Statistical Model for Radiated Susceptibility of TWPs under Complex Random Excitation

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***Abstract*— A statistical model for the prediction of radiated susceptibility (RS) of unshielded twisted wire pairs (TWPs) running above ground, and illuminated by a random field in a complex electromagnetic environment, is presented. The incident field is modelled as a superposition of plane waves with random amplitude, phase, and polarization. The expected value of the average power of RS noise at terminal loads is derived in closed form, as well as the probability density function of the induced voltage.**

Keywords: Statistical EMC models, radiated susceptibility, twisted wire pairs, Field-to-Wire Coupling.

I. Introduction

Radiated susceptibility (RS) is a typical EMC phenomenon that cannot be exhaustively predicted and analyzed by resorting to deterministic models. As a matter of fact, the lack of information about the source of interference (electromagnetic field with partially/totally unknown characteristics), and the victim device (uncertainty of geometrical and electric data) can be successfully overcome only with the introduction of statistical approaches [1].

A promising application is the prediction of RS of cables and wiring harness illuminated by an electromagnetic field described by random parameters [1]. As an example, in [2] a statistical model for the RS of an electrically-short two-conductor transmission line (TL) exposed to a single random plane-wave field is proposed. This result is extended in [3] by considering an incident field composed of a superposition (continuous or discrete) of random waves. In [4], an RS model for electrically-long two-conductor TLs excited by a Rayleigh-channel field is investigated.

In order to be relevant for EMC, such prediction models should be extended to treat wiring structures of practical interest. In line with this aim, this work focuses on unshielded twisted-wire pairs (TWPs), which are among the more diffuse interconnects used in today's communication cable technology. While deterministic models for RS of TWPs are available in the literature [5]-[9], statistical models are still unexplored.

In this work, the general approach reported in [3] is applied to a TWP running above ground, and illuminated by an incident field resulting from the superposition of infinite plane-waves (that is, a plane-wave integral) with random amplitude, phase, and polarization [10].

The proposed model allows for the derivation in closed form of (a) the expected value of the average power of RS noise at terminal loads; (b) the probability density function (pdf) of the magnitude of the voltage induced at terminations.

II. Structure Under Analysis

The structure under analysis is sketched in Fig. 1 and is composed of a TWP running at height h above a perfectly-conducting ground plane. The TWP is composed of two lossless bare conductor with radius r_w and separation s, which are wound into a bifilar helix of pitch $p >> s$ [5], [6].

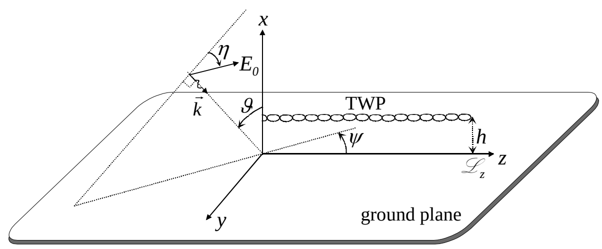
The total distance the TWP extends along the z-axis is $\mathscr{L}_z$ (see Fig. 1), which is related to the wires' length $\mathscr{L} > \mathscr{L}_z$ by the relationship:

$$\mathscr{L}_z = \alpha p \mathscr{L} / (2\pi) \tag{1}$$

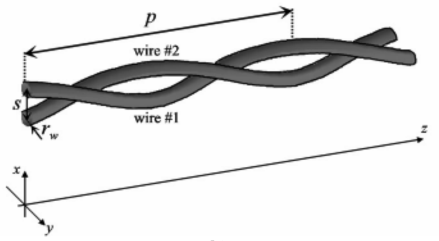
where

$$\alpha = \left[\left(\frac{s}{2} \right)^2 + \left(\frac{p}{2\pi} \right)^2 \right]^{-1/2} \tag{2}$$

is the helix rotation parameter [5], [8].



(a)



(b)

Fig. 1 TWP running above a ground plane

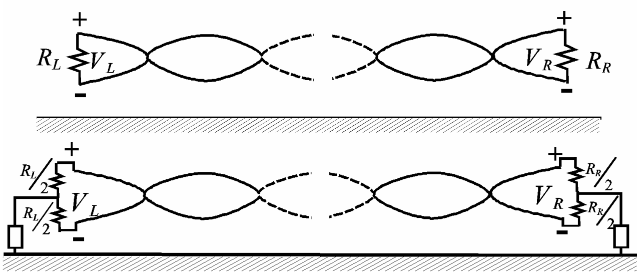


Fig. 2 Balanced terminal loads

The terminations are connected to perfectly-balanced devices (modeled with resistors R_S and R_R). As shown in Fig. 2, such loads can be floating, or grounded with arbitrary impedances (or a combination floating-grounded).

The TWP is illuminated by an incident electromagnetic field resulting from the superposition of uniform plane waves. Each impinging wave is characterized by an electric-field $E = E_0 e^{j\varphi}$ (where E_0 is the field strength, φ is the phase), incidence angles ϑ, ψ, and polarization angle η [see Fig. 1(a)].

III. DETERMINISTIC MODEL

Under the limitation $h < \lambda/10$ (λ being the wavelength), the TWP terminal response to a single plane-wave with known parameters can be expressed in closed form by applying a field-to-wire coupling model based on TL theory [11], [5], [8], [9]. Care has been taken in modeling not only the TWP, but also the so-called differential risers (DRs), i.e., the short wire segments connecting terminal loads to the wire pair. By doing so, an expression of the differential mode (DM) voltage induced at the left (L) and right (R) terminations (see Fig. 2) is obtained in the form:

$$V_{\substack{R\\L}} = \Theta_{\substack{R\\L}}(j\omega, E_0, \varphi, \vartheta, \psi, \eta, R_L, R_R) \qquad (3)$$

For the sake of conciseness, and since this work is mainly devoted to statistical analysis, the explicit expression of (3) (which is very complex) is not reported here.

By exploiting (3), an approximate upper bound (UB) to the magnitude of DM voltages can be derived under the following simplifying assumptions:

(a) According to the common practice, terminal loads are matched, i.e., $R_S = R_R = Z_C$ where Z_C is the DM characteristic impedance of the TWP.

(b) $\omega << 2\pi f_{res}$, where $f_{res} = c_0\alpha/(2\pi + \alpha p\cos\psi\sin\vartheta)$ is a resonance frequency which depends on twisting parameters, ω is the angular frequency, c_0 is the light velocity. This is not a real limitation, because f_{res} exceeds even the limit of validity of the TL model ($h < \lambda/10$) for typical values of twisting parameters.

(c) The TWPs is composed of $N + 1/4$ twists, i.e., an arbitrary integer number N of twists, plus a quarter of residual twist. One can demonstrate that this hypothesis is in line with a worst-case EMC analysis (that is, it leads to a maximized TWP response).

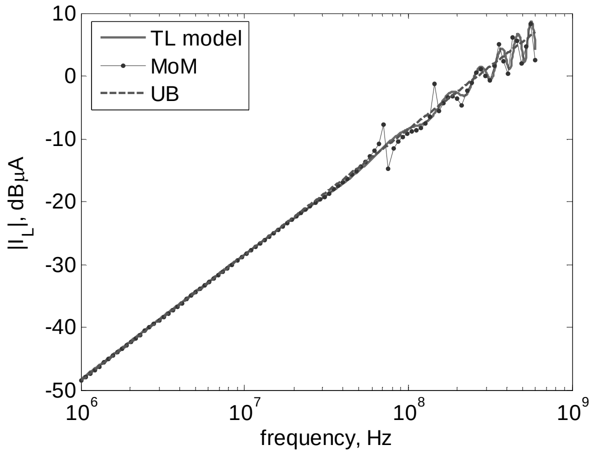


Fig. 3 Current induced in the left TWP termination

By virtue of these assumptions, an UB to the DM voltage induced at the left (L) and right (R) terminations can be cast as:

$$\left|V_{\substack{L\\R}}\right| \cong E_0\, s\,\frac{\omega}{c_0}\left|\underbrace{\frac{e_x}{\alpha} \pm \left(\frac{p}{2\pi} m_y - h\,m_z\right)}_{TWP} + \underbrace{\frac{s}{8} e_x \pm h\,m_z}_{DR}\right| \qquad (4)$$

where

$$e_x = \cos\eta\sin\vartheta \qquad (5)$$

$$m_y = \cos\eta\cos\psi + \cos\vartheta\sin\eta\sin\psi \qquad (6)$$

$$m_z = \cos\eta\sin\psi - \cos\vartheta\sin\eta\cos\psi \qquad (7)$$

In (3), the different terms due to the TWP and the DR contributions have been highlighted. One can demonstrate that (3) is an exact expression in case of electrically-short TWPs (i.e., for $\mathscr{L}_z/\lambda << 1$), whereas it represents an approximate UB for electrically-long lines (with an error of underestimation which is typically less than 5 dB).

As an example, the DM current induced in the left termination [derived from expression (3), solid line] and the UB [derived from expression (4), dashed line] are plotted in Fig. 3. The structure is characterized by $h = p = 5\,\mathrm{cm}$, $s = 4\,\mathrm{mm}$, $r_w = 0.5\,\mathrm{mm}$, $\mathscr{L}_z = 2{,}0125\,\mathrm{m}$, $N = 40$, $E_0 = 1\,\mathrm{V/m}$, $\vartheta = \psi = \eta = 45^o$. Additionally, the full-wave solution of a commercial software tool based on the method of moments (MoM) is plotted in Fig. 3 for comparison and validation (solid line with circular markers) [12]. In the MoM code, each TWP wire has been finely represented as a helix composed of 1003 straight-wire segments (2006 total segments). It is worth mentioning that evaluation of a single frequency point via MoM requires meanly 1 minute on a Pentium IV - 1.6 GHz PC equipped with 1 GB RAM, whereas a few seconds suffice for thousands of frequency points by exploiting the computationally-efficient TL model.

IV. Statistical Model

A. Plane-Wave Integral Representation of Fields

Let us suppose that the TWP is illuminated by a complex field resulting from the superposition of infinite impinging plane waves of infinitesimal amplitude, whose incidence angles describe any possible direction in the upper half space ($0 < \vartheta < \pi/2$, $0 < \psi < 2\pi$). In the following, the shorthand Ω will be used to denote the incidence angles (ϑ, ψ).

Namely, the infinitesimal field amplitude can be written as $dE_0 = \Gamma(\Omega) d\Omega$, where $\Gamma(\Omega)$ is the plane wave angular density, and $d\Omega = \sin\vartheta\, d\vartheta\, d\psi$ is the elemental solid angle. Also, we can introduce both the field phase $\varphi(\Omega)$ and the polarization angle $\eta(\Omega)$ as functions of Ω [3], [10].

In order to model a random field, quantities $\Gamma(\Omega)$, $\varphi(\Omega)$ and $\eta(\Omega)$ are treated as uncorrelated random variables (RVs) in the space Ω [13]. In particular, the following statistical properties are postulated:

$$\langle \Gamma(\Omega_1)\Gamma(\Omega_2)\rangle = D_\Gamma \delta(\Omega_1 - \Omega_2) \tag{8}$$

$$\langle \cos\varphi \rangle = \langle \sin\varphi \rangle = \langle \cos\eta \rangle = \langle \sin\eta \rangle = 0 \tag{9}$$

$$\langle \cos^2\varphi \rangle = \langle \sin^2\varphi \rangle = \langle \cos^2\eta \rangle = \langle \sin^2\eta \rangle = 1/2 \tag{10}$$

where symbol $\langle \cdot \rangle$ is used to denote the expectation operator and $\delta(\cdot)$ is the Dirac distribution. Eq. (8) defines the autocorrelation function of $\Gamma(\Omega)$, corresponding to the autocorrelation of a white stationary random process characterized by a so-called "average intensity" D_Γ which is independent of Ω , i.e., all the waves have equal energy regardless of the incoming direction [3], [13]. As an example, in ideal reverberation chambers, one has $D_\Gamma = E_0^2/(4\pi)$, where E_0^2 is the electric field mean square value.

Concerning (9)-(10), these expressions are satisfied (as a specific example) if φ , η are independent and uniformly-distributed random variables ranging in the interval $[0, 2\pi]$.

B. Statistical Characterization

By exploiting the approximate UB in (4) and the random plane-wave integral representation of fields, the integral representation of the induced voltage can be cast as

$$\left| V_{\substack{L\\R}} \right| = s\frac{\omega}{c_0}\iint_{2\pi} \Gamma(\Omega) \left| A_\pm(\Omega)\cos\eta + B_\pm(\Omega)\sin\eta \right| d\Omega \tag{11}$$

where

$$A_\pm(\Omega) = \sin\vartheta\left(\frac{1}{\alpha} + \frac{s}{8}\right) \pm \frac{p}{2\pi}\cos\psi \tag{12}$$

$$B_\pm(\Omega) = \pm\frac{p}{2\pi}\cos\vartheta\sin\psi \tag{13}$$

are deterministic functions of Ω . Note that in (13), the integration is extended over the upper half space (2π solid angle).

By following the approach reported in [3], one can easily show that $|V_X|$, $X = R, L$, behaves as a Rayleigh-distributed RV whose pdf is

$$f_{|V_X|}(|V_X|) = \frac{|V_X|}{\sigma^2}\exp\left(-\frac{|V_X|^2}{2\sigma^2}\right) \tag{14}$$

where

$$\sigma^2 = \left\langle |V_X|^2 \right\rangle \big/ 2 \tag{15}$$

By virtue of (8)-(11), the mean square voltages in (15) can be cast as

$$\left\langle |V_{\substack{L\\R}}|^2 \right\rangle = \frac{1}{2}\left(s\frac{\omega}{c_0}\right)^2 D_\Gamma \iint_{2\pi} [A_\pm^2(\Omega) + B_\pm^2(\Omega)] d\Omega \tag{16}$$

Evaluation of (16) in closed-form yields:

$$\left\langle |V_L|^2 \right\rangle = \left\langle |V_R|^2 \right\rangle = \frac{1}{2}\left(s\frac{\omega}{c_0}\right)^2 D_\Gamma \times \frac{\alpha^2(16p^2 + s^2\pi^2) + 64\pi^2 + 16s\alpha\pi^2}{48\pi\alpha^2} \tag{17}$$

Consequently, the expected value of the average power due to RS noise at loads is:

$$\langle P_{av} \rangle = \left\langle |V_R|^2 \right\rangle \big/ Z_C = \left\langle |V_L|^2 \right\rangle \big/ Z_C \tag{18}$$

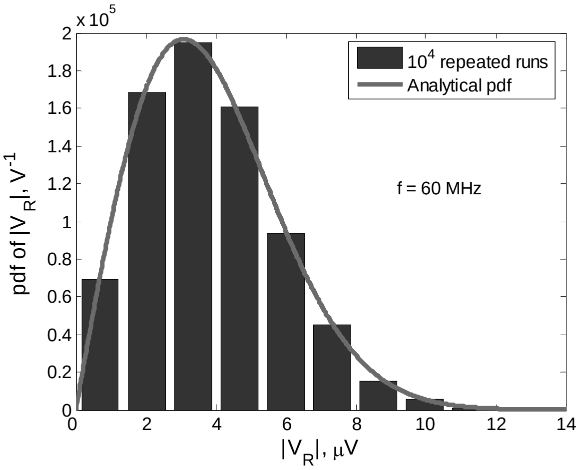
C. Model Validation

The normalized histogram of $|V_R|$, obtained by processing 10^4 repeated-runs [3], and the analytical Rayleigh pdf in (14) are compared in Fig. 4(a) (at 60 MHz) and Fig. 4(b) (at 300 MHz). These simulations refer to a structure characterized by: $h = 2\,\text{cm}$, $p = 1\,\text{cm}$, $s = 1\,\text{mm}$, $r_w = 0.25\,\text{mm}$, $D_\Gamma = 1\,\text{V}^2/\text{m}^2$, $N = 40$, $\mathscr{L}_z = 2.0125\,\text{m}$.

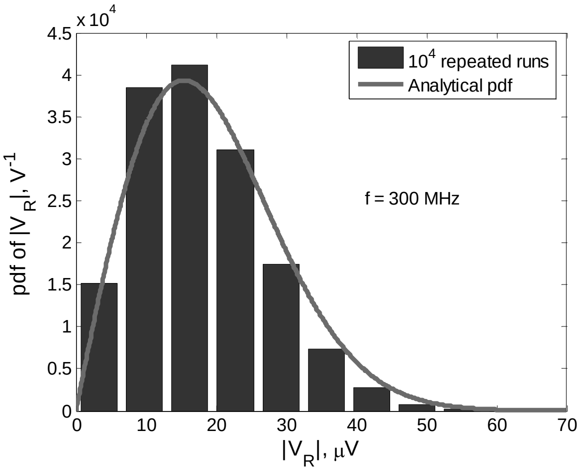
It is worth recalling that analytical expressions (14)-(18) derive from the simplified UB given in (4), whereas numerical simulations exploit the complete TL model in (3). As mentioned above, the UB underestimates the real RS level for electrically-long lines. Notwithstanding this, a good agreement was found between the proposed statistical model and repeated-run analysis.

V. Discussion and Conclusion

The expected value of the average power P_{av} dissipated in terminal loads has been derived in (17)-(18). Concerning this parameter, one can observe that: (a) it provides a concise description of RS noise in statistical terms; (b) it increases with the square of frequency; (c) it does not depend on the TWP length and height above ground; (d) it depends on the



(a)



(b)

Fig.4 Normalized histogram and pdf of the voltage induced in the right load

twist pitch *p* and the wires' separation *s*. The latter aspect is exemplified in Fig. 5, where P_{av} is plotted versus *p* for different values of *s*, and for $f = 600\,\text{MHz}$, $D_\Gamma = 1\,\text{V}^2_{\text{rms}}/\text{m}^2$, $r_w = 0.25\,\text{mm}$. Given that, in practical applications, the separation is a fixed parameter (determined by the diameter of the wire insulation and the desired characteristic impedance), reduction of the twist pitch is the sole expedient that the installer may adopt to reduce RS noise. Fig. 5 shows that the expected average power decreases up to 20-30 dB passing from $p = 15\,\text{cm}$ to $p = 0.5\,\text{cm}$.

The proposed model has been developed with reference to a plane-wave integral extended to the whole space. However, the derivation can be easily extended to treat either a continuous or discrete wave superposition (i.e., an integral or a sum of plane waves) illuminating the line from a specific element of the solid angle [3], allowing for better modeling of different electromagnetic environments.

To conclude, one can note that a number of simplifying assumptions should be removed in order to extend the applicability of results. For instance, losses and dielectric insulation are not accounted for, as well as the effect of

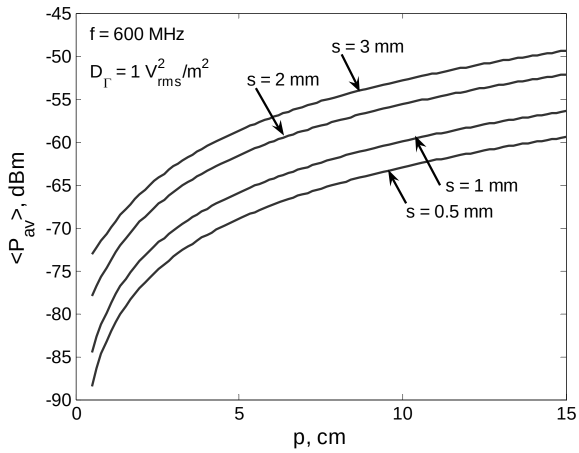


Fig. 5 Expected value of the average power due to RS noise as a function of the twist pitch *p* and wires' separation *s*.

possible unbalance of terminal loads, which gives rise to conversion between common and differential mode [7]. These issues represent hints for future works.

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