the probability distribution function to determine a selected set of points. In MDT, these points are called design values. In the UT approach they are the sigma points S_i.

The main idea of the UT is to approximate the effect of an arbitrary nonlinear mapping by the mapping of the set of sigma points. Once the mapping is completed, the statistical moments are available through a weighted average of the mapped values at the sigma points.

B. Calculation of the Sigma Points

Let $\hat{u}$ be a zero mean random variable with known probability distribution, and $\overline{U}$ is the average value of the quantity being mapped. Both are subjected to a continuous nonlinear mapping $G(\overline{U}+\hat{u})$. The mapping may be expressed using the Taylor polynomial expansion:

$$G(\overline{U}+\hat{u}) = G(\overline{U}) + \frac{dG}{du}\hat{u} + \frac{1}{2!}\frac{d^2G}{du^2}\hat{u}^2 + \frac{1}{3!}\frac{d^3G}{du^3}\hat{u}^3 + \ldots \tag{1}$$

The formulation representing the Taylor series (2) as a polynomial is more compact, therefore it will be used from this point on.

$$G(\overline{U}+\hat{u}) = G(\overline{U}) + p(\hat{u}) \tag{2}$$

The expected value of (2) is:

$$\overline{G} = E\{G(\overline{U}+\hat{u})\} = E\{G(\overline{U})\} + E\{p(\hat{u})\} = G(\overline{U}) + \overline{P} \tag{3}$$

In Equation (3) $\overline{P}$ is the expected value of the Taylor polynomial. The variance of (2) is:

$$\sigma_g^2 = E\{[G(\overline{U}+\hat{u}) - \overline{G}]^2\} = E\{p(\hat{u})^2\} - \overline{P}^2 \tag{4}$$

The Taylor representation is also usable for the sigma points.

$$G(\overline{U}+S_i) = G(\overline{U}) + p(S_i) \tag{5}$$

The polynomial is the same because the sigma points S_i belong to the probability distribution of $\hat{u}$. The comparison of the expected value and variance of (5) with (3) and (4) results in the set of equations for the sigma points.

$$w_0 = 1 - \sum_i w_i$$
$$\sum_i w_i S_i^k = E\{\hat{u}^k\} \tag{6}$$

The order of approximation is k, depends on how one truncates the polynomial. Therefore, the truncation of the Taylor polynomial determines the number and value of the sigma points S_i as well as the weights w_i of the UT.

The set of equations for the sigma points (6) is nonlinear. Therefore, there is a number of possible choices for sigma points satisfying the equation system. However, there is a set of solutions that are roots to the polynomials of the Gaussian quadrature integration scheme [3]. This simplifies the solution of (6), since the weights w_i and sigma points S_i are more easily calculated from the quadrature integration scheme. Naturally, the interpolation polynomial is dependent on the probability distribution of $\hat{u}$ (equivalent to the weight function $w(\hat{u})$ of the integration). Table I presents the normalized sigma points. The normalization factor is the standard deviation in the Gaussian distribution case. In the case of the uniform distribution, the normalization factor is the interval of the distribution [-1,1]. and weights for the uniform and gaussian distributions.

TABLE I
SIGMA POINTS AND WEIGHTS FOR UNIFORM AND NORMAL DISTRIBUTIONS

Order	Normalized Sigma Points and Weights		
	Weights	**Sigma Points**	**Probability Distribution**
1	0.500 0.500	-0.577 0.577	$w(\hat{u}) = \begin{cases} \frac{1}{2} & \lvert\hat{u}\rvert < 1 \\ 0 & \lvert\hat{u}\rvert > 1 \end{cases}$
2	0.278 0.444 0.278	-0.775 0 0.775	
4	0.119 0.239 0.284 0.239 0.119	-0.906 -0.538 0 0.538 0.906	
1	0.500 0.500	-1 1	$w(\hat{u}) = \frac{1}{\sqrt{2\pi}} e^{\frac{\hat{u}^2}{2}}$
2	0.167 0.666 0.167	-1.73 0 1.73	
4	0.011 0.222 0.534 0.222 0.011	-2.857 -1.356 0 1.356 2.857	

C. Calculation of the moments of the mapped distribution

Once the sigma points are known, it is straightforward to apply them to the nonlinear mapping. The statistical moments are calculated using:

$$E\left\{G\left(\overline{U} + \hat{u}\right)^n\right\} = \sum_i w_i G\left(\overline{U} + S_i\right)^n \tag{7}$$

The central moments are calculated using the expected value of the result (calculated using (7) with n=1). The general expression is:

$$E\left\{\left[G\left(\overline{U} + \hat{u}\right) - \overline{G}\right]^n\right\} = \sum_i w_i \left[G\left(\overline{U} + S_i\right) - \overline{G}\right]^n \tag{8}$$

The calculation of the resulting statistical moments is linked with the nonlinear function. This function may have analytical or numerical form. If an analytical equation is available, the denormalized values of the sigma points are to be used in the calculation. As an example, one may use the UT to calculate the first resonant frequency of a cavity filled with a dielectric which has the permittivity as a random variable. Since all other parameters are fixed, the nonlinear mapping function is:

$$G\left(\varepsilon_r + \hat{u}\right) = \frac{f_0}{\sqrt{\varepsilon_r + \hat{u}}} \tag{9}$$

Where f_0 is the resonant frequency of an empty filled cavity with the same dimensions, ε_r is the mean value of the permittivity, and $\hat{u}$ is a zero mean random variable. The distribution of the random variable determines the weigths and sigma points. As discussed in the previous section, the denormalization factor depends in the type of distribution. In the case of the uniform distribution , the variable will be denormalized by the size of the interval. In the case of the normal distribution, the denormalization factor is the standard deviation. The nonlinear function (9) is calculated for each sigma point and the moments are obtained using (7) and (8).

If the nonlinear function is numerical, such as resulting from a numerical simulation, then the same process has to be repeated for each sigma point.

D. Accuracy of the Unscented Transform

The accuracy of the UT is dependent on the order of the approximation as presented on Table I. If the sigma points are obtained from the quadrature scheme, then the accuracy will be the same of the chosen interpolation polynomial. As an example, one can compare the expected value and central moments if an exact calculation is possible. This is the case of the nonlinear mapping shown in (9). In the example the cavity is tuned to f_0 is 300 MHz, and it is filled with a dielectric with mean permittivity of 4.

The random variable may have a uniform or gaussian distribution. In this example, the relative permittivity was modeled as a random variable that could vary between 3 and 5. This translates to an interval of [4-1,4+1] in the case of the uniform distribution. In the case of the gaussian distribution this results in a standard deviation of 1/3 (considering 99% confidence interval). Therefore, the sigma points are:

- For the uniform distribution (3 sigma points)

$$\begin{aligned} G(4-0.775) &= \frac{300}{\sqrt{4-0.775}} = 167.054 \\ G(4) &= \frac{300}{\sqrt{4}} = 150 \\ G(4+0.775) &= \frac{300}{\sqrt{4+0.775}} = 137.289 \end{aligned} \tag{10}$$

Therefore the expected value, calculated with (7) is 151.206 MHz

- For the Gaussian distribution (2 sigma points)

$$G\left(4-\frac{1}{3}\right) = 156.670 \quad G\left(4+\frac{1}{3}\right) = 144.115 \tag{11}$$

Using (7) shows that the calculated expected value is 150.393 MHz. Table II shows the comparison of the UT calculations with exact analytical results. In this table, the moments are calculated using 2, 3 and 5 sigma points (as presented in Table I).

TABLE II
COMPARISON OF DIFFERENT MOMENTS OF TWO DISTRIBUTIONS FOR VARIOUS APPROXIMATION ORDERS AND THE EXACT RESULTS

Type of moment	Type of distribution			
	Uniform (UT)	**Uniform (Exact)**	**Normal (UT)**	**Normal (Exact)**
Expected Value (MHz)	151.189	151.205	150.393	150.397
	151.206		150.398	
	151.205		150.400	
Standard Deviation (MHz)	10.962	11.143	6.277	6.360
	11.143		6.364	
	11.144		6.360	
Skewness	0.000	0.266	0.000	0.390
	0.257		0.371	
	0.266		0.388	
Kurtosis	-2	-1.109	-2.000	0.322
	-1.190		0.040	
	-1.109		0.320	

E. The Multivariate Unscented Transform

The multiple random variables case is also modeled by the UT . It is possible to include either independent or correlated variables. Although the approach allows modeling correlated variables, it is best to calculate the sigma points for independent random variables. Once these points are known, further processing is a matter of linear transformation using a covariance matrix.

In multivariate cases, the choice of sigma points and weights is not unique, and it is usually necessary to use additional sigma points [4]. There are many possible sets that may be used. One set that is simple to calculate is the combination of sigma points provided by the appropriate quadrature scheme. In this set, the weights are calculated by the product of individual weights of each random variable.

In most multivariate problems, there may be dominance of a set of random variables over the others. This may be ascertained by a careful analysis of the moments of the marginal distributions (one input random variable at a time) or by the analysis of the correlation between input and output variables.

F. Estimation of Variable Influence in the Multivariate Case

A numerical problem with several random variables may be well characterized by a smaller subset of variables. Using the concept of marginal distribution probability function [5], it is possible to determine what are the most important variables. These distributions are essentially one-variable distributions, where the calculation is performed for each variable separately using (8). The resulting expected value and variance provide information on the significance of each of the variables. Since the UT is based on a Taylor approximation of the nonlinear mapping, the calculation of the marginal statistical moments provide a good estimate of the influence of each parameter in the output result:

$$I_{x_n} = \frac{E\left\{G(\overline{U}_n)^2\right\} - E\left\{G\left(\overline{U}_n\right)\right\}^2}{E\left\{G(\overline{U}_1,..,\overline{U}_n)^2\right\} - E\left\{G\left(\overline{U}_1,..,\overline{U}_n\right)\right\}^2} \quad (12)$$

Where I_{Xn} is the relative percentage influence of variable $\overline{U}_n$ in the variance of the result.

G. Calculation of the Probability Density Function

In addition to the calculation of the statistical moments of the solution, the UT can be used to obtain the probability density function (PDF) of the solution. This may be very useful since one may use the PDF to calculate the confidence intervals of the solution.

The UT is based on a polynomial approximation for an arbitrary mapping. This allows a simple form of calculation of the probability density function. The Jacobian approach to calculating a mapped distribution function requires the inverse function of the mapping. Since the UT uses a polynomial to approximate the mapping, the problem is reduced to a root finding procedure. The case of the second order approximation has a closed form representation for the Gaussian probability density function (PDF). The results of the mapping are three points $G\left(\overline{X} - \sqrt{3}\sigma\right)$, $G\left(\overline{X}\right)$ and $G\left(\overline{X} + \sqrt{3}\sigma\right)$. The coefficients of the polynomial are shown in (12).

$$\begin{aligned} a_0 &= G\left(\overline{X}\right) \\ a_1 &= \frac{1}{2\sqrt{3}}\left[G\left(\overline{X}+\sqrt{3}\sigma\right) - G\left(\overline{X}-\sqrt{3}\sigma\right)\right] \\ a_2 &= \frac{1}{6}\left[G\left(\overline{X}+\sqrt{3}\sigma\right) - 2G\left(\overline{X}\right) + G\left(\overline{X}-\sqrt{3}\sigma\right)\right] \end{aligned} \quad (13)$$

The resulting probability density distribution is calculated by (13).

$$p(G) = \frac{1}{\sqrt{2\pi}}\left\{\frac{e^{-\frac{1}{2}\left[\frac{1}{2}\frac{-a_1+\sqrt{a_1^2-4a_2a_0+4a_2G}}{a_2}\right]^2}}{\sqrt{a_1^2-4a_2a_0+4a_2G}} + \frac{e^{-\frac{1}{2}\left[\frac{1}{2}\frac{-a_1-\sqrt{a_1^2-4a_2a_0+4a_2G}}{a_2}\right]^2}}{\sqrt{a_1^2-4a_2a_0+4a_2G}}\right\} \quad (14)$$

The procedure described in (13) and (14) is well suited for problems with one random variable. However, the calculation of the PDF for the multivariate case is also possible. In such case, the algorithm for determination of the output PDF is summarized as follows :

a) Calculate mapped denormalized sigma points;

b) Using the Moore-Penrose pseudo-inverse, calculate the coefficients of the second order polynomial;

c) Generate the total probability density for all random variables using the polynomial calculated in (b);

d) Integrate the total probability density with respect to all variables resulting in a univariate cumulative density function (CDF) of the solution;

e) Differentiate the CDF to obtain the PDF of the solution.

H. Application of the UT to Electromagnetic Compatibility

The application of the Unscented Transform in EMC is very similar to the use of Monte Carlo (MC) method in such problems. The main advantage of the UT is that the result is calculated with far less computational effort and time compared to MC. Therefore, the UT can model EMC simulations containing different sources of uncertainties as well as different associated probability distributions. It may model position, electrical parameter or manufacturing uncertainties.

This work presents the modeling of uncertainties associated with the shielding effectiveness of cabinets. The UT is used to model uncertainties effects due to the position of the sampled field, and the size and position of the aperture.

III. Results

The problem consists of the statistical characterization of the shielding effectiveness (SE) of a metallic box with an aperture shown in Fig. 1. The box had dimensions (x,y,z) 30 x 12 x 30 cm. The aperture was in the z=0 plane with dimensions (x,y) 10 x 4 cm. The field was sampled in the middle of the box (z=15 cm).

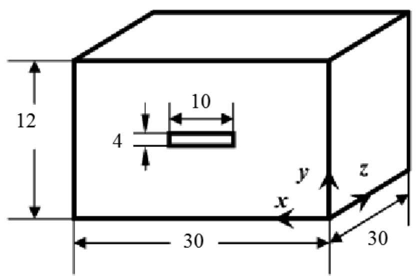

Fig. 1 Metallic box with an aperture

In this problem the random variables were the dimensions of the aperture. The set of sigma points were (10,4), (10,6), (10,2), (12,4), (8,4), (12,6), (12,2), (8,6), and (8,2). The distribution was assumed to be normal.

A. Shielding effectiveness of cabinet with size of the aperture as random variable.

The calculation of the SE was performed with the Mefisto TLM simulator for 400000 timesteps. The discretization was uniform with a spatial size of 1cm. Fig.2 shows the expected value of the Shielding Effectiveness and one standard deviation interval.

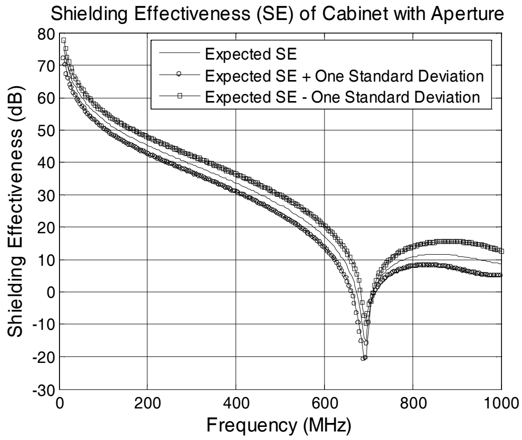

Fig. 2 Average Shielding Effectiveness with one standard deviation margin.

B. Influence of width and height of the aperture

The influence of each parameter of the aperture is shown in Fig,3. The calculation of the influences using (12) shows that the width (x dimension) is the dominant effect at higher frequencies, although the effect of height (y dimension) cannot be neglected in the lower range. The oscillations in the response are caused by the truncation of the time-domain response.

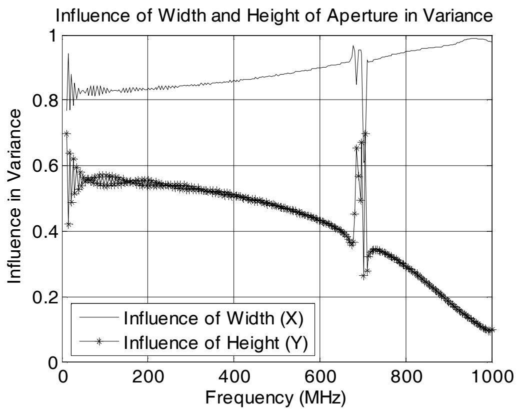

Fig. 3 Influence of aperture dimensions in the variance of the calculated response.

C. Probability Density Function of Minimum SE

The Probability Density (PDF) and Cumulative Probability (CDF) functions shown in Fig.4 were calculated with the steps discussed in section II.G. The cumulative probability function shows that the probability of obtaining a shielding effectivess of less than -16 dB is about 1.5%. The CDF also shows that 95% of the results will be between -20.76dB and -17 dB.

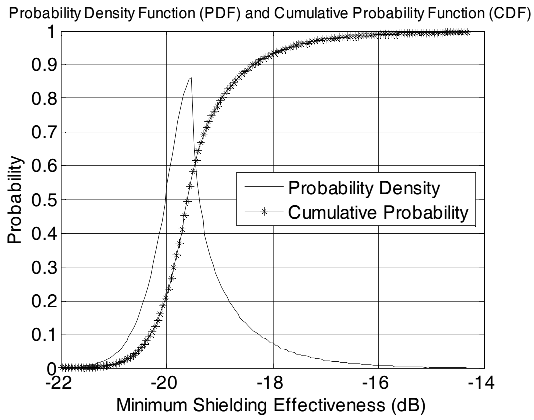

Fig. 4 Probability Functions – PDF and CDF of the minimum shielding effectiveness given the aperture variation.

IV. CONCLUSION

The application of the UT technique in establishing the uncertainty in outputs due to uncertain input parameters was described. It was shown that the moments of the statistical distribution as well as the pdf may be obtained from a small number of simulations which are weighted appropriately. Both the single and multivariate cases were discussed. The accuracy of the UT was presented for the higher order statistical moment calculation. Examples were shown for the shielding effectiveness of cabinets with uncertain slot parameters.

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