Diffraction from rotationally symmetric ensembles of open and closed arbitrarily shaped screens

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***Abstract*—This paper analyses scalar scattering and diffraction from ensembles of coaxial axi-symmetric closed or open screens that are arbitrarily shaped. Plane wave and complex point source excitations are examined in the frequency domain by the semi-analytic Method of Regularisation. The Dirichlet boundary value problem for the Helmholtz equation is reduced analytically to a well-conditioned coupled infinite system of algebraic equations. This system is then effectively solved by a truncation method with any pre-specified accuracy, strictly depending on the truncation number. Applications related to plane wave excitation include accurate studies of coupled cavity resonators, of open cavities with insertions and of configurations where targets lie in the shadow zone. Our investigation employing beam-like excitation focuses on the accurate analysis of a front-fed parabolic reflector antenna. The tapered illumination is simulated by a complex-point source, supported by a back plate. The maximum permissible extent of the supporting back plate that does not unduly compromise the operation of the reflector antenna is determined. Numerical results based on the solution and related field plots are presented and discussed.**

I. Introduction

Though the full 3-D scalar diffraction problem is rather complicated and, in contrast to the corresponding 2-D problem, does not correspond exactly to any electromagnetic problem, its analysis highlights the essential features of multiple diffraction phenomena arising when plane wave illumination or beam-like radiation strikes an ensemble of targets. In this context, it is also important to develop solution methods that allows us to examine any combination of the open and closed (solid) shells or screens, where the walls of screens that may surround open cavities may be either idealised and infinitesimally or more physically realistic and of finite thickness.

Our method is entirely based on the Method of Regularization (MoR) [1] whose accuracy has been well established when solving analogous problems for isolated single screens. Its generalization to a rotationally symmetric ensemble of screens may be found in [2]. Specialising the ensemble allows us to investigate some rather diverse classes of problems including the following: a) the degree of detectability of the targets that lie in the shadow zone of an obstacle [3]; b) the accurate calculation of the complex eigenfrequences of open resonators [4]; c) accurate calculation of the complex diffraction pattern of axisymmetric arbitrarily shaped cavities with insertions [5]. In all these practical problems the ensembles of screens are illuminated by either by a propagating plane wave (in (a) and (c)) or an omnidirectional point source (in the case of coupled resonators).

Replacing the plane wave illumination by a beam-like radiation provides new opportunities for practical applications: for example, the construction of a rigorous theory, and associated numerical analysis, of the transmitting front-fed thick (or thin) parabolic antenna with realistic feeder which employs a supporting back plate of arbitrary shape.

The tapered illumination source is simulated by the so-called complex-point source (CPS) which produces the beam-like radiation in the far-field zone [1]. Along with finding the optimal source position on the optical axis of reflector that provides the maximum directivity with minimal side-lobes, we rigorously show that the size of the back plate D_0 may be extended (up to the value $D_0/D=0.25$) without significant impact on antenna characteristics. The description of our analysis of the problem of shadowing of the main beam by the back plate is given in Section 3. In the Conclusion further development of the method is discussed.

II. Brief Description of Method

We consider an ensemble of N arbitrary coaxial surfaces of revolution $S_1^0,\ldots,S_N^0$ formed by the rotation of generating curves $l_1^0,\ldots,l_N^0$ about the z-axis. Each curve l_n^0 is assumed to be part of a smooth and non-self-crossing curve l_n with ends placed on the z-axis. Full surfaces $S_1,\ldots,S_N$ cannot intersect but can contain each other.

Each curve l_n is defined by $\{\rho_n(\tau),0,z_n(\tau)\}$ in the cylindrical coordinate system, where parameter $\tau\in[0,\pi]$. The curve l_n^0 corresponds to the interval $[0,\tau_n^0]\subseteq[0,\pi]$. The parameterizations are assumed to be smooth, that is,

$$z_n,\rho_n\in C^2(0,\pi),$$

one-to-one and proper, so that

$$0<\ell_n(\tau)=\sqrt{\{z_n'(\tau)\}^2+\{\rho_n'(\tau)\}^2}<\infty,\quad \tau\in[0,\pi].$$

The full surfaces $S_1,\ldots,S_N$ are closed and do not have sharp edges at the poles, and therefore

$$\rho_n(0) = \rho_n(\pi) = 0, \; z_n'(0) = z_n'(\pi) = 0.$$

The physical wave scattering process is modelled by the mathematical problem of solving the Helmholtz equation subject to Dirichlet boundary conditions imposed on the surfaces. We seek a solution that satisfies the Sommerfeld radiation condition and the condition of the finiteness of the energy within any bounded volume V of space.

The solution to this problem exists and can be represented as a sum of surface potential integrals containing the unknown surface distributions. Enforcing the boundary conditions on this form of solution leads a system of integral equations for the unknown surface distributions. Using the analytical regularization method we transform this system to an infinite system of linear algebraic equations of the second kind

$$(\mathbf{I} + \mathbf{H})\mathbf{x} = \mathbf{u}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{H}$ is the matrix of a compact operator.

Such a system can be effectively solved numerically by a truncation method, where the infinite system is truncated to finite system. Solutions of the truncated systems converge to the solution of the original system, and the condition number stabilizes quickly. The conversion of the system to one of second kind is the key step in ensuring guaranteed accuracy of computed solutions. More details of this briefly described solution can be found in the papers [2, 6].

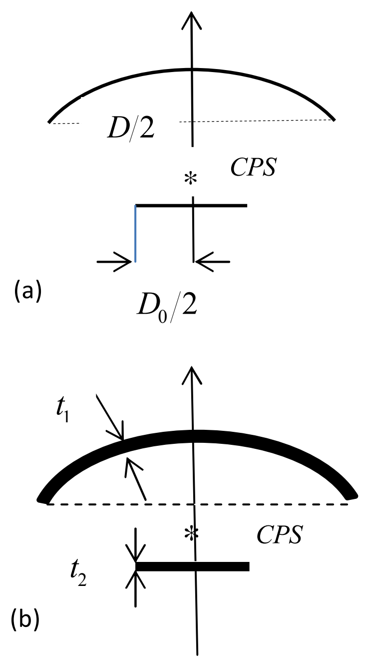

Fig. 1. Excitation of the parabolic reflector by the CPS: a) thin reflector; b) thick reflector.

III. Transmitting Front-Fed Parabolic Antenna with a Supported Back Plate of Arbitrary Shape

Numerical analysis of the rotationally symmetric reflector system shown in Fig.1 has been performed for the electrical size of reflector D/λ varying in the range $5 \le D/\lambda \le 50$. The ratio D_0/D was varied in the range $0 \le D_0/D \le 0.5$ and the thicknesses t_1 and t_2 were varied in the range $0 \le t_1, t_2 \le \lambda/2$. The position of the CPS was varied around the optical focus f of the parabolic dish. The numerical results so obtained reveal that the shadowing effect on the main beam is insignificant even for rather wide back plates of various (rotationally symmetric) shapes such as discs, parabolas, and so on, of sizes up to $D_0/D = 0.25$. Further numerical results based on the solution and related field plots will be presented and discussed, as will extensions of the applications outlined in papers [3]-[5].

IV. Conclusion

The general scalar diffraction problem for a coaxial ensemble of arbitrarily shaped bodies of revolution (open cavities and solid targets) subject to illumination by a plane wave or complex point source is rigorously solved. The problem is analytically reduced to the numerical solution of a well-conditioned coupled linear system of algebraic equations of the second kind. Numerical implementation shows the rapid convergence of the developed algorithm allowing us to guarantee the pre-specified accuracy of computations. The power of the method is demonstrated in the solution of the various practical problems described. Although the method focuses on scalar problems it appears capable of development to problems of electromagnetic diffraction.

Numerical results based on the solution and related field plots will be presented and discussed at the meeting.

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