Muffler Resonator Partially Filled with DNG Metamaterial

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***Abstract*—A metallic cylindrical resonator of elliptical cross-section containing two metallic baffles and half-filled with anti-isorefractive DNG metamaterial is analyzed by separation of variables, in the frequency domain. Numerical results based on the obtained resonance condition and related field plots will be presented and discussed.**

I. Introduction

A resonator partially filled with double-positive (DPS) material and partially with double-negative (DNG) metamaterial may perform independently of its size at those frequencies where the metamaterial behaves as postulated, thus allowing for miniaturization. The idea of utilizing phase compensation between the DPS and the DNG portions of the structure was introduced by Engheta [1], [2] for a parallel-plate structure and subsequently applied by Uslenghi [3], [4], [5] to parallelepipedal cavities and by Daniele *et al.* [6] to a circular cylindrical cavity. Couture *et al.* [7], [8] introduced a different method, based on a metamaterial with zero permittivity. Studies were also performed that did not lead to phase compensation [9], [10]

In this paper, a metallic PEC cylindrical resonator of elliptical cross-section is considered. The electric field is assumed parallel to the resonator axis, hence the boundary conditions on the top and bottom bases of the cylinder are satisfied and the cylinder height is not relevant to this study. The dual configuration, consisting of a cylinder with perfect magnetic (PMC) walls and the magnetic field parallel to its axis, is not explicitly considered, but is trivially obtained.

The shape of the resonator resembles the muffler of an automobile, hence its name. A cross section of the resonator in any plane perpendicular to the generatrices of its elliptical wall is shown in Fig. 1. The rectangular coordinates (x, y, z) and the elliptic-cylinder coordinates (u, v, z) are related by

$$\begin{cases} x = \frac{d}{2}\cosh u \cos v \\ y = \frac{d}{2}\sin u \sin v \\ z = z \end{cases} \tag{1}$$

where $\xi = \cosh u$ and $\eta = \cos v$ are also commonly used, and $0 \leq u < \infty$, $0 \leq v \leq 2\pi$ and $-\infty < z < \infty$. The interfocal distance $F_1F_2 = d$ is the cross section of a flat strip that separates the two equal halves of the resonator; the half volume $y > 0$ is filled with DPS material and the half volume $y < 0$ with DNG metamaterial. The portion of the $y = 0$ plane outside the interfocal strip F_1F_2 consists of two thin metallic strips (AF_1 and F_2B in cross section), whose presence makes it possible to solve the boundary-value problem exactly, by separation of variables. The interfocal strip corresponds to $u = 0$, whereas the elliptical wall of the cylinder corresponds to $u = u_0$.

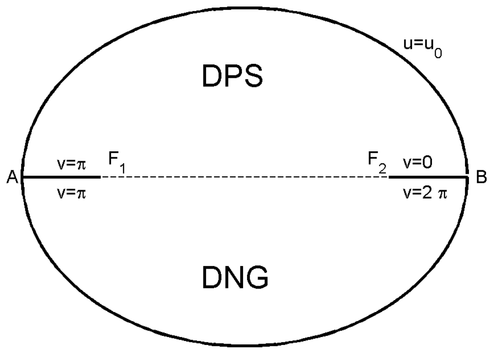

Fig. 1. Cross section of the resonator.

The DPS region is filled with a linear, uniform and isotropic material characterized by a real positive electric permittivity ϵ and a real positive magnetic permeability μ, or alternatively by a real positive wavenumber $k = \omega\sqrt{\epsilon\mu}$ and a real positive intrinsic impedance $Z = \sqrt{\mu/\epsilon}$, where ω is the angular frequency. The DNG region is filled with a linear, uniform and isotropic material characterized by a real negative permittivity $-\epsilon$ and a real negative permeability $-\mu$, or alternatively by a real negative wavenumber $-k$ and a real positive intrinsic impedance Z. Thus, the DPS and DNG regions of the resonator are filled with materials having real refractive indexes of opposite sign and the same real intrinsic impedance. The analysis is conducted in the phasor domain with time-dependence factor $\exp(j\omega t)$. The results obtained are valid at those frequencies where the DNG metamaterial behaves as postulated. Because of the dispersive properties of passive

DNG materials, broadbanding may be achievable only by the use of active (non-Foster) metamaterials.

II. Resonance Condition

We are looking for modes with a z-oriented electric field inside the muffler resonator given by

$$E_{\ell z} = \left[a_{\ell n}^{(e)} \mathrm{Re}_n^{(1)}(\pm c, u) + b_{\ell n}^{(e)} \mathrm{Re}_n^{(4)}(\pm c, u)\right] \mathrm{Se}_n(c, v) + \left[a_{\ell n}^{(o)} \mathrm{Ro}_n^{(1)}(\pm c, u) + b_{\ell n}^{(o)} \mathrm{Ro}_n^{(4)}(\pm c, u)\right] \mathrm{So}_n(c, v), \quad (2)$$

where the notation for the even and odd radial and angular Mathieu functions is that of Stratton [11], [12]; $\ell = 1$ for the DPS volume and $\ell = 2$ for the DNG volume. The corresponding magnetic field is

$$\mathbf{H}_\ell = \frac{(-1)^{\ell+1} j}{cZ\sqrt{\xi^2 - \eta^2}} \left[\frac{\partial E_{\ell z}}{\partial v}\hat{\mathbf{u}} - \frac{\partial E_{\ell z}}{\partial u}\hat{\mathbf{v}}\right], \quad (3)$$

where $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ are the unit vectors oriented perpendicular to the surfaces $u = \text{constant}$ and $v = \text{constant}$, and directed toward increasing values of u and v, respectively. In the previous formulas, the parameter c is

$$c = \frac{kd}{2}, \quad (4)$$

and the + sign (- sign) in (2) applies to $\ell = 1$ ($\ell = 2$), respectively. The order n of the Mathieu functions is chosen to be an integer, thereby ensuring that the boundary conditions on the two metallic baffles can be satisfied. Note that Se_n and So_n are even functions of c, while the relationship between any radial function with parameter c and the same function with parameter $-c$ is note simple [13].

Imposition of the boundary condition that E_z be zero at all metallic surfaces implies that all even modal expansion coefficients are zero, and that the odd modal expansion coefficients $b_{\ell n}^{(o)}$ are related to $a_{\ell n}^{(o)}$, resulting in

$$E_{1z} = a_{1n}^{(o)} \left[\mathrm{Ro}_n^{(1)}(c, u) - \frac{\mathrm{Ro}_n^{(1)}(c, u_0)}{\mathrm{Ro}_n^{(4)}(c, u_0)} \mathrm{Ro}_n^{(4)}(c, u)\right] \mathrm{So}_n(c, v), \quad (5)$$

$$E_{2z} = a_{2n}^{(o)} \left[\mathrm{Ro}_n^{(1)}(-c, u) - \frac{\mathrm{Ro}_n^{(1)}(-c, u_0)}{\mathrm{Ro}_n^{(4)}(-c, u_0)} \mathrm{Ro}_n^{(4)}(-c, u)\right] \mathrm{So}_n(c, v). \quad (6)$$

It remains to impose the continuity of the tangential components of the electric and magnetic fields across the strip $u = 0$ separating the DPS and DNG regions. This yields the system

$$\begin{cases} \frac{\mathrm{Ro}_n^{(1)}(c,u_0)}{\mathrm{Ro}_n^{(4)}(c,u_0)} \mathrm{Ro}_n^{(4)}(c, 0) a_{1n}^{(o)} \\ -\frac{\mathrm{Ro}_n^{(1)}(-c,u_0)}{\mathrm{Ro}_n^{(4)}(-c,u_0)} \mathrm{Ro}_n^{(4)}(-c, 0) a_{2n}^{(o)} = 0 \\ \left[\mathrm{Ro}_n^{(1)\prime}(c, 0) - \frac{\mathrm{Ro}_n^{(1)}(c,u_0)}{\mathrm{Ro}_n^{(4)}(c,u_0)} \mathrm{Ro}_n^{(4)\prime}(c, 0)\right] a_{1n}^{(o)} \\ -\left[\mathrm{Ro}_n^{(1)\prime}(-c, 0) - \frac{\mathrm{Ro}_n^{(1)}(-c,u_0)}{\mathrm{Ro}_n^{(4)}(-c,u_0)} \mathrm{Ro}_n^{(4)\prime}(-c, 0)\right] a_{2n}^{(o)} = 0 \end{cases}$$

in the modal coefficients $a_{1n}^{(o)}$ and $a_{2n}^{(o)}$, where the prime means derivative with respect to u. For non-zero fields to exist in the resonator, the determinant of the coefficients of the previous system must be zero, yielding the condition

$$\begin{aligned} &\mathrm{Ro}_n^{(1)}(c, u_0)\mathrm{Ro}_n^{(4)}(-c, u_0)\mathrm{Ro}_n^{(4)}(c, 0)\mathrm{Ro}_n^{(1)\prime}(-c, 0) \\ &- \mathrm{Ro}_n^{(1)}(c, u_0)\mathrm{Ro}_n^{(1)}(-c, u_0)\mathrm{Ro}_n^{(4)}(c, 0)\mathrm{Ro}_n^{(4)\prime}(-c, 0) \\ &- \mathrm{Ro}_n^{(4)}(c, u_0)\mathrm{Ro}_n^{(1)}(-c, u_0)\mathrm{Ro}_n^{(1)\prime}(c, 0)\mathrm{Ro}_n^{(1)}(-c, 0) \\ &+ \mathrm{Ro}_n^{(1)}(c, u_0)\mathrm{Ro}_n^{(1)}(-c, u_0)\mathrm{Ro}_n^{(4)\prime}(c, 0)\mathrm{Ro}_n^{(4)}(-c, 0) = 0. \end{aligned} \quad (7)$$

This resonance condition contains the two parameters c and u_0 or, equivalently, the interfocal distance and the major axis of the ellipse of Fig. 1 in terms of wavelength.

III. Conclusion

A resonance condition has been obtained for the fields inside a muffler resonator. For given dimensions of the cavity, the possible resonance frequencies are determined by condition (7). These frequencies and associated field distributions will be studied numerically.

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