Diffraction of a Skew Incident Electromagnetic Surface Wave at an Impedance Wedge

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***Abstract*—By making use of an earlier work [1] with due modifications, we present an exact solution, although not in explicit form, to diffraction of a skew incident electromagnetic surface wave of either E or H type at an impedance wedge. The asymptotic expression for the diffracted field in far field suggests an alternative method of measurement that allows for determining experimentally a large class of surface impedances.**

I. Introduction

An exact closed-form solution to diffraction of a normally incident electromagnetic surface wave by an impedance wedge is known since 1958 [2], on use of the eponymous Sommerfeld-Malyuzhinets technique (e.g. [3]); based on [2], a detailed study has been reported in [4] and [5]. Recently, these results [2]–[4] have been applied to the study of conversion of surface plasmon polaritons into photons at the edge of a metallic wedge [6] and a new accurate method of measuring the surface impedance of metals in the infrared (IR) range has been proposed and demonstrated in [7] and [8].

In 2006, exact but non-explicit solutions to diffraction of a skew-incident plane electromagnetic wave at an impedance wedge appeared ([9], [10], [1]); see also a paper published in 2008 [11] and a recent monograph [12]. This progress, with due modifications where necessary, allows us to study diffraction of a skew-incident electromagnetic surface wave of either E or H type at an impedance wedge. The far-field expression of the diffracted wave suggests an alternative method for measuring the surface impedance by making use not the scattering diagram, but rather the propagation constant in the direction perpendicular to the edge of the wedge.

To carry out such an investigation, we make use of [1], because of its demonstrated efficiency.

II. Analysis

A. Formulation of the Problem

The canonical body under study, namely, an impedance wedge, occupies the domain $\Phi < |\varphi| \leq \pi$ and $|z| < \infty$ in the cylindrical co-ordinate system (r, φ, z); see Fig. 1.

The electric properties of the upper and lower faces of the wedge at $\varphi = \pm\Phi$ are characterised by the (with respect to the intrinsic impedance of the ambient homogeneous medium Z_0) normalised impedances $\eta^{\pm}$. Therefore, the Leontovich conditions hold good on the faces of the wedge

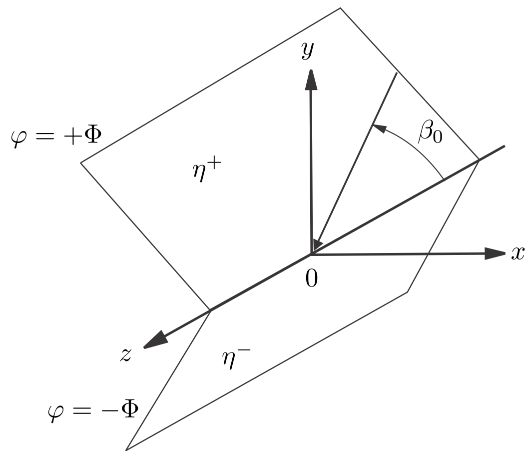

Fig. 1. An impedance wedge under skew incidence of an electromagnetic surface wave

$$\begin{aligned} E_r(r, \pm\Phi, z) &= \mp\eta^{\pm} Z_0 H_z(r, \pm\Phi, z), \\ E_z(r, \pm\Phi, z) &= \pm\eta^{\pm} Z_0 H_r(r, \pm\Phi, z), \end{aligned} \tag{1}$$

with $\eta^+ = \mathrm{i}\,\mathrm{Im}\,\eta^+$ being purely imaginary.

Let an electromagnetic surface wave travel on the upper face of the wedge towards the origin of the co-ordinate system 0, with the angle between the direction of propagation and the z-axis being β_0 (see Fig. 1). For convenience it is assumed that there are $0 < \beta_0 \leq \pi/2$ and $\Phi > \pi/2$. The z-components of such a wave are given by

$$[Z_0 H_z^{\mathrm{inc}}(r, \varphi; z)\; E_z^{\mathrm{inc}}(r, \varphi; z)]^{\mathrm{T}} = \overline{U}_0 \mathrm{e}^{-\mathrm{i}k_0' r\cos(\varphi-\varphi_0)+\mathrm{i}k_0'' z}, \tag{2}$$

with $\overline{U}_0 = [U_{10}\; U_{20}]^{\mathrm{T}}, k_0' = k_0 \sin\theta_0, k_0'' = k_0 \cos\theta_0$.

For an inductive upper face, that is $\mathrm{Im}\,\eta^+ < 0$, the surface wave is of the E type because of the only non-vanishing field component in the direction of propagation being electric. In this case we have

$$\overline{U}_0 = [-Z_0 H_0 \sin\beta_0 \;\; -\eta^+ Z_0 H_0 \cos\beta_0]^{\mathrm{T}}, \quad (3)$$

$$\theta_0 = \arccos(\gamma' \cos\beta_0), \quad \gamma' = \sqrt{1-(\eta^+)^2}, \quad (4)$$

$$\varphi_0 = \Phi - \tilde{\varphi}, \quad \tilde{\varphi} = \arcsin(\eta^+/\sin\beta_0), \quad (5)$$

The branch of γ' is chosen in such a way that $\mathrm{Im}\,\gamma' > 0$ and $\mathrm{Re}\,\gamma' \geq 0$ hold.

If the upper face is capacitive ($\mathrm{Im}\,\eta^+ > 0$), it supports a surface wave of the H type with the only non-vanishing field component in the direction of wave motion being magnetic. The quantities that differ from the ones for the E type are

$$\overline{U}_0 = [E_0 \cos\beta_0/\eta^+ \;\; -E_0 \sin\beta_0]^{\mathrm{T}}, \quad (6)$$

$$\gamma' = \sqrt{1-1/(\eta^+)^2}, \quad \tilde{\varphi} = \arcsin\left(\frac{1}{\eta^+ \sin\beta_0}\right). \quad (7)$$

Therefore, the angles of incidence φ_0 and θ_0 are complex-valued with $|\mathrm{Re}\,\varphi_0| \leq \Phi$, $0 \leq \mathrm{Re}\,\theta_0 \leq \pi/2$ and $\mathrm{Im}\,\theta_0 \leq 0$ in line with the assumed range for β_0.

The z-components of the total field takes the following form

$$[Z_0 H_z(r,\varphi;z) \;\; E_z(r,\varphi;z)]^{\mathrm{T}} = \overline{U}(r,\varphi) \exp(\mathrm{i}k_0'' z) \quad (8)$$

with $\overline{U}(r,\varphi) = [U_1(r,\varphi) \;\; U_2(r,\varphi)]^{\mathrm{T}}$ solving the two-dimensional Helmholtz equation outside the wedge and satisfying the respective conditions derived from (1) on the faces of the wedge. Furthermore, it is subject to edge and radiation conditions; see [1] and [12].

B. Integral Equations for the Spectra

As is well known, $\overline{U}(r,\varphi)$ can be expressed in terms of the Sommerfeld integrals:

$$\overline{U}(r,\varphi) = \frac{1}{2\pi\mathrm{i}} \int_\gamma \overline{f}(\alpha+\varphi)\, \mathrm{e}^{-\mathrm{i}k_0' r \cos\alpha} \mathrm{d}\alpha, \quad (9)$$

where γ denotes the Sommerfeld double-loop and $\overline{f}(\alpha) = [f_1(\alpha) \;\; f_2(\alpha)]^{\mathrm{T}}$ the spectra to be determined. The radiation condition demands that $\overline{f}(\alpha) - \overline{U}_0/(\alpha - \varphi_0)$ be regular in the strip $|\mathrm{Re}\,\alpha| \leq \Phi$, where φ_0 is defined in (5).

Inserting (9) into the boundary condition (1) and inverting the Sommerfeld integrals, one obtains a system of equations for the spectra. For example, the equation for $f_1(\alpha)$ reads

$$f_1(\alpha + 2\Phi) - \frac{b_2^+(\alpha)}{b_2^-(\alpha} f_1(\alpha - 2\Phi) = q_1(\alpha) f_1(\alpha), \quad (10)$$

with the coefficients $b_2^+(\alpha), b_2^-(\alpha)$ and $q_1(\alpha)$ given in [1].

On use of $f_1(\alpha) = F_0(\alpha)\mathcal{F}_1(\alpha)$, the above functional equation can be simplified to

$$\mathcal{F}_1(\alpha + 2\Phi) + \mathcal{F}_1(\alpha - 2\Phi) = Q_1(\alpha)\mathcal{F}_1(-\alpha), \quad (11)$$

with $Q_1(\alpha) = q_1(\alpha)F_0(-\alpha)/F_0(\alpha + 2\Phi)$ and the auxiliary function $F_0(\alpha)$ given in [1].

By making use of the S-integrals and taking into account the edge and radiation conditions, an integral equivalent of (11) in the strip $|\mathrm{Re}\,\alpha| \leq 2\Phi$ reads

$$\mathcal{F}_1(\alpha) = \frac{\nu U_{10}/F_0(\varphi_0)}{\sin\nu(\alpha - \varphi_0)} + A_1^+ \mathrm{e}^{-\mathrm{i}\nu\alpha} + A_1^- \mathrm{e}^{\mathrm{i}\nu\alpha} - \frac{\mathrm{i}}{8\Phi} \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{Q_1(-t)\mathcal{F}_1(t)}{\cos\nu(\alpha + t)} \mathrm{d}t, \; \nu = \frac{\pi}{4\Phi}. \quad (12)$$

The constants $A_1^\pm$ are fixed by deleting non-physical poles

$$f_1(\pm\Phi - \pi/2) = b_1^\pm(\mp\Phi - \pi/2) f_1(\pm\Phi + \pi/2), \quad (13)$$

The coefficients $b_1^\pm(\alpha)$ are given in [1].

Relation (12), together with (13), amounts to an integral equation for $\mathcal{F}_1(\alpha)$ on the imaginary axis of the complex α-plane. These values can be obtained by solving numerically the integral equation and then extrapolated into the strip $|\mathrm{Re}\,\alpha| \leq 2\Phi$ on use of (12). Similarly, the second spectrum $f_2(\alpha)$ can be deduced. Inserting them into the Sommerfeld integrals (9) leads to an exact solution, although not in explicit form, to the problem under study.

C. Far-Field Expansion

Deforming the path of integration γ in (9), $\overline{U}(r,\varphi)$ can be rewritten as

$$\overline{U}(r,\varphi) = \overline{U}^{\mathrm{go}}(r,\varphi) + \overline{U}^{\mathrm{sw}}(r,\varphi) + \overline{U}^{\mathrm{d}}(r,\varphi). \quad (14)$$

For large $|k_0' r|$, the diffracted part $\overline{U}^{\mathrm{d}}(r,\varphi)$ is given by

$$\overline{U}^{\mathrm{d}}(r,\varphi) \sim \overline{Q}(\varphi)\, \mathrm{e}^{\mathrm{i}k_0' r}/\sqrt{r} \quad (15)$$

with the non-uniform diffraction coefficient (scattering diagram)

$$\overline{Q}(\varphi) = \left[\overline{f}(\varphi - \pi) - \overline{f}(\varphi + \pi)\right] \sqrt{\mathrm{i}/(2\pi k_0')}. \quad (16)$$

A uniform expression for the diffracted field can be given in a similar way as in [1]; the same is true for the geometrical-optics and surface-wave ingredients of the total field.

D. An Alternative Measurement Method

Below we confine our analysis to the case of an incident electromagnetic surface wave of the E type. The case of an incident electromagnetic surface of the H type can be studied in a similar fashion.

For $|\eta^+/\sin\beta_0| \ll 1$, it turns out from (5) that there is $\tilde{\varphi} \approx \eta^+/\sin\beta_0$. Therefore, in line with (16) and (12), the scattering diagram $\overline{Q}(\varphi)$ near the "shadow boundary" of the incidence at $\varphi = \Phi - \pi$ is proportional to

$$1/(\Delta\varphi + \eta^+/\sin\beta_0), \quad \Delta\varphi = \varphi - (\Phi - \pi). \quad (17)$$

In the infrared and optic regions, the surface impedance of good conductors like copper is almost purely imaginary. As indicated by (17), in a small neighbourhood of the shadow boundary of incidence at $\Delta\varphi = 0$ the angular intensity distribution of the diffracted field is then a Lorenzian curve, whose width is $2/\sin\beta_0$ times the imaginary part of the surface impedance. Based precisely on this fact, a new accurate method has been proposed for determining the surface impedance by measuring the scattering diagram at normal incidence with $\beta_0 = \pi/2$ ([7], [8]).

To measure in addition those surface impedances which are no longer confined to $|\eta^+| \ll 1$, the expression for the diffracted field (15) suggests an alternative way: to measure the wave number $k_0 \sin\theta_0$ in the radial direction r and then to calculate the surface impedance η^+ by solving relation (4),

$$\eta^+ = -\mathrm{i}\sqrt{\left(1 - \sin^2\theta_0\right) / \cos^2\beta_0 - 1}. \tag{18}$$

As indicated clearly by the first relation of (4), it is the skew incidence with $\beta_0 \neq \pi/2$ that enables such an alternative method of measurement for surface impedances.

Detailed results and their discussion will be presented at the conference.

III. Conclusion

In this paper we have reported an exact solution, although not in explicit form, to diffraction of a skew-incident electromagnetic surface wave of either E or H type at an impedance wedge. Thereby we have made use of an earlier work [1]. The unknown field is represented in terms of the Sommerfeld integrals. By inverting the Leontovich condition on the faces of the wedge, a system of equations for the spectra (complex plane-wave amplitudes) turns out. An integral equivalent of this system of functional equations is then established with the aid of the S-integrals. By solving the integral equation along the imaginary axis of the complex angle α the spectra there are obtained and then extrapolated using the same integral expression to a strip that contains at its centre the imaginary axis. By evaluating the Sommerfeld integrals at large distance from the edge we have derived an asymptotic expression for the far field. The asymptotic expression for the diffracted field in far field suggests an alternative method of measurement which allows for the determination of a large class of surface impedances.

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