# Study on the Principal Axis of Anisotropic Absorber Panel by the Use of Diagonalization Method

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Abstract— Evaluation method of anisotropic absorber panel at microwave frequencies is discussed. Anisotropic absorber panel has the axis of anisotropy (principal axis). In order to specify the principal axis, we proposed the evaluation method by the use of diagonalization of reflection and transmission coefficient matrices. The principal axis of several materials is evaluated in this paper.

Key words: Anisotropic absorber panel, Diagonalization method, Principal axis, Reflection coefficient, Transmission coefficient, Microwave frequencies

## I. INTRODUCTION

Recently, absorber panels have been frequently used to reduce undesired EM-waves. Though the absorber panels are fabricated to have isotropic reflections, the panels, in closer inspection, usually show anisotropic characteristics slightly. For example, short metal-fiber dispersed absorbing material, non-woven fabric absorber containing metal wires [1], and wedge-shaped carbon-impregnated absorbing plastics [2] show the anisotropy in reflections. These absorbers have evaluated by the average reflection so far. The authors propose here the evaluation method for the anisotropic absorber.

Anisotropic absorber panels have the axes of anisotropy. Reflection coefficients at the axes show maximum or minimum value. In here, we defined that they are *the principal axes*. As conventional method, the principal axis was specified by rotate measurement sample every several degrees.

We propose here an evaluation method by diagonalizing reflection coefficient matrix. Eigenvalues and eigenvectors of the matrix, which correspond to co-polarization reflection coefficients and the direction of principal axis, are obtained [3]. By this method, the principal axes can be specified easily compared to the conventional evaluation method. Also, the principal axes of EM-wave shielding materials can be obtained by diagonalizing procedure of the transmission coefficient matrix.

Calculated eigenvalues and the principal axes direction of several materials by the diagonalization method are described in this paper.

## II. EVALUATION METHOD OF ANISOTROPIC ABSORBER PANEL

## A. Diagonalization of Reflection Coefficient Matrix

Suppose that the linearly polarized waves hit absorber panel. Figure 1 shows the relationship between incident, reflected, and transmitted waves. As shown in Fig. 1, the absorber panel is parallel to the x-y plane.  $\xi$ ,  $\eta$  are the principal axes of the panel ( $\xi$  axis perpendicular to  $\eta$  ones), and  $\phi$  1,  $\phi$  2 are the angle of  $\xi$ ,  $\eta$  axes from x axis.  $\theta$  is the angle of incident electric field  $E_i$  from x axis.

The reflected electric field  $E_r$  from the absorber panel can be represented as [4],

$$\begin{pmatrix}
E_{rx} \\
E_{ry}
\end{pmatrix} = \begin{pmatrix}
\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & \Gamma_{yy}
\end{pmatrix} \begin{pmatrix}
E_{ix} \\
E_{iy}
\end{pmatrix}$$
(1)

where  $E_i = (E_{ix}, E_{iy})$ ,  $E_r = (E_{rx}, E_{ry})$ ,  $\Gamma_{xx}$ ,  $\Gamma_{yy}$  are the copolarized wave reflection coefficients, and  $\Gamma_{xy}$ ,  $\Gamma_{yx}$  are the cross-polarized wave ones.

When x, y axes coincide to  $\xi$ ,  $\eta$  axes, (1) becomes,

$$\begin{pmatrix}
E_{r\xi} \\
E_{r\eta}
\end{pmatrix} = \begin{pmatrix}
\Gamma_{\xi} & 0 \\
0 & \Gamma_{\eta}
\end{pmatrix} \begin{pmatrix}
E_{i\xi} \\
E_{i\eta}
\end{pmatrix}$$
(2)

where  $\Gamma_{\xi}$ ,  $\Gamma_{\eta}$  are the reflection coefficient at  $\xi$ ,  $\eta$  axes.

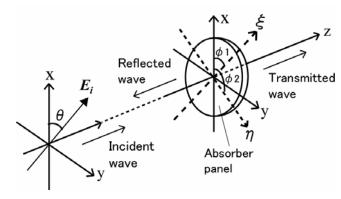


Fig. 1 Relationship between incident, reflected, and transmitted waves

Since  $\Gamma_{\xi}$ ,  $\Gamma_{\eta}$  are either the maximum or minimum value of the reflection, (2) is the evaluation equation of the anisotropic absorber panel.

Once the reflection coefficient matrix of equation (1) is given, equation (2) can be obtained by rotating the coordinate [5]. The transformation from equation (1) to (2) is the diagonalization of matrix. From the measured reflection coefficient matrix of (1), the eigenvalues and eigenvectors of the matrix, which correspond to co-polarization reflection coefficients and the axes of the anisotropy, are obtained by the diagonalization.

In order to diagonalize the matrix, it is necessary to decide all the elements of equation (1).  $\Gamma_{xx}$ ,  $\Gamma_{yy}$  can be measured directly, however,  $\Gamma_{xy}$ ,  $\Gamma_{yx}$ , the cross-polarized coefficients, can't measure directly because cross-polarization components do not exist in the reflection by the metal plate, a reference of reflection. In here,  $\Gamma_{xy}$ ,  $\Gamma_{yx}$ , are obtained through the following procedure.

From equation (1), the reflected component  $E_{rx}$  is given as,

$$E_{rr} = \Gamma_{rr} E_{ir} + \Gamma_{rv} E_{iv} = (\Gamma_{rr} \cos \theta + \Gamma_{rv} \sin \theta) E_{i}$$
 (3)

Consider the new reflection coefficient  $\Gamma_x^{\theta}$  which equal to  $E_{rx}/E_{ix}$ . By using  $E_{rx}$  of equation (3),  $\Gamma_x^{\theta}$  can be represented as  $\Gamma_{xx} + \Gamma_{xy} \tan \theta$ .  $\Gamma_{xy}$  can be given as,

$$\Gamma_{xy} = \left(1/\tan\theta\right) \left(\Gamma_x^{\theta} - \Gamma_{xx}\right) \tag{4}$$

In the same manner,  $\Gamma_{vx}$  is represented by,

$$\Gamma_{yx} = \tan\theta \left( \Gamma_{y}^{\theta} - \Gamma_{yy} \right) \tag{5}$$

where  $\Gamma_{y}^{\theta}$  equals to  $E_{ry}/E_{iy}$ .

Since  $\Gamma_x^{\ \theta}$ ,  $\Gamma_y^{\ \theta}$ ,  $\Gamma_{xx}$ ,  $\Gamma_{yy}$  are the measured values,  $\Gamma_{xy}$ ,  $\Gamma_{yx}$  can be calculated by equations (4), (5).

# B. Diagonalization of Transmission Coefficient Matrix

The transmitted electric field  $E_t$  in Fig. 1 can be represented by following equation.

$$\begin{pmatrix}
E_{tx} \\
E_{ty}
\end{pmatrix} = \begin{pmatrix}
T_{xx} & T_{xy} \\
T_{yx} & T_{yy}
\end{pmatrix} \begin{pmatrix}
E_{tx} \\
E_{ty}
\end{pmatrix}$$
(6)

 $T_{xx}$ ,  $T_{yy}$  are the co-polarized wave transmission coefficients, and  $T_{xy}$ ,  $T_{yx}$  are the cross-polarized wave ones. By the same reason stated for the reflection,  $T_{xy}$ ,  $T_{yx}$  can't measured directly.  $T_{xy}$ ,  $T_{yx}$  can be calculated by,

$$T_{xy} = (1/\tan\theta) (T_x^{\theta} - T_{xx})$$

$$T_{yx} = \tan\theta (T_y^{\theta} - T_{yy})$$
(8)

 $T_x^{\theta}$  and  $T_y^{\theta}$  equal to  $E_{tx} / E_{ix}$ ,  $E_{ty} / E_{iy}$ , respectively.

When x, y axes coincide to  $\xi$ ,  $\eta$  ones, equation (6) can be given,

$$\begin{pmatrix}
E_{t\xi} \\
E_{t\eta}
\end{pmatrix} = \begin{pmatrix}
T_{\xi} & 0 \\
0 & T_{\eta}
\end{pmatrix} \begin{pmatrix}
E_{t\xi} \\
E_{t\eta}
\end{pmatrix}$$
(9)

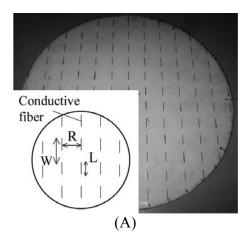
## III. MEASUREMENT RESULTS

In order to confirm the validity of diagonalization method, samples of TABLE I are used. Figure 2 shows the photo of the sample (A), (B) whose diameters are 30cm.

Sample (A) is conductive wire array sheet. Finite length metal wires are aligned periodically on a thin dielectric film. The wire dimensions are L=10mm, R=18mm, W=25mm. Thickness of sample (A) is 235  $\mu$  m. This array structure shows the resonant type dispersion in permittivity and its real part becomes negative at frequencies higher than the resonant

TABLE I MEASUREMENT SAMPLES

	Measurement sample
(A)	Wire array sheet (finite length metal wires are aligned periodically on a thin dielectric film, thickness: 235 $\mu$ m)
(B)	Non-woven fabric absorber sheet containing metal fibers (thickness: 9mm)



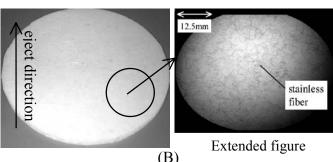


Fig. 2 Photo of the sample (A), (B)

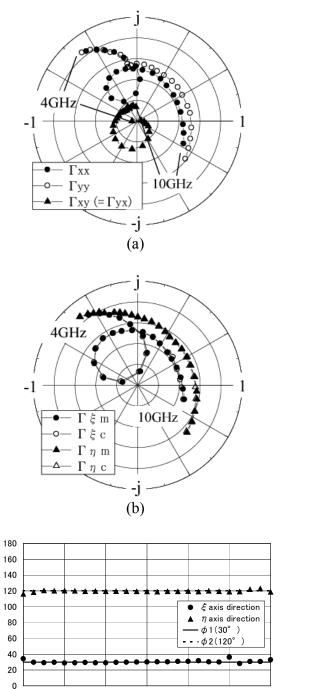


Fig. 3 (a)  $\Gamma_{xx}$ ,  $\Gamma_{xy}$ ,  $\Gamma_{yx}$ ,  $\Gamma_{yy}$  values, (b)  $\Gamma_{\xi c}$ ,  $\Gamma_{\xi m}$ ,  $\Gamma_{\eta c}$ ,  $\Gamma_{\eta m}$  values (c) principal axis direction of sample (A)

(c)

Frequency (GHz)

φ1, φ2(°)

frequency  $f_0$  [6]. The principal axis of sample (A) is parallel to wire direction.

Sample (B) is non-woven fabric absorber sheet containing

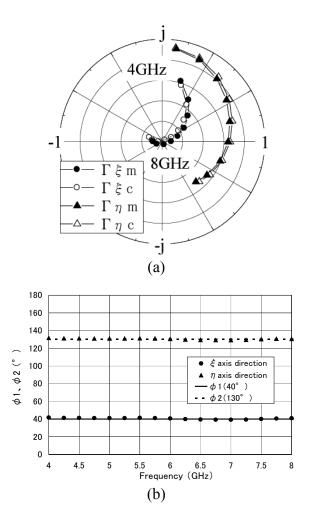
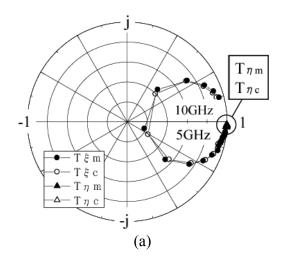


Fig. 4 (a)  $\Gamma_{\xi c}$ ,  $\Gamma_{\xi m}$ ,  $\Gamma_{\eta c}$ ,  $\Gamma_{\eta m}$  values, (b) principal axis direction of sample (B)

metal fibers. Thickness of sample (B) is 9mm. The principal axis of sample (B) is parallel to the fiber direction.

Reflection coefficients were measured by the free space measurement setup [7]. The distance between transmitting, receiving antennas (horn antennas) and sample is 1.5m. Also, transmission coefficient measured in the free space [8]. The distance between transmitting, receiving antennas (horn antennas) and sample is 0.74m.

Figure 3(a) shows the measured  $\Gamma_{xx},~\Gamma_{yy}$  values and calculated  $\Gamma_{xy}$  (=  $\Gamma_{yx}$ ) values by (4), (5) (  $\Gamma_{xy},~\Gamma_{yx}$  are calculated by (  $\Gamma_{xy}+\Gamma_{yx}$ ) / 2 because  $\Gamma_{xy},~\Gamma_{yx}$  show the same value) of sample (A). In here, ferrite/rubber mixture is used to provide absorption property between sample (A) and metal plate. Frequency range is from 4GHz to 10GHz. In the calculation,  $\theta$  equals 45°, and  $\phi$  1 equals 30°,  $\phi$  2 equals 120°.  $\Gamma_{\xi\,c},~\Gamma_{\eta\,c}$  in Fig.3(b) are the eigenvalues which calculated by diagonalizing  $\Gamma_{xx},~\Gamma_{xy},~\Gamma_{yx},~\Gamma_{yy}$  of Fig.3(a).  $\Gamma_{\xi\,m},~\Gamma_{\eta\,m}$  are the measured reflection coefficient when the incident wave coincide to  $\xi$  and  $\eta$  axes. From Fig.3(b),  $\Gamma_{\xi\,c},$ 



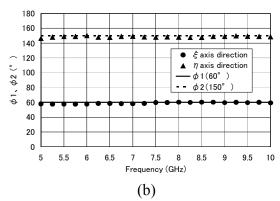


Fig. 5 (a) T  $_{\xi}$  c, T  $_{\xi}$  m, T  $_{\eta}$  c, T  $_{\eta}$  m values, (b) principal axis direction angle of sample (A)

 $\Gamma_{\eta c}$  agree well to  $\Gamma_{\xi m}$ ,  $\Gamma_{\eta m}$ , respectively. Also, Fig.3(c) shows the calculated values of principal axes direction. Calculated  $\xi$ ,  $\eta$  axes angles almost agree well to  $\phi$  1,  $\phi$  2.

Figures 4(a), (b) show  $\Gamma_{\xi c}$ ,  $\Gamma_{\eta c}$ ,  $\Gamma_{\xi m}$ ,  $\Gamma_{\eta m}$  values, and the calculated principal axes directions of sample (B). Frequency range is from 4GHz to 8GHz.  $\theta$  equals 45°, and  $\phi$  1 equals 40°,  $\phi$  2 equals 130°. From these figures,  $\Gamma_{\xi c}$ ,  $\Gamma_{\eta c}$  agree well to  $\Gamma_{\xi m}$ ,  $\Gamma_{\eta m}$ . Also, calculated  $\xi$ ,  $\eta$  axes angles agree well to  $\phi$  1,  $\phi$  2.

Figure 5(a) shows T  $_{\xi\,c}$ , T  $_{\eta\,c}$ , T  $_{\xi\,m}$ , T  $_{\eta\,m}$  values of sample (A). Frequency range is from 5GHz to 10GHz. In the calculation,  $\theta$  equals 45°, and  $\phi$ 1 equals 55°,  $\phi$ 2 equals 145°. T  $_{\xi\,c}$ , T  $_{\eta\,c}$  agree well to T  $_{\xi\,m}$ , T  $_{\eta\,m}$ , and T  $_{\eta\,c}$ , T  $_{\eta\,m}$  are independent on frequency. Figure 5(b) shows the calculated  $\xi$ ,  $\eta$  axes direction.  $\xi$ ,  $\eta$  axes angles almost agree well to  $\phi$ 1,  $\phi$ 2. For example, T  $_{xx}$ , T  $_{xy}$ , T  $_{yx}$ , T  $_{yy}$ , and T  $_{\xi\,c}$ , T  $_{\eta\,c}$ , T  $_{\xi\,m}$ , T  $_{\eta\,m}$  at 8GHz are given as,

$$\begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} = \begin{pmatrix} 0.84 - j \, 0.30 & 0.098 + j \, 0.15 \\ 0.098 + j \, 0.15 & 0.94 - j \, 0.14 \end{pmatrix}$$

$$\begin{pmatrix} T_{\zeta c} & 0 \\ 0 & T_{\eta c} \end{pmatrix} = \begin{pmatrix} 0.78 - j \, 0.38 & 0 \\ 0 & 0.99 - j \, 0.051 \end{pmatrix}$$
 
$$\begin{pmatrix} T_{\zeta m} & 0 \\ 0 & T_{\eta m} \end{pmatrix} = \begin{pmatrix} 0.76 - j \, 0.39 & 0 \\ 0 & 0.99 - j \, 0.035 \end{pmatrix}$$

From above results, we confirm that  $\Gamma_{\xi}$ ,  $\Gamma_{\eta}$ ,  $T_{\xi}$ ,  $T_{\eta}$  and the principal axis direction can be specified with certain accuracy by using diagonalization of reflection and transmission coefficient matrix.

#### IV. CONCLUSION

The evaluation method of anisotropic absorber panels by diagonalization method is proposed. In general, cross-polarized wave components  $\Gamma_{xy}$ ,  $\Gamma_{yx}$ ,  $T_{xy}$ ,  $T_{yx}$  can't be measured directly. In this paper, the calculation method of  $\Gamma_{xy}$ ,  $\Gamma_{yx}$ ,  $T_{xy}$ ,  $T_{yx}$  is proposed.

From diagonalization of reflection coefficient matrix of sample (A), (B), we find that calculated  $\Gamma_{\xi c}$ ,  $\Gamma_{\eta c}$  almost agree well to measured  $\Gamma_{\xi m}$ ,  $\Gamma_{\eta m}$ , and calculated  $\xi$ ,  $\eta$  axes direction angle agree well to  $\phi$  1,  $\phi$  2. Also, from diagonalization of transmission coefficient matrix of samples (A),  $T_{\xi c}$ ,  $T_{\eta c}$  almost agree well to measured  $T_{\xi m}$ ,  $T_{\eta m}$ .

From consideration in this paper, we find that reflection coefficients  $\Gamma_{\xi}$ ,  $\Gamma_{\eta}$ , transmission coefficients  $T_{\xi}$ ,  $T_{\eta}$  and the principal axis direction can be obtained by a simple procedure using diagonalization.

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