Antenna Pattern Reconstruction by Spherical Vector Waves for Spherical Antenna Measurement

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***Abstract*—The acquisition of three-dimensional radiation pattern is important for characterizing antenna. Spherical antenna measurement is a practical approach to measured antenna pattern; however, in most cases, the measured samples can not be used directly, and reconstruction is needed to achieve useful data. Spherical vector waves are preferred as tool for pattern reconstruction, and the band-limited property of spherical wave expansion enables to analyse reconstruction by the sampling theorem. This research identifies the limitation of conventional algorithm, and proposes novel approaches to overcome the limitation.**

I. Introduction

The fundamental requirement for characterizing antenna is the acquisition of its full pattern, that is, the complex polarimetric antenna response in angular domain. Antenna pattern reconstruction refers to the determination of the radiation pattern of an antenna under test (AUT), by measuring the amplitude and phase of the electromagnetic signal received from the AUT.

Spherical antenna measurement is the unique approach to measure antenna pattern in spherical geometry. The complex electric field radiated from vertical or horizontal polarized AUT be measured through scanning over a spherical surface in both the azimuthal and elevation dimensions. However, due to the limitation of measurement set-up, the scanning in measurement may not be taken on whole sphere but only on some part of it; on the other hand, due to the limitation of measurement time, only coarse sampling interval is practical. The measured samples could be either incomplete or coarse, therefore, extrapolation and interpolation are needed to reconstruct the antenna pattern.

This research focuses on analytic and continuous antenna pattern reconstruction. Generally, the plane wave representation and spherical wave representation of antenna radiation pattern are utilized for the reconstruction. While the plane wave model has infinite angular resolution regardless of the antenna size, the spherical wave model has finite angular resolution determined by the antenna size [1]; in this research, the spherical wave model is preferred.

The constrained iterative restoration algorithm in [2] has been extensively explored and applied for a long time; several years ago, in [3] [4], the spherical wave model and its band-limited property are connected with the classical iterative algorithm, to restore signal and reconstruct radiation pattern. Since the measured samples are given as condition, figuring out what kind of samples could be reconstructed, in other words, how incomplete (measurement range) and how coarse (sampling interval) samples could be tolerated by the algorithm, are very important.

The main objective of this work is to study on the antenna pattern reconstruction by spherical vector waves in the perspective of sampling theorem. Limitations of conventional iterative algorithms are identified, and proposed algorithm will be introduced and validated by numerical investigations.

II. Spherical Wave Expansion

Spherical wave functions (SWF) [5], are homogeneous solutions to vector Helmholtz equation in the spherical coordinates. The radiation pattern outside the minimum sphere of antenna can be expanded into a weighted sum of spherical wave functions, that is, spherical wave expansion (SWE) [5] :

$$\mathbf{E}(r,\theta,\phi) = k\sqrt{\eta}\sum_{s=1}^{2}\sum_{n=1}^{N}\sum_{m=-n}^{n} Q_{smn}\mathbf{F}_{smn}(r,\theta,\phi) \quad (1)$$

where:

m: mode indices for ϕ direction
n: mode indices for θ direction
$\mathbf{F}_{smn}(r,\theta,\phi)$: SWF, TE ($s=1$) and TM ($s=2$) mode SWF compose of a complete orthogonal set
Q_{smn}: spherical wave coefficient (SWC)
$k\sqrt{\eta}$: coefficient to ensure the normalization condition that unit SWC corresponds to 1 $\mathrm{W}^{\frac{1}{2}}$, k is the wavenumber
N: truncation number

The expanded series can be truncated at a finite number determined by antenna's minimum sphere. The minimum sphere of an antenna is defined as the smallest possible spherical surface, which is centred at the coordinate origin and could enclose the antenna completely.

The highest spatial frequency for a radiating field is determined by the wavelength as $1/\lambda$; according to the sampling theorem, its minimum sampling interval should be $\lambda/2$, thus its sampling number is no more than $\lfloor 2kr_0 \rfloor$ per circumference of the minimum sphere, where r_0 is the radius of the minimum sphere. To represent the radiation field, the spherical wave number should be no less than $\lfloor 2kr_0 \rfloor$, therefore the bandwidth (truncation number) for the expansion should be at least $\lfloor kr_0 \rfloor$.

By increasing the bandwidth, the evanescent components of the field can be retained. Therefore, in this work, truncation number is defined as $N \geq \lfloor kr_0 \rfloor$, and the brackets indicate the floor function, the largest integer smaller than or equal to kr_0.

Generally SWC are unknown, and field at certain observing sphere is achieved to calculate SWC; once the SWC has been solved, the field can be calculated at anywhere outside the antenna minimum sphere.

III. Antenna Pattern Reconstruction for Spherical Antenna Measurement

The measured samples are conventionally reconstructed by iterative SWE algorithm [3] [4]. For a specific reconstruction algorithm, how coarse and how incomplete the samples are will influence the accuracy of the reconstruction pattern. Investigation will be conducted to figure out the influences of sampling interval and measurement range on the reconstruction accuracy by conventional iterative SWE algorithm.

A. Influence of Sampling Interval on Reconstruction Accuracy

To simply study on the influence of sampling interval on reconstruction accuracy, a set of complete pattern samples over the whole scanning sphere is necessary: $\mathbf{E}(\theta_{l_1}, \phi_{l_2}), \theta_{l_1} \in [0 : \Delta\theta : \pi], \phi_{l_2} \in [0 : \Delta\phi : (2\pi - \Delta\phi)]$, where $\Delta\theta$ and $\Delta\phi$ are the sampling interval in elevation and azimuthal dimensions respectively. Conventionally, SWC can be calculated from the achieved samples, and interpolation can be conducted along with the reconstruction of antenna pattern by SWE.

According to the sampling theorem and the band-limited property of SWE, the sampling number per circumference should be at least twice the truncation number $2N$, that is, the sampling interval in measurement should be less than $\frac{\pi}{N}$. Since the truncation number is determined by the minimum sphere radius of AUT, the requirement for sampling interval is determined by the minimum sphere radius as well.

In practical measurement, AUT is not necessarily located in the center of the global coordinate. For example, the pivot of circular antenna array is usually located in the coordinate center, therefore each antenna element is deviated from the center. For the same AUT, the minimum sphere radius when it is deviated from coordinate center is obviously bigger than the radius when it is centred. Therefore, the minimum sampling interval required (the required sampling number per circumference) for the deviated case should be smaller (bigger) than that for the centred case.

Taken the open-ended rectangular waveguide (OERW) as an example of AUT for illustration, the normalized mean square errors of the reconstructed pattern from samples with different sampling intervals are investigated, to assess the differences between the reconstructed pattern and the reference pattern, as is shown in Fig. 1.

With small enough sampling intervals, the reconstruction error will stay the same as the truncation error; the bigger the truncation number is, the smaller the reconstruction error will be. On the other hand, the Nyquist sampled, $2N$ points

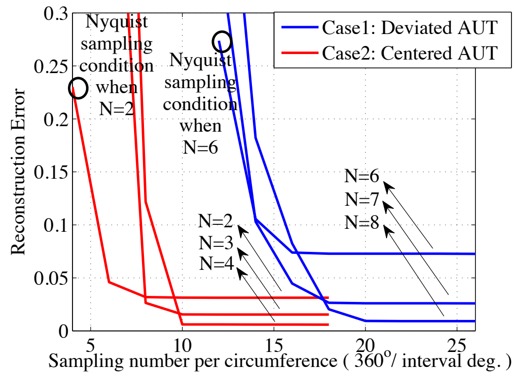

Fig. 1. Error of reconstructed pattern by conventional algorithm for both deviated OERW and centred OERW

TABLE I
OERW SIMULATION PARAMETERS

Wavelength	27.3mm
Aperture size (a× b)	$0.84\lambda \times 0.37\lambda$
Aperture location	1) deviated case: move along x-axis by λ from center (N_{min} =6) 2) center-located (N_{min} =2)
Observing distance	45.5λ

per circumference in the spherical measurement ($\frac{\pi}{N}$ sampling interval), is not definitely sufficient for the algorithm to reconstruct accurately; $2(N+1)$ points or more could be enough, therefore the sampling interval should be no larger than $\frac{2\pi}{2(N+1)}$. In addition, comparing between both cases when truncated at $N = \lfloor kr_0 \rfloor$, for deviated case, although the truncation number used is larger than the centred case, the reconstruction error is even bigger; what is worse, the required sampling interval is smaller.

There is a trade between required sampling interval and the truncation number used; for larger N, the truncation error is smaller but more samples are needed to represent the higher modes.

B. Influence of Measurement Range on Reconstruction Accuracy

To simply study on the influence of measurement range on reconstruction accuracy, a set of incomplete samples on some part of the scanning sphere is the target. Here is a typical incomplete measured data set: $\mathbf{E}(\theta_{l_1}, \phi_{l_2})$, $\theta_{l_1} \in [0 : \Delta\theta : \theta_{scan}], \phi_{l_2} \in [0 : \Delta\phi : (2\pi - \Delta\phi)]$, where θ_{scan} represents the measurement range.

Conventional iterative algorithm involving SWE [3] [4] can be concluded as :

1) extend incomplete samples by zero-padding to cover entire sphere
2) calculate SWC using extrapolated data
3) calculate new pattern by SWE using SWC achieved in former step

4) update calculated pattern data by replacing the measured samples within the scanned area

Steps 2 to step 4 are repeated iteratively.

Considering the practical measurement again, AUT is not necessarily located in the center of global coordinate system. Take the deviated and centred OERW in Table I. again as example; given absolutely sufficient sampling interval $\Delta\theta = \Delta\phi = \frac{\pi}{60}$, the normalized mean square error of the reconstructed pattern from samples with different measurement range will be investigated. Fig. 2 shows the error of reconstructed patterns for both the deviated and the centred cases, with samples achieved through spherical antenna measurement with different measurement range.

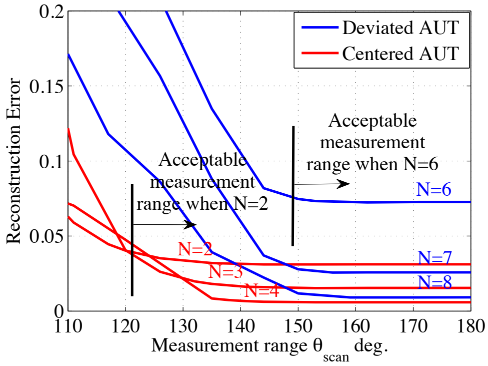

Fig. 2. Error of reconstructed pattern by conventional algorithm for both deviated OERW and centred OERW

It is found that the reconstruction error will stay the same as truncation error with adequate measurement range, and the bigger the truncation number is, the smaller the convergent reconstruction error. Besides, when truncated at $N = \lfloor kr_0 \rfloor$, for the deviated case, the algorithm require larger measurement range than the centred case, but the reconstruction accuracy is even lower. According to sampling theorem, the unmeasured range on scanning sphere should be no more than the biggest allowable sampling interval $\frac{2\pi}{2(N+1)}$. Therefore, the measurement range for elevation and azimuthal dimension should be no less than $\pi(1 - \frac{1}{N+1})$ and $2\pi(1 - \frac{1}{2(N+1)})$ respectively. Again, the requirement measurement range is decided by the truncation number, furthermore determined by the minimum sphere radius. Obviously, the bigger the radius of AUT minimum sphere is, the larger the measurement range will be required by the conventional algorithm. The trade between truncation number and measurement range should be handled properly.

C. Limitation of Conventional Algorithm

Due to the trade between truncation number, and required measurement range and sampling interval, the conventional algorithm is limited by the radius of the AUT minimum sphere in the global coordinate of measurement set-up. To overcome the limitation, algorithm enabling to minimize the AUT minimum sphere radius in the iterative SWE process will be proposed.

D. Proposed Algorithm

1) Applying Translation and Rotation of Spherical Waves: For the deviated case, the samples are measured in the coordinate system that AUT is not centred; there are ways to translate the measured samples into the primed coordinate system which center the AUT, and that is the problem of coordinate translation. Generally, coordinate translation could be accomplished by Cartesian method and Spherical method, and the former method will not be illustrated here.

Arbitrary translations of coordinate system can be accomplished by a succession of three operations: rotation, axial translation, and inverse rotation. To achieve the radiation pattern in primed coordinate systems, the representative spherical waves function could be rotated, translated and inverse rotated. In Hansen's book [5], both the rotation and translation of spherical waves are provided; although only z-directed axial translation is described, it is sufficient for the coordinate translation.

[5] Euler angles $(\chi_o, \theta_o, \phi_o)$ are introduced to describe the rotation from the unprimed to the primed coordinate system. Rotation angle about z-axis is denoted as ϕ_o, and rotation angle about y-axis is denoted as θ_o, and rotation angle about z-axis is denoted as χ_o. Through rotation, the spherical wave function $\mathbf{F}^{(c)}_{smn}(r, \theta, \phi)$ in the unprimed coordinate system (r, θ, ϕ) can be achieved as the combination of spherical waves defined in the primed system (r', θ', ϕ'):

$$\mathbf{F}^{(c)}_{smn}(r, \theta, \phi) = \sum_{\mu=-n}^{n} e^{jm\phi_o} d^n_{\mu m}(\theta_o) e^{j\mu\chi_o} \mathbf{F}^{(c)}_{s\mu n}(r', \theta', \phi') \tag{2}$$

where the rotation coefficient $d^n_{\mu m}(\theta_o)$ is a real function of θ.

[5] Suppose the primed coordinate system (r', θ', ϕ') is translated a distance A in the positive direction of z-axis of the unprimed coordinate system (r, θ, ϕ), then spherical wave function $\mathbf{F}^{(c)}_{smn}(r, \theta, \phi)$ in the unprimed coordinate system will be achieved through:

$$\mathbf{F}^{(c)}_{s\mu n}(r, \theta, \phi) = \sum_{\sigma=1}^{2} \sum_{\upsilon=|\mu|, \upsilon\neq 0}^{\infty} C^{sn(c)}_{\sigma\mu\upsilon}(kA) \mathbf{F}^{(1)}_{\sigma\mu\upsilon}(r', \theta', \phi') \tag{3}$$

when $r' < |A|$, and

$$\mathbf{F}^{(c)}_{s\mu n}(r, \theta, \phi) = \sum_{\sigma=1}^{2} \sum_{\upsilon=|\mu|, \upsilon\neq 0}^{\infty} C^{sn(1)}_{\sigma\mu\upsilon}(kA) \mathbf{F}^{(c)}_{\sigma\mu\upsilon}(r', \theta', \phi') \tag{4}$$

when $r' > |A|$. Function $C^{sn(c)}_{\sigma\mu\upsilon}(kA)$ is the translation coefficients.

Measured samples in the primed coordinate system can be achieved by utilizing the rotation and translation of spherical waves; then pattern reconstruction could be conducted by iterative SWE algorithm in the primed coordinate system where AUT is centred. The disadvantage of this approach is that the calculation is quite complex.

2) Applying Translational Phase Shift: Referring to the antenna array theory, a translation in space becomes a phase shift in the Fourier domain, and the relative displacements of antenna elements with respect to each other introduce relative phase shifts in the radiation vector. In the case of deviated AUT, by translational phase shift of measured samples, the amplitude pattern stays the same, and the phase pattern is smoothed enabling to be reconstructed with small truncation number. This is much easier than the approach applying rotation and translation of spherical waves, but with assumption that the scanning sphere radius is much bigger than the shifted distance. Through shifting the deviated AUT pattern to coordinate center, the minimum sphere radius will always be minimized; both the tolerance for sampling interval and measurement range will be extended. The proposed algorithm guarantees the same requirements of both sampling interval and measurement range for the same AUT, and the antenna pattern reconstruction will not be limited by the AUT location any more. Fig. 3 gives the proposed algorithm scheme.

Given the incomplete measured samples distributed as $0 : \frac{\pi}{6} : \frac{2\pi}{3}$ in θ dimension and $0 : \frac{\pi}{6} : \frac{11\pi}{6}$ in ϕ dimension, for the deviated OERW case in Table 1, Fig. 4 and Fig. 5 provide a set of examples of the E-plane amplitude of reconstructed pattern by conventional algorithm and proposed algorithm respectively; obviously, the proposed algorithm could well reconstruct the incomplete samples while the conventional algorithm could not.

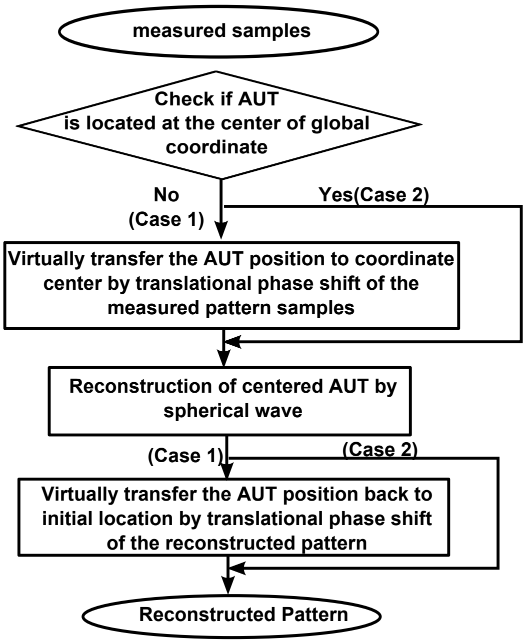

Fig. 3. Proposed algorithm for both complete and incomplete spherical measurement

IV. Conclusion

This research mainly concerns the analytical reconstruction of radiation pattern from measured samples in the spherical antenna measurement. Spherical vector waves are the main tool, and the band-limited property of SWE makes it possible to evaluate the reconstruction by sampling theorem. It is found that the conventional algorithm is limited by the AUT location. To overcome the limitation, algorithm applying coordinate translation technique by rotation and translation of spherical waves, as well as the translational phase shift techniques are proposed. This research mainly benefit the characterization of AUT under given severe sampling condition.

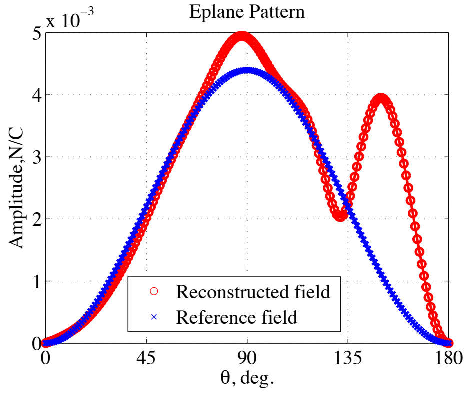

Fig. 4. E-plane of reconstructed pattern by conventional algorithm

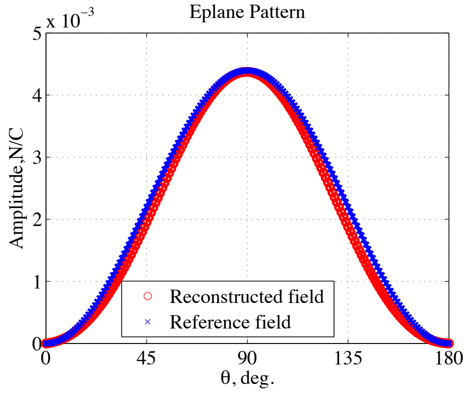

Fig. 5. E-plane of reconstructed pattern by proposed algorith

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