Using Spatial Processing Based on the Double Weighted Fourier Transform for Eliminating the Multipath Effect

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***Abstract*—Numerical simulation has been used to demonstrate the possibility of eliminating the multipath effect arising during wave propagation in an inhomogeneous medium with the aid of the spatial field processing based on the double weighted Fourier transform. We have examined scattering by both one and two local inhomogeneities scales of which are less than the Fresnel radius.**

I. INTRODUCTION

In solving problems of radio wave scattering, refraction, and diffraction by local inhomogeneities, we often have to account for the multipath effect which may result from refraction by both one (Fig. 1) and many inhomogeneities (Fig. 2).

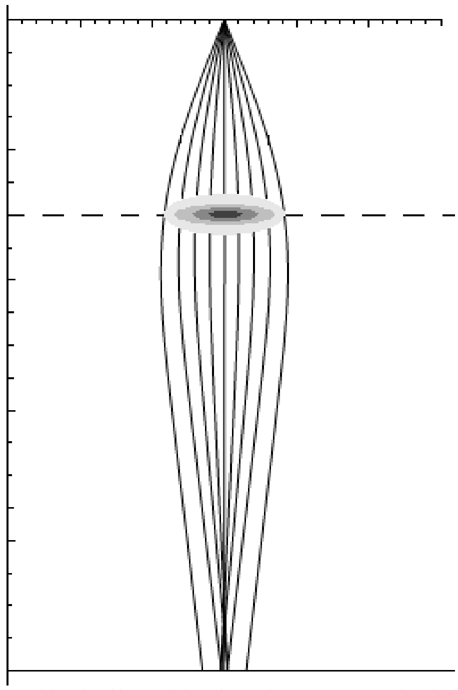

Fig. 1. The multipath effect arising from the wave scattering by a local inhomogeneity

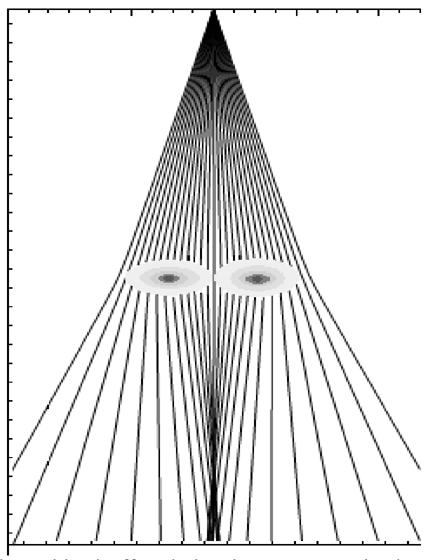

Fig. 2. The multipath effect during the wave scattering by two local inhomogeneities

In inverse problems, this phenomenon complicates finding physical characteristics of an inhomogeneous medium [1].

In radio tomography, scattered field data, collected by a receiver system, make it possible to find permittivity of small-scale inhomogeneous plasma whose scale is less than the Fresnel radius [2]. Key information on the inhomogeneity parameters is in the wave phase. To determine permittivity of the medium, we should get a number of phase projections with the use of different transmission angles by means of a receive-transmit system. However, the solution of this tomographic problem is strongly hampered by the multipath effect.

In earlier papers [3]–[4], we put forward an integral representation for the field of a wave, scattered by an inhomogeneity, in the form of the double weighted Fourier transform (DWFT). Based on this integral expression, we obtained spatial processing of field measurements [3]–[4] in transverse receiver/transmitter coordinates. As is shown in [4]–[6], the DWFT processing provides super Fresnel resolution without information on inhomogeneity coordinates during strong and weak phase fluctuations.

Next we will consider how the spatial DWFT processing influences results of field measurements in case of multipath effect.

II. DWFT and its Inversion for the Wave Field

Let a source and a receiver be at points $\mathbf{r_0} = (z_0, x_0, y_0) = (z_0, \boldsymbol{\rho_0})$ and $\mathbf{r} = (z_t, x, y) = (z_t, \boldsymbol{\rho})$, where $\boldsymbol{\rho_0} = (x_0, y_0)$ and $\boldsymbol{\rho} = (x, y)$ - are two-dimensional vectors in planes $z = z_0$ and $z = z_t$. Between the source and the receiver is an inhomogeneous medium [4]. In this case, the wave field in the DWFT method is as follows [3–6]:

$$U(\boldsymbol{\rho}, \boldsymbol{\rho_0}) = \frac{-Ak^2}{4\pi^3 Z^3} \exp\left[ik\left(Z + (\boldsymbol{\rho}+\boldsymbol{\rho_0})^2/2Z\right)\right] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\xi d^2\xi_0 \times \exp\left(ik\left[2(\boldsymbol{\xi\xi_0} - \boldsymbol{\xi\rho_0} - \boldsymbol{\xi_0\rho})/Z + \Phi(\boldsymbol{\xi}, \boldsymbol{\xi_0}, z_t, z_0)\right]\right). \quad (1)$$

Where $Z = z_t - z_0$ - is the distance between the planes with the source and receiver, A_0 - is the incident spherical wave amplitude, $k = \omega / c$, $\omega = 2\pi f$ - is the radiation frequency, c - is the velocity of light in free space, $\Phi(\boldsymbol{\xi}, \boldsymbol{\xi_0}, z_t, z_0)$ - is the linear integral from $\tilde{\varepsilon}(\mathbf{r})$, calculated from equation

$$\Phi(\boldsymbol{\xi}, \boldsymbol{\xi_0}) = 1/2 \int_{-\infty}^{+\infty} \tilde{\varepsilon}(\boldsymbol{\xi}(z' - z_0)/Z + \boldsymbol{\xi_0}(z_t - z')/Z, z')dz, \quad (2)$$

$\tilde{\varepsilon}(\mathbf{r}) = \varepsilon(\mathbf{r}) - 1$ - is the permittivity variation $\varepsilon(\mathbf{r})$.

By applying integral operator (3) to the field $U(\boldsymbol{\rho}, \boldsymbol{\rho_0})$

$$\tilde{U}\left(\boldsymbol{\rho}^*, \boldsymbol{\rho_0}^*\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\rho d^2\rho_0 U(\boldsymbol{\rho}, \boldsymbol{\rho_0}) \exp\{-ikZ\} \times \exp\frac{ik}{Z}\left\{2(\boldsymbol{\rho}^*\boldsymbol{\rho_0} + \boldsymbol{\rho_0}^*\boldsymbol{\rho}) - (\boldsymbol{\rho} + \boldsymbol{\rho_0})^2/2\right\}. \quad (3)$$

We obtain

$$\tilde{U}(\boldsymbol{\rho}^*, \boldsymbol{\rho_0}^*) = \hat{L}\left[U(\boldsymbol{\rho}, \boldsymbol{\rho_0})\right] = -\frac{A_0 \pi Z}{4k^2} \exp\left\{ik\left[2\boldsymbol{\rho}^*\boldsymbol{\rho_0}^*/Z + \Phi(\boldsymbol{\rho}^*, \boldsymbol{\rho_0}^*)\right]\right\}. \quad (4)$$

Thus, after processing (3), we can find phase

$$k\tilde{\Phi}(\boldsymbol{\rho}^*, \boldsymbol{\rho_0}^*) = \arg\left[\tilde{U}(\boldsymbol{\rho}^*, \boldsymbol{\rho_0}^*)\right] - 2\boldsymbol{\rho}^*\boldsymbol{\rho_0}^*/Z = \frac{k}{2}\int_{-\infty}^{\infty} \tilde{\varepsilon}\left[\boldsymbol{\rho}^*(z' - z_0)/Z + \boldsymbol{\rho_0}^*(z_t - z')/Z, z'\right]dz'. \quad (5)$$

The phase $k\tilde{\Phi}(\boldsymbol{\rho}^*, \boldsymbol{\rho_0}^*)$ in (5) has the same form as in the GO approximation. It is significant that wave field (4) after spatial DWFT processing (3) has no amplitude fluctuations which appear because of wave focusing by inhomogeneities [7].

To analyze the multipath effect on the resolution capability of the DWFT method, we should substitute the direct problem solution $U(\boldsymbol{\rho}, \boldsymbol{\rho_0})$ in (3). Unfortunately there is no strict solution to the problem of spherical wave propagation with local inhomogeneity. So, it is worthwhile taking an approximated solution for the processed field $U(\boldsymbol{\rho}, \boldsymbol{\rho_0})$ in (3). To examine the resolution capability of the inverse DWFT, for the processed field model we employ the solution derived with the phase screen method [8].

III. Numerical Simulation Results

Let us explore the possibilities of spatial DWFT processing (3) in case of multipath effect during strong phase fluctuations when

$$\max\left|k\tilde{\Phi}(\boldsymbol{\rho})\right| \geq 2\pi. \quad (6)$$

For the sounding signal model we use a phase screen approximation [9]. Permittivity of the inhomogeneity will be described by a sum of Gaussian functions

$$\tilde{\varepsilon}(\boldsymbol{\rho}, z) = \varepsilon_{mi} \sum_{i=1}^{N} \exp\left(-\left[(\boldsymbol{\rho} - \boldsymbol{\rho_{mi}})^2 + (z - z_{mi})^2\right]/2l_i^2\right). \quad (7)$$

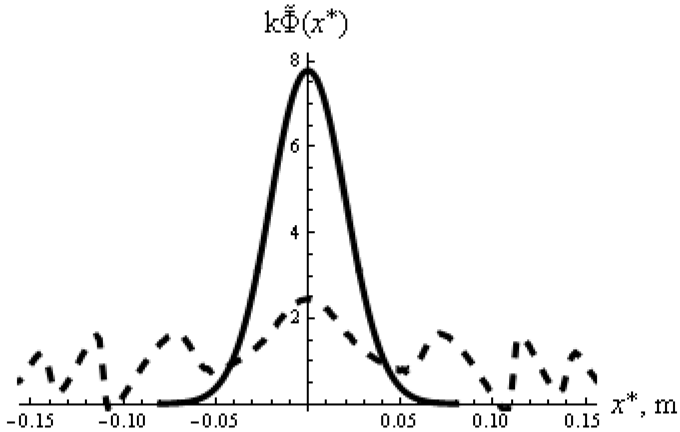

Fig. 3. The behavior of the phase $k\tilde{\Phi}(x^*) = k\tilde{\Phi}(x^*, x^*, 0, 0)$ for Gaussian inhomogeneity (7) at $N = 1$: phase measurements without processing (dashed line), measurements after processing (3) (solid line)

Fig. 3 presents the results of calculations of the phase $k\tilde{\Phi}(x^*) = k\tilde{\Phi}(x^*, x^*, 0, 0)$ as function of $x^* = x_0^*$ in the cross-section $y^* = y_0^* = 0$ at the following parameters: $x_m = z_m = 0$, $\varepsilon_m = 0.1$, $l = 2cm$, $z_t = -3m$, $z_0 = 3m$, $\lambda = 2mm$, $z_s = 0m$. In this case, the Fresnel radius $a_F = 5,4cm$ exceeds sizes of the inhomogeneity.

In case of multipath effect during propagation through the only inhomogeneity (Fig. 1), it is difficult to gain any information on parameters of the inhomogeneity from the phase behavior without spatial processing (see the dashed line in Fig. 3). Processing (3) (the solid line in Fig. 3) allowed us to reveal the inhomogeneity profile and eliminate the multipath effect from the measurement results.

Next let us consider the case of the multipath effect arising during wave scattering by two small-scale inhomogeneities (Fig. 2.)

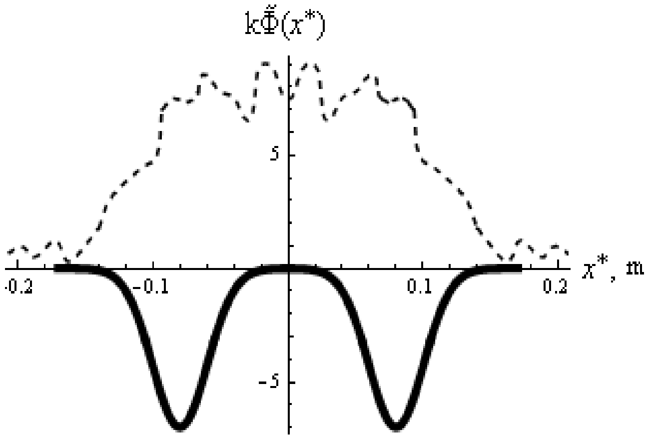

Fig. 4. The behavior of phase $k\tilde{\Phi}(x^*) = k\tilde{\Phi}(x^*, x^*, 0, 0)$ for two Gaussian inhomogeneities (7) at $N = 2$: phase measurements without processing (dashed line), measurements after the DWFT processing (solid line)

Fig. 4 presents the results of calculations of the phase $k\tilde{\Phi}(x^*) = k\tilde{\Phi}(x^*, x^*, 0, 0)$ as function of $x^* = x_0{}^*$ in the cross-section $y^* = y_0{}^* = 0$ at the following parameters: $x_m = z_m = 0$, $\varepsilon_{m1} = -0.09$, $\varepsilon_{m2} = -0.09$, $l_1 = 2cm$, $l_2 = 2cm$, $z_t = -3m$, $z_0 = 3m$, $\lambda = 2mm$, $z_s = 0m$. In this case, the Fresnel radius $a_F = 5,4cm$ exceeds sizes of the inhomogeneities.

The dashed line in fig. 4 shows that if the Fresnel radius exceeds typical scales of an inhomogeneous medium, the Fresnel limit of resolution precludes determining two inhomogeneities in measurements. The multipath effect caused by the presence of the two inhomogeneities led only to a considerable broadening of the phase projection. The solid line in (Fig. 4) indicates that the use of the DWFT spatial processing brought about the resolution of the two inhomogeneities for strong phase fluctuations.

IV. Conclusion

We have demonstrated that the multipath effect on results of scattered field measurements can be eliminated with the aid of an additional spatial DWFT field processing. This appreciably simplifies determination of physical characteristics of small-scale inhomogeneous media (scales of which are less than the Fresnel radius) in solving an inverse problem. The numerical simulation results have shown that the DWFT processing, used even for strong phase fluctuations during wave scattering by one or two small-scale inhomogeneities, eliminates the multipath effect.

Acknowledgment

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