Gaussian Beam Diffraction by a Fast Moving Wedge

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***Abstract*—This contribution is concerned with deriving the scattering of a time harmonic Gaussian beam from a moving PEC wedge. The incident wave object serve as the basis wave propagators of the phase-space beam summation method which is a general framework for analyzing propagation of scalar and electromagnetic fields from extended sources. By utilizing the Lorentz Transformation and applying Maxwell's boundary conditions in the (scatterer) co-moving frame, the exact solution for the total fields is obtained in both the laboratory and the co-moving frames in the form of spectral integrals. By using high frequency asymptotics the total electromagnetic field is recast in the form of GO and diffraction terms. Finally, a low velocity approximation is applied to the fields.**

I. Introduction

The subject of electromagnetic (EM) field scattering from uniformly moving objects is of significance due to the numerous applications in different fields such as communication, RADAR and object recognition. By obtaining solutions to canonical problems, a generalization for approximated models can be made in order to address the more generic and complex scatterers. Such canonical problem is scattering from moving PEC half plane and PEC wedges.

Various solutions for wedge diffraction by a plane wave were derived, by applying eigenfunction and integral representation in [1], and asymptotic solution in [2] for a wedge with angle which is less than π. In [3], [4] a high frequency solutions were obtained. However these solutions are valid only outside the shadow and reflection boundaries (transition regions). A uniform asymptotic solutions were derived in [5]–[7]. Dyadic diffraction coefficients obtained in form Fresnel integral [8].

Beam summation representation of the edge field of a half plane due to a plane wave obtained in [9], and in [10] and [11] a 2D and 3D GB scattering were investigated, respectively. In this method the diffraction field is represented by a sum of Gaussian beams that propagate in all directions. Scattering problems of moving half planes and wedges were solved for pulse [12], [13], plane wave [14]. In [15] a plane wave diffraction by a multiple wedges was investigated and affects of reflections, transmission and doppler shifts were examined. In [16] Idemen and Alkumru had examined the scattering by a moving half plane.

Decomposition of EM fields by a spectrum of beams is very important due to the mutual spatial-directional locality

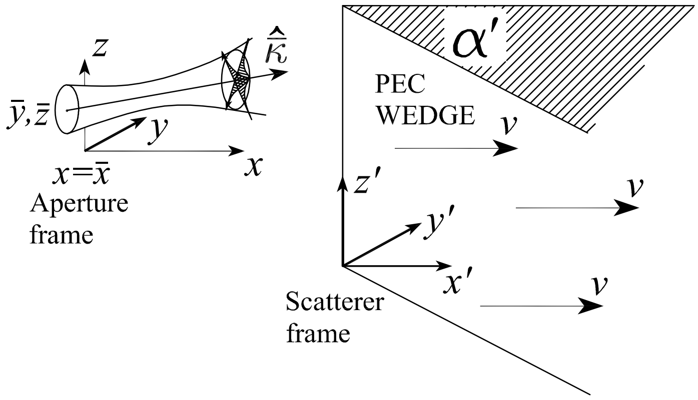


Fig. 1.
Physical configuration. A GB propagator is impinging on a PEC wedge that is moving with velocity v in the direction of the x-axis. The propagator is emanating from the point $(y, z) = (\bar{y}, \bar{z})$ over the stationary plane at $x = \bar{x}$. The scattering object is a PEC wedge with a head angle of α' in the co-moving frame.

of these wave objects. Using the locality properties, complex configurations can be reduced to simpler local beam-scatterer interactions. In the phase-space (PS) representation, the aperture field is expanded into a summation of Gaussian beam propagators (GBPs) that emanate in a discrete set of points and direct [17]–[19]. The propagating waveobjects are localized solutions of Maxwell's equations which carry a Gaussian decay away from the beams' axes. Therefore, by applying locality considerations for some generic scattering problems, the scattered field can be obtained from a class of canonical problems, such as the one that is presented here.

II. Problem Definition

We consider the scattering of the time-harmonic EM beam propagator by a PEC wedge that uniformly translates in the direction of the x-axis with the velocity: $\mathbf{v} = v\mathbf{u}_x$. The edge is located at $x = 0$ at time $t = 0$ (see Fig. 1). The wedge is infinite in the z direction, with a head angle of $\alpha' < \pi/2$.

The incident EM beam propagator with a $\exp(j\omega t)$ time dependence is given by the plane wave spectral representation over the spectral variables (k_y, k_z) as follows [18]:

$$\mathbf{E}^i(\mathbf{r},t) = \frac{1}{(2\pi)^2}\int dk_z dk_y \psi(k_y,k_z)\mathbf{E}_0(k_y,k_z)\times \exp\left\{j\left[\omega t - \mathbf{k}\cdot(\mathbf{r}-\bar{x}\mathbf{u}_x)\right]\right\} \quad (1)$$

where $\mathbf{u}$ denotes a unit vector in cartesian coordinates, $k = \omega/c$ is the free-space wavenumber,

$$\mathbf{k} = k_x\mathbf{u}_x + k_y\mathbf{u}_y + k_z\mathbf{u}_z,\ k_x = \sqrt{k^2 - k_y^2 - k_z^2},$$
$$\mathrm{Re}k_x \geq 0,\ \mathrm{Im}k_x \leq 0,$$
$$\mathbf{E}_0(\mathbf{k}_t) = a_z\mathbf{u}_z + a_y\mathbf{u}_y - (a_z k_z + a_y k_y)k_x^{-1}\mathbf{u}_x \quad (2)$$

and the (spectral) beam-expansion analysis window is given by [17], [18]

$$\psi(k_y,k_z) = -2\pi j/(k\Gamma)\exp[j(k_z\bar{z} + k_y\bar{y})]\times \exp\left\{j[(k_z-\bar{k_z})^2 + (k_y-\bar{k_y})^2]/(2k\Gamma)\right\} \quad (3)$$

In (2), a_z, a_y and $\bar{k}_z, \bar{k}_y$ are the incident EM beam propagator parameters which determine the field magnitude and beam axis, respectively, and Γ with $\mathrm{Im}\Gamma < 0$ is the synthesis window complex parameter that determines the beam complex curvature.

The EM beam propagator in (1) originates from the point $(\bar{x}, \bar{y}, \bar{z})$ and propagates in the direction of the unit vector $\hat{\boldsymbol{\kappa}} = \left(\bar{k_x}, \bar{k_y}, \bar{k_z}\right)/k$, where $\bar{k}_x = \sqrt{k^2 - \bar{k}_y^2 - \bar{k}_z^2}$ with $\mathrm{Re}\bar{k}_x \geq 0$, $\mathrm{Im}\bar{k}_x \leq 0$. We consider only propagating (non-evanescent) EM beam propagators for which we assume that $\bar{k}_y^2 + \bar{k}_z^2 \leq k^2$.

III. Plane Wave Diffraction by a Wedge

Recalling the so-called plane wave invariance principle of Special Relativity, we consider first the scattering of a plane wave from a stationary wedge. The resulting scattered fields are used later for obtaining the scattering of the EM beam propagator from a moving wedge. The incident plane wave is $\mathbf{E}^{pw} = \mathbf{E}_0\exp\left[j(\omega t - k\hat{\boldsymbol{\kappa}}\cdot\mathbf{r})\right]$, $\mathbf{H}^{pw} = \eta_0^{-1}\hat{\boldsymbol{\kappa}}\times\mathbf{E}^{pw}$, where the unit vector $\hat{\boldsymbol{\kappa}} = -\cos\phi_0\sin\theta_0\mathbf{u}_x - \sin\phi_0\sin\theta_0\mathbf{u}_y + \cos\theta_0\mathbf{u}_z$

The total EM field solution due to a plane wave incidence on a PEC wedge is given in the integral form [5]

$$E_z^{pw} = E_{z0}\left[g(\rho,\Phi^-) - g(\rho,\Phi^+)\right]\exp(-jk_z z),$$
$$H_z^{pw} = H_{z0}\left[g(\rho,\Phi^-) + g(\rho,\Phi^+)\right]\exp(-jk_z z) \quad (4)$$

where $\Phi^{\mp} = \phi \mp \phi_0$ and

$$g(\rho,\Phi) = \frac{1}{4\pi jN}\int_{L-L'} d\xi \cot\left(\frac{\xi+\Phi}{2N}\right)\times \exp[j(k\sin\theta_0\rho\cos\xi)], \quad (5)$$

with $N = (2\pi - \alpha')/\pi$. The contours L and L' are given in [5]. The transversal components of the EM fields can be obtained by applying simple differential operators [20].

For large values of $k\rho\sin\theta_0$, we may write $(E/H)_z^{pw} = (E/H)_z^i + (E/H)_z^r + (E/H)_z^d$, where the incident, $(E/H)_z^i$, and the reflected, $(E/H)_z^r$, fields are given by [5], [7]

$$E_z^{i,r} = \sum_m \pm E_{z0}U(\pi - |\phi_0 \mp \phi + 2m\pi N|)\times \exp\{-jk[\cos\theta_0 z - \sin\theta_0\rho\cos(2mN\pi - \phi \pm \phi_0)]\},$$
$$H_z^{i,r} = \sum_m H_{z0}U(\pi - |\phi_0 \mp \phi + 2m\pi N|)\times \exp\{-jk[\cos\theta_0 z - \sin\theta_0\rho\cos(2mN\pi - \phi \pm \phi_0)]\} \quad (6)$$

Here $U(\cdot)$ denotes the unit step function and m is an integer that satisfies

$$|\phi \mp \phi_0 - 2m\pi N| < \pi \quad \text{for} \quad m = ... -2, -1, 0, 1, 2, .. \quad (7)$$

The diffracted field E_z^d is given by

$$E_z^d = -E_{z0}\exp(-jk\rho\sin\theta_0)\exp(-jk_z z)\times \frac{\exp\left(-j\frac{\pi}{4}\right)}{2N\sqrt{2\pi k\rho}}\frac{D_e}{\sqrt{\sin\theta_0}},$$
$$H_z^d = -H_{z0}\exp(-jk\rho\sin\theta_0)\exp(-jk_z z)\times \frac{\exp\left(-j\frac{\pi}{4}\right)}{2N\sqrt{2\pi k\rho}}\frac{D_m}{\sqrt{\sin\theta_0}} \quad (8)$$

where D_e denoted the diffraction coefficient in [5], [7].

IV. Beam Diffraction by a Moving Wedge

By applying the Lorentz transformation [21] to the incident EM field in (1), we obtain the z-component of incident field in the scatterer frame in the form

$$(E/H)''^i_z(\mathbf{r}',t') = \int dk_y dk_z (E/H)'_{z0}(k_y,k_z)\psi(k_y,k_z)\times \exp(jk_x\bar{x})\exp[j(\omega't' - \mathbf{k}'\cdot\mathbf{r}')] \quad (9)$$

$$\omega' = \gamma\omega\left(1 - \frac{k_x}{k}\beta\right), \quad k' = \gamma k\left(1 - \frac{k_x}{k}\beta\right),$$
$$\mathbf{k}' = \left[\gamma k\left(\frac{k_x}{k} - \beta\right), k_y, k_z\right]. \quad (10)$$

where the scatterer frame vector wave number $\mathbf{k}'$ and ω' are given in (10). The integral representation in (9) is a plane wave spectral integral representation over the spectral variables (k_y, k_z). Each spectral plane wave is characterized by a spectral temporal frequency and vector wave number.

A. Exact Solution

Next we apply linearity to the representation in (9) and substitute the $\exp[j(\omega't' - \mathbf{k}'\cdot\mathbf{r}')]$ term in (9) with E_z^{pw} (with the corresponding plane wave parameters). Thus, the total (z component) EM field due to an incident EM beam propagator is given by the spectral plane wave representation:

$$
\begin{aligned}
&(E/H)'_z(\mathbf{r}',t') = \int dk_y dk_z (E/H)'_{z0}(k_y,k_z)\psi(k_y,k_z)\times \\
&\exp(jk_x\bar{x} + j\omega' t' - jk_z z)/(4\pi j N')\times \\
&\left\{\int_{L-L'} d\xi \cot\left(\frac{\xi+\Phi'^{-}}{2N'}\right)\exp(jk'_t\rho'\cos\xi)\ \times\right. \\
&\left.\mp\int_{L-L'} d\xi \cot\left(\frac{\xi+\Phi'^{+}}{2N'}\right)\exp(jk'_t\rho'\cos\xi)\right\}, \\
&\Phi'^{\mp} = \phi' \mp \phi'_0,\ k'_t = \sqrt{k'^2_x + k^2_y},\ \rho' = \sqrt{x'^2+y^2}, \\
&\phi' = \tan^{-1}(y/x') \qquad (11)
\end{aligned}
$$

where $\phi'_0 = \tan^{-1}(k_y/k'_x) + \pi(2r-1)$ with r being the integer which satisfies $0 \le \phi'_0 \le 2\pi$. The integral representation in (11) is the *exact* solution for a beam diffraction by a moving wedge.

B. High Frequency Asymptotics

For large values of $k'_t\rho'$, we can evaluate the plane waves scattering integral solution asymptotically by substituting the exact (plane wave) solution with the asymptotic one. Thus, the high frequency spectral representation for the total field is given by $E = E^{GO} + E^d$ where GO denotes the Geometrical Optics contribution and d denotes the diffraction term. The GO term reads

$$
\begin{aligned}
E'^{GO}_z(\mathbf{r}',t') &= \sum_m \pm \int dk_y dk_z E'_{z0}(k_y,k_z)\psi(k_y,k_z)\times \\
&\exp\left[-j\Omega^{GO}(k_y,k_z)\right]\exp[j(\omega' t' + k_x\bar{x})]\times \\
&U\left[\pi - \left|-(\phi'\mp\phi'_0) + 2m\pi N'\right|\right], \\
H'^{GO}_z(\mathbf{r}',t') &= \sum_m \int dk_y dk_z H'_{z0}(k_y,k_z)\psi(k_y,k_z)\times \\
&\exp\left[-j\Omega^{GO}(k_y,k_z)\right]\exp[j(\omega' t' + k_x\bar{x})]\times \\
&U\left[\pi - \left|-(\phi'\mp\phi'_0) + 2m\pi N'\right|\right] \qquad (12)
\end{aligned}
$$

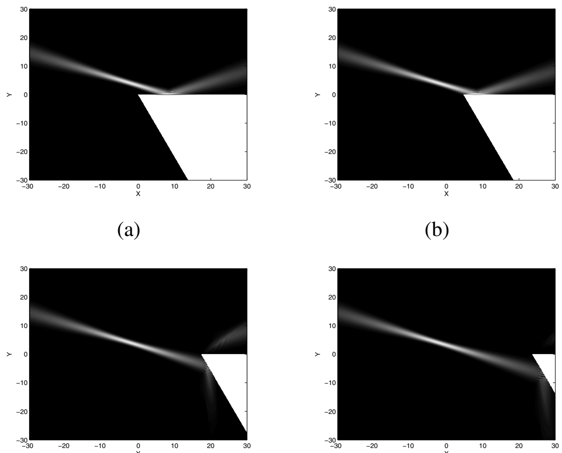
where

$$
\Omega^{GO}(k_y,k_z) = k_z z - k'_t\rho'\cos\left[2mN'\pi - (\phi'\mp\phi'_0)\right]. \quad (13)
$$

The diffraction field is given by

$$
\begin{aligned}
E'^d_z(\mathbf{r}',t') &= \int dk_y dk_z E'_{z0}(k_y,k_z)\psi(k_y,k_z)\times \\
&\exp[j(\omega' t' - k_z z - k'_t\rho' + k_x\bar{x})]\frac{\exp(-j\frac{\pi}{4})}{2N'\sqrt{2\pi k'_t\rho'}}D_e, \\
H'^d_z(\mathbf{r}',t') &= \int dk_y dk_z H'_{z0}(k_y,k_z)\psi(k_y,k_z)\times \\
&\exp[j(\omega' t' - k_z z - k'_t\rho' + k_x\bar{x})]\frac{\exp(-j\frac{\pi}{4})}{2N'\sqrt{2\pi k'_t\rho'}}D_m. \quad (14)
\end{aligned}
$$

Next, we apply the inverse Lorentz transform the integrals in (12) and (14) and obtain the integral representation of the scattered fields in the aperture frame. The integrals are in the form of Fourier integrals and can be evaluated asymptotically by applying a near-beam-axis approximation. The resulting expressions are quite lengthy and due to space limitations the



(a) (b) (c) (d)

Fig. 2. Asymptotic results of the Ez fields components in K-frame. The wedge is moving at speed of $0.7c$ and the beam propagating from the point $(k\bar{y}, k\bar{z} = (120,0))$ in $\hat{\bar{\boldsymbol{\kappa}}}$. Different time samples are presented as indicated in each figure. (a) The total field at $t = 0$. Here the the beam is reflection from the upper surface. (b) The beam is near the edge. (c) The beam is impinging near the edge. The Field consists of partial reflection from the lower surface and (time-dependent) diffraction field. Some of the field that propagating from the upper surface of the wedge is due to earlier reflection which occurred at Fig. 2(b). (d) Reflection from the lower surface.

final expressions are not presented here. Instead we present in Fig. 2 simulations of the asymptotic field and we discuss next the spacial case of normal incident (z-directed) electric field in the low velocity regime in which $v << c, O(\beta^2) \sim 0$.

V. Low Velocity Regime

In this section we introduced the EM scattered fields in the aperture frame under the low velocity approximation. Here we approximate the asymptotic fields for $\beta \ll 1$ in the following manner: The amplitudes are sampled at $\beta = 1$ and the phases are approximated via a first order Taylor series.

A. Outside Transition Regions

Outside transition regions, we can evaluate each integral separately. The resulting E_z fields are given by (m=0):

$$
\begin{aligned}
E'^{GO}_z(r,t) &\sim \mp E'_{z0}(\bar{k}_y,\bar{k}_z)U\left\{\pi - \left|\phi \mp (\bar{\phi}-\pi)\right|\right\}\times \\
&\sqrt{\frac{\Gamma_x(x)}{\Gamma_x(0)}}\sqrt{\frac{\Gamma_y(x)}{\Gamma_y(0)}}\exp\left[j\omega t - jk\Omega^{GO}(r_b)\right], \\
\Omega^{GO}(r_b) &= z_b + \frac{1}{2}\left[x_b^2\Gamma_x(x) + y_b^2\Gamma_y(x)\right] \qquad (15)
\end{aligned}
$$

with

$$
\begin{aligned}
&x_b = -\sin\bar{\phi}(x-\bar{x}) + \cos\bar{\phi}(\pm y - \bar{y}), \quad y_b = z - \bar{z}, \\
&z_b = \cos\bar{\phi}\,(x-\bar{x}) + \sin\bar{\phi}\,(\pm y - \bar{y}) \qquad (16)
\end{aligned}
$$

and

$$
\Gamma_x(x) = \left(\frac{\cos^2\bar{\kappa}_x}{\Gamma} + \frac{x-\bar{x}}{\cos\bar{\kappa}_x}\right)^{-1},\ \Gamma_y(x) = \left(\frac{1}{\Gamma} + \frac{x-\bar{x}}{\cos\bar{\kappa}_x}\right)^{-1} \quad (17)
$$

and (m=1)

$$E_z'^{GO}(r,t) \sim -E'_{z0}(\bar{k}_y,\bar{k}_z)U(\pi - |\phi + \phi_0' + 2\pi N'|) \times \sqrt{\frac{\Gamma_x(x_1)}{\Gamma_x(0)}}\sqrt{\frac{\Gamma_y(x_1)}{\Gamma_y(0)}} \exp\left[j\omega(t-\beta x/c) - jk\Omega^{GO}(r_b)\right],$$

$$\Omega^{GO}(r_b) = z_{b1} + \frac{1}{2}\left[x_{b1}^2\Gamma_x(x_1) + y_{b1}^2\Gamma_y(x_1)\right] \quad (18)$$

where $\bar{x}(t) = \bar{x} - vt$ and

$$\begin{aligned} x_{b1} =& -\sin\bar{\phi}\left[\cos\alpha' x_1 - \sin\alpha' y_1 - \bar{x}(t)\right] - \\ & \cos\bar{\phi}(\cos\alpha' y_1 + \sin\alpha' x_1 + \bar{y}), \\ y_{b1} =& z - \bar{z}, \\ z_{b1} =& \cos\bar{\phi}\left[\cos\alpha' x_1 - \sin\alpha' y_1 - \bar{x}(t)\right] - \\ & \sin\bar{\phi}(\cos\alpha' y_1 + \sin\alpha' x_1 + \bar{y}) \end{aligned} \quad (19)$$

with

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha' & -\sin\alpha' \\ \sin\alpha' & \cos\alpha' \end{bmatrix} \begin{bmatrix} x - vt \\ y \end{bmatrix}. \quad (20)$$

The diffraction field is given by

$$E_z'^{d}(\mathbf{r}',t') = E'_z(\bar{k}_y,\bar{k}_z)\frac{\exp(-j\frac{\pi}{4})}{2N'\sqrt{2\pi k_t'\rho}}D_e \times \frac{(k\Gamma)^{-1}}{\sqrt{\det(\mathbf{q}2)}}\exp\left\{-j\left[q_0 - \frac{1}{2}\left(\mathbf{q}_1^t\mathbf{q}_2^{-1}\mathbf{q}_1\right)\right]\right\}. \quad (21)$$

with

$$q_0 = k_t'\rho' - \omega t(1 - k_x/k\beta) + k\beta x - \bar{k}_x\bar{x} - \bar{k}_y\bar{y},$$

$$\mathbf{q}_1 = \begin{bmatrix} \tan\bar{\phi}(\bar{x} - vt) - \bar{y} + \beta\rho'\tan\bar{\phi} \\ z - \bar{z} \end{bmatrix},$$

$$\mathbf{q}_2 = \begin{bmatrix} \frac{\rho'\beta}{k_t'\cos^3\bar{\phi}} - \frac{\bar{x}-vt}{k_t\cos^3\bar{\phi}} - \frac{1}{k\Gamma} & 0 \\ 0 & -\frac{\rho}{k} - \frac{1}{k\Gamma} + \frac{\bar{x}-vt}{k_x} \end{bmatrix}. \quad (22)$$

B. Inside Transition Regions

Inside the transition regions the integral of the GO and the diffraction fields are not continues. By using a small argument approximation of the diffraction field [5], the new expression for the phase of the diffracted field can be written as

$$\Omega_d \sim j\left\{k'\rho'\cos\left[2mN'\pi - (\phi' \mp \phi_0')\right] - k_z z\right\} \quad (23)$$

which is the same as for the GO field. Thus we combine the diffraction and GO field integrals.

VI. Conclusions

In this paper we introduced an exact and asymptotic solutions for GB diffraction by a PEC wedge using the frame hopping method. The exact fields are is given by spectral representations in integral form. An asymptotic analysis and simulations were presented as well the special case of low velocity scattering.

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