Imaging of Objects Near Rough Surfaces Based on Time-reversal Signal Processing and Surface Flattening Transform

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***Abstract* – This paper considers the imaging of objects located close to rough surfaces such as ocean or terrain. If transmitters and receivers are also located close to rough surfaces, incident wave is no longer a plane wave, nor a spherical wave in free space and it is necessary to consider Green's functions with the point source located close to the surface, similar to the Sommerfeld dipole problem.**

This paper considers the near surface imaging by making use of time-reversal imaging and surface flattening transform. Surface flattening transform convert the rough surface problem into flat surface with inhomogeneous random medium. MCF (Mutual Coherence Function) is obtained and used to obtain imaging of point target near rough surface, making use of the multi-static data matrix, time-reversal matrix, the eigenvectors, and the steering matrix. Numerical examples are given.

An important point is that integration of stochastic wave propagation and signal processing is necessary to obtain imaging through complex clutter environment.

I. Introduction

Near rough surface imaging is an important problem with many applications in the detection and imaging of objects near ocean and terrain surfaces. Objects may be located above or below rough surfaces and the detecting sensors are all located near or on the surface. This is related to the problem of radio wave propagation over smooth earth. However, if the surface is rough, several additional problems need to be considered. Since the sensors are near the surface, the wave incident on rough surface is no longer spherical or beam wave, usually assumed for LGA (Low Grazing Angle) scattering. Instead, we need to consider stochastic Green's function with the source located near the surface. Both coherent and incoherent waves travel over the rough surface interacting with rough surface.

The development of the stochastic Green's function near rough surface has been studied, in particular when rms height is small based on Dyson and Bethe-Salpeter's equations. In this paper, we present an alternative approach which may be applicable to rough surfaces with small and large rms. We also consider not only the rough surface scattering, but also the inclusion of the array signal processing in the near surface imaging.

First, we discuss surface flattening transform. In the late 1970s, Tappert proposed to solve the rough surface problem by transforming rough surface to flat surface (e.g. [4]-[8]). By this transformation, free space above the rough surface becomes inhomogeneous, but the rough surface becomes flat surface. The rough surface problem can then be solved by solving inhomogeneous medium with flat surface boundary.

In recent years, the coordinate transformation has been proposed to transforming the medium and the boundary to create inhomogeneous and anisotropic medium, in which the wave behaves as if it is in free space, thus creating "cloaking" and much interest has been generated. We make use of this technique to solve our scattering problem.

The surface flattening transform is equivalent to "transformation EM". It transforms the rough surface into flat surface, but the free space above rough surface is transformed to inhomogeneous random medium with flat boundary.

This paper first discusses the surface flattening transform (e.g. [1]-[8]) and then present a solution based on modified Rytov transform. The relations with transformation electromagnetics are discussed (e.g. [9]-[11]). We then present time-reversal imaging of objects based on flattening transform using multi-static data matrix, time-reversal matrix, the eigenvectors and the steering vectors (e.g. [12]-[17]). Some numerical examples are given.

II. Surface Flattening Transform and Relation with Transformation Electromagnetics

Let us consider the wave equation in free space.

$$(\nabla^2 + k^2)G = -\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

The rough surface height is given by $\zeta(x,y)$. The surface is now transformed to the transformed space in (x', y', z').

$$z' = z - \zeta(x,y), x' = x, y' = y$$

This will transform the wave equation from (x, y, z) to (x', y', z').

In transformed space with flat surface, the wave equation is transformed to

$$(\nabla'^2 + k^2 + F)G = -\delta(x'-x_0)\delta(y'-y_0)\delta\left(z' - \left[z_0 - \zeta(x,y)\right]\right)$$

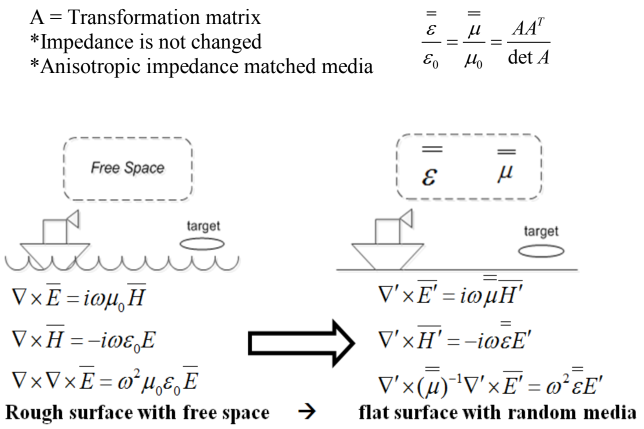
$$F = -2\nabla\zeta \cdot \nabla' \frac{\partial}{\partial z'} - \left(\nabla^2\zeta\right)\frac{\partial}{\partial z'} + \left|\nabla\zeta\right|^2 \frac{\partial^2}{\partial z'^2} = random$$

Boundary condition: G=0 on flat surface.

Note that F represents the inhomogeneous medium containing the slope $\nabla\zeta$, the radius of the curvature $\nabla^2\zeta$, and the magnitude of the slope $|\nabla\zeta|^2$

This formulation is equivalent to the transformation of Maxwell's equation. Several papers have been published for solving this equation including path integral, numerical simulation and others. However, our purpose is not simply to solve this equation, but we need to obtain MCF (mutual coherent function) and use the solution to form signal processing using time reversal imaging function.

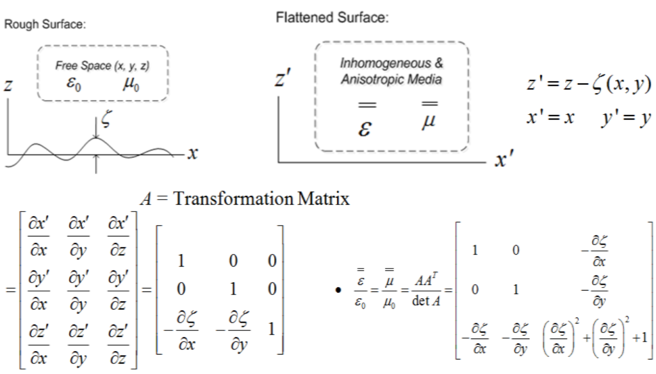
Relations with transformation electromagnetics are shown in Fig. 1 and Fig. 2. Fig. 3 shows Green's function and modified Ryton solution.



$\frac{\bar{\bar{\varepsilon}}}{\varepsilon_0} = \frac{\bar{\bar{\mu}}}{\mu_0} = \frac{AA^T}{\det A}$

$\nabla\times\bar{E} = i\omega\mu_0\bar{H}$, $\nabla\times\bar{H} = -i\omega\varepsilon_0 E$, $\nabla\times\nabla\times\bar{E} = \omega^2\mu_0\varepsilon_0\bar{E}$ → $\nabla'\times\bar{E}' = i\omega\bar{\bar{\mu}}\bar{H}'$, $\nabla'\times\bar{H}' = -i\omega\bar{\bar{\varepsilon}}E'$, $\nabla'\times(\bar{\bar{\mu}})^{-1}\nabla'\times\bar{E}' = \omega^2\bar{\bar{\varepsilon}}E'$

Fig.1. Relations with transformation EM(Cloaking)



$z' = z - \zeta(x,y)$, $x' = x$, $y' = y$

$$A = \begin{bmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} & \frac{\partial x'}{\partial z} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial z} \\ \frac{\partial z'}{\partial x} & \frac{\partial z'}{\partial y} & \frac{\partial z'}{\partial z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{\partial\zeta}{\partial x} & -\frac{\partial\zeta}{\partial y} & 1 \end{bmatrix} \quad \bullet\ \frac{\bar{\bar{\varepsilon}}}{\varepsilon_0} = \frac{\bar{\bar{\mu}}}{\mu_0} = \frac{AA^T}{\det A} = \begin{bmatrix} 1 & 0 & -\frac{\partial\zeta}{\partial x} \\ 0 & 1 & -\frac{\partial\zeta}{\partial y} \\ -\frac{\partial\zeta}{\partial x} & -\frac{\partial\zeta}{\partial y} & \left(\frac{\partial\zeta}{\partial x}\right)^2 + \left(\frac{\partial\zeta}{\partial y}\right)^2 + 1 \end{bmatrix}$$

Fig.2. Relations with transformation EM (Continued)

Green's Function

$(\nabla^2 + k^2)U = -FU$

- Path Integral
- Modified Rytov
- $U = U_c \exp\psi = U_0 + U_1 + U_2 + \cdots$
- $\psi = \psi_1 + \psi_2 \ldots = \frac{U_1}{U_0} + \left(\frac{U_2}{U_0} - \frac{1}{2}\psi_1^2\right) + \cdots$
- $< U_a U_b^* >= U_{a0} U_{b0}^* < \exp(\psi_a + \psi_b^*) >$
  $= U_{a0} U_{b0}^* \exp[< \psi_a > + < \psi_b^* > + \frac{1}{2} < (\psi_a + \psi_b^*)^2 >]$
- Energy Conservation
  2*Re$< \frac{U_2}{U_0} > = - < |\psi_{a1}|^2 >$ (Yura)
- $< U_a U_b^* >= U_{a0}\, U_{b0}^* \exp\left(-\frac{1}{2}D\right)$
  where $D = < |\psi_{a1}|^2 > + < |\psi_{b1}|^2 > -2 < \psi_{a1}\psi_{b1}^* >$

$(\nabla^2 + k^2)U = -FU$

$U = U_c \exp\psi$

$$\psi \approx \int \frac{G(\bar{r} - \bar{r}')F(\bar{r}')U_0(\bar{r}')}{U_0(\bar{r})} dv'$$

$$F = -2\underbrace{\left(\frac{\partial\xi}{\partial x}\frac{\partial}{\partial x'} + \frac{\partial\xi}{\partial y}\frac{\partial}{\partial y'}\right)\frac{\partial}{\partial z'}}_{\text{Slope}} - \underbrace{\left(\frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial x^2}\right)\frac{\partial}{\partial z'}}_{\text{Radius of Curvature}} + \underbrace{[\left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2]\frac{\partial^2}{\partial z'^2}}_{\text{(Slope)}^2}$$

Fig.3. Green's Function

III. MCF AND TIME-REVERSAL IMAGING

From the Green's function shown in Fig.3, we first calculate the Mutual Coherence Function (MCF), $< U_a U_b^* >$.For the Time-Reversal imaging, we use the first equation in Fig.6, where $<G_i G_j^*>$ is MCF, and G_{si} and G_{sj} are the steering vector. MCF contains the effects of the slope, the radius of curvature, and (slope)^2 as fhsown in Fig.3. This is then incorporated into theTR imaging function in Fig.6.

Fig. 4 and Fig. 5 show example of MCF (Mutual Coherence Functions) and time-reversal imaging. Fig. 6 summarizes our recent work on integrating signal processing and propagation.

In Fig.6, we also include the imaging function for TR-MUSIC (Multiple-Signal-Classification), Capon minimum variance, Modified Beam former, and SAR imaging functions. They are all expressed using the same notation and the multi-static data, time-reversal matrix, the eigenvector, and the steering matrix.

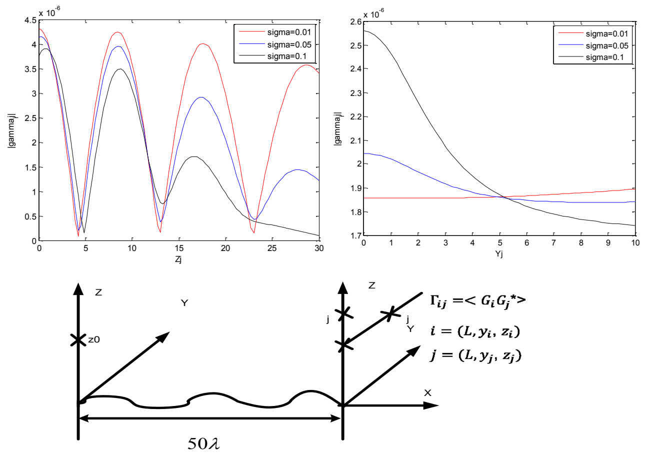


Fig.4. Mutual Coherence Function based on Surface Flattening Transform. Point source is located at (z_0=3λ, x=0). Vertical MCF is measured at x_0=50λ as function of z. Horizontal MCF is measured at z0=3λ as function of y.

Horizontal Array Imaging Function

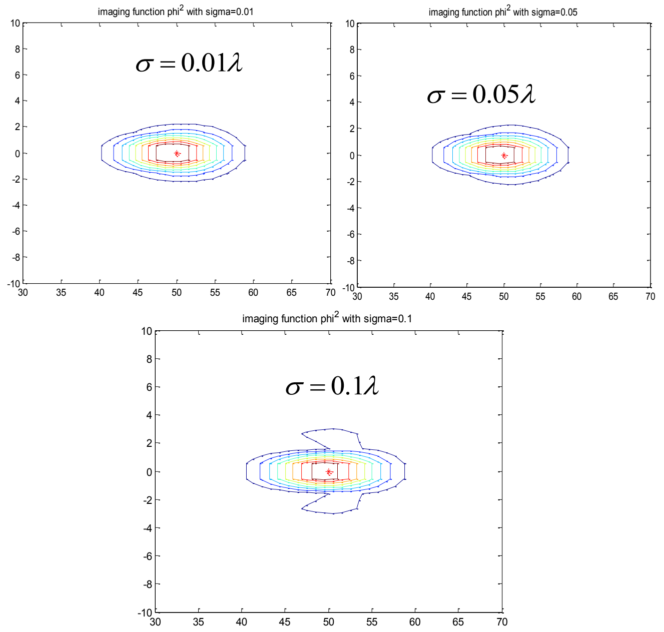


Fig.5. Time Reversal Imaging Function for Different (*rms* height). Target is located at (x=50λ,z=3λ,y=0). Horizontal array: image is on x-z plane, y=0. Only the horizontal array imaging is shown. *N*=40 (number for array elements) $d = 0.5\lambda, z_0 = 3\lambda$

Summary Equations

$$\text{TR imaging} = \int d\omega_1 d\omega_2 U_1^2 U_2^2 \lambda_1^2 \sum_i \sum_j \langle G_i G_j^* \rangle \left(G_{si}^* G_{sj} \right)$$

$$\text{TR MUSIC} = \psi_{MUSIC} = \frac{1}{\sum_{p=M+1}^{N-1} \int d\omega U^2 \left| [g_s]' v_p \right|}$$

$$\text{Capon} = \psi_{Capon} = \frac{1}{\int\int d\omega |U|^4 [g_s(\omega_1)]' [R]^{-1} [g_s(\omega_2)]}$$

$$\text{Modified beamform} = \int d\omega_1 d\omega_2 U_1^2 U_2^2 \sum_i \sum_j \sum_p \sum_q \langle G_i G_j G_p^* G_q^* \rangle \left(G_{si} G_{sj} G_{sp}^* G_{sq}^* \right)$$

$$\text{SAR} = \int d\omega_1 d\omega_2 |U_1|^2 |U_2|^2 \sum_i \sum_j \langle G_i^2(\omega_1) G_j^{*2}(\omega_2) \rangle \left(G_{si}^{*2}(\omega_1) G_{sj}^2(\omega_2) \right)$$

G_i=Stochastic Green's function
G_{si}=Steering vector
V_p=Eignevector of time-reversal matrix
ω_1=Frequency spectrum of incident wave

Image resolution comparison

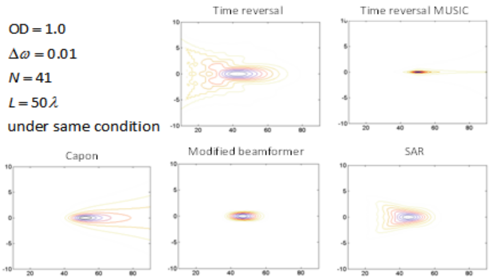


Fig.6.Propagation and Signal Processing- Integration and Comparison New Imaging Techniques (from Ishimaru, A., S. Jaruwatanadilok, Y. Kuga, Feb 2012)

IV. Conclusion

This paper discusses integration of surface flattening transform, propagation, and signal processing for imaging of objects near rough surfaces. Relations with transformation electromagnetics are also discussed.

This paper emphasizes the importance of the integrating the signal processing and stochastic wave propagation and scattering to obtain the imaging of objects in the presence of complex clutter environments and presents a unified theoretical basis for imaging functions through complex random media.

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