Directive Radiation of a Line-Source Inside an Anisotropic Material Slab via Transformation Electromagnetics

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***Abstract*— A transformation electromagnetics technique for transforming a cylindrical wave into a plane wave is provided. The electromagnetic material parameters in the final coordinate space require only a single spatially varying term of the permittivity tensor for converting the wave front of TE waves. The detailed derivation of the spatially varying constitutive parameters and matching conditions for the plane wave are provided. An example case with a magnetic current line source is reported and the proposed final antenna is proved to be directive using full wave simulations.**

I. Introduction

Transformation optics proved to be an efficient way of controlling wave propagation by tailoring the anisotropy and inhomogeneity of the medium where the constitutive parameters of the medium (permeability and permittivity tensors) are spatially dependent. Transformation optics has been considered to design devices such as invisibility cloaks [1], electromagnetic concentrators [2], superabsorber [3] and superscatterer[4], rotators [5] and lens antenna [6].

In this study we utilize transformation optics for achieving directive radiation using a single source. Other metamaterial designs have been proposed for the same purpose using for example low refractive index [7] - [10] and left-handed media [11]. Similarly Fabry-Perot cavity based antennas have been reported to achieve high directivity [12], [13]. The optical transformation method for enhancing directivity, similar to the approach presented here, was demonstrated in [14], [15]. However, the spatially varying permittivity and permeability tensor entries constitute a challenge for implementing these designs because several material electromagnetic parameters should vary spatially. It is of great importance to reduce the number of spatially varying parameters, therefore in this article we propose further development of the cylindrical to planar transformation already used in [14], [15] in order to achieve single spatially varying tensor entry of permittivity or permeability.

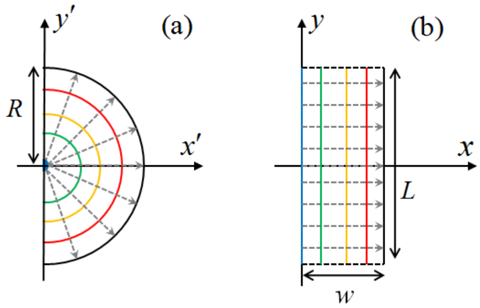

Fig. 1. (a) The virtual space, (b) the real space with color-coded lines denoting the transformed wave fronts.

The organization of the paper is as follows. In Sec. II the derivation of the constitutive parameters and the method of transformation are presented, providing the simple design formulas. In Sec. III we provide the simulation results using finite-element methods for validation of the designed parameters.

II. Optical Transformation

The coordinate transformation scheme for mapping from cylindrical to Cartesian should be in the form outlined in (1). The concentric circles in the virtual space in Fig. 1 (denoted with the primed coordinate variables, x' and y') are mapped to the vertical lines in the physical space (denoted with the non-primed coordinate variables, x and y) given as

$$\begin{aligned} x &= f(\rho') = f(\sqrt{x'^2 + y'^2}) \\ y &= g(\phi') = g(\arctan(\frac{y'}{x'})) \\ z &= z' \end{aligned} \qquad (1)$$

Note that different functions f and g would result in different transformations. Our aim is to explore a transformation scheme which will greatly reduce the complexity of the spatial dependence of necessary permittivity and permeability tensor entries. According to the basic transformation electromagnetic theory [1], the relative constitutive parameters in the physical space can be constructed by using the relations

$$\underline{\boldsymbol{\varepsilon}} = \frac{J\underline{\boldsymbol{\varepsilon}}'J^T}{\det(J)}, \quad \underline{\boldsymbol{\mu}} = \frac{J\underline{\boldsymbol{\mu}}'J^T}{\det(J)} \tag{2}$$

where J is the Jacobian transformation matrix between the two coordinate systems, explicit expression can be found in [16]. If we assume the virtual space to be a homogeneous isotropic medium, say free space ($\underline{\boldsymbol{\varepsilon}}' = \underline{\mathbf{I}}$ and $\underline{\boldsymbol{\mu}}' = \underline{\mathbf{I}}$, where $\underline{\mathbf{I}}$ is the unit matrix), the relative constitutive parameters in the physical space reduce to

$$\underline{\boldsymbol{\varepsilon}} = \underline{\boldsymbol{\mu}} = \mathrm{diag}\left(\frac{\rho' \frac{\partial f}{\partial \rho'}}{\frac{\partial g}{\partial \phi'}}, \frac{\frac{\partial g}{\partial \phi'}}{\rho' \frac{\partial f}{\partial \rho'}}, \frac{\rho'}{\frac{\partial f}{\partial \rho'}\frac{\partial g}{\partial \phi'}} \right) \tag{3}$$

where "diag" denotes the diagonal matrix. When assuming that the system is invariant along the z axis, Maxwell's equations for the TE and TM waves (with respect to the z axis) can be decoupled. In that case, the TE waves depend only on $\varepsilon_{xx}, \varepsilon_{yy}, \mu_{zz}$ whereas the TM waves depend only on $\mu_{xx}, \mu_{yy}, \varepsilon_{zz}$. Then TE (TM) waves' wavenumbers (thus the trajectory) will depend solely on the product of $\varepsilon_{xx}\mu_{zz}$ and $\varepsilon_{yy}\mu_{zz}$ ($\mu_{xx}\varepsilon_{zz}$ and $\mu_{yy}\varepsilon_{zz}$). Thus we can modify the permittivity or permeability tensor entries by keeping $\varepsilon_{yy}\mu_{zz}$ and $\varepsilon_{yy}\mu_{zz}$ ($\mu_{yy}\varepsilon_{zz}$ and $\mu_{yy}\varepsilon_{zz}$) constant, and still preserve the purpose of the transformation and the trajectory of the wave, consequently. Notice that this modification comes with the cost of changing the wave impedances, however matching the wave impedances is further addressed in the following. From this point on we will report the desired conditions for TE waves, nonetheless similar steps can be also applied for TM waves.

Now consider the three constitutive parameters $\varepsilon_{xx}, \varepsilon_{yy}, \mu_{zz}$ that the TE$_z$ waves' trajectories depend on. Assume that we multiply $\varepsilon_{xx}, \varepsilon_{yy}$ and divide μ_{zz} by the same amount μ_{zz}/χ, then the resulting modified constitutive parameters become

$$\varepsilon_{xx}^m = \frac{\rho'^2}{\chi\left(\frac{\partial g}{\partial \phi'}\right)^2} \quad \varepsilon_{yy}^m = \frac{1}{\chi\left(\frac{\partial f}{\partial \rho'}\right)^2} \quad \mu_{zz}^m = \chi \tag{4}$$

where the superscript m stands for "modified". It is straightforward from (4) that only two parameters $\left(\varepsilon_{xx}^m, \varepsilon_{yy}^m\right)$ have spatial dependence in general.

However the choice of the functions f and g can further simplify these spatial dependence. Since the derivatives of f and g appear in the formula, we can simply assume $x = f(\rho') = \rho'(w/R)$ and $y = g(\phi') = \phi'(L/\pi)$ where L and w are the height and the width of the designed metamaterial slab in the physical space whereas R is the radius of the circle in the virtual space (Fig. 1). Eventually the final expressions for the modified parameters can be expressed as

$$\varepsilon_{xx}^m = x^2\frac{\pi^2}{\chi L^2}, \quad \varepsilon_{yy}^m = \frac{R^2}{\chi w^2}, \quad \mu_{zz}^m = \chi \tag{5}$$

Note that only ε_{xx}^m has a spatial dependence. We apply the transformation in (1) from a half disc to a slab inside free space as in Fig. 1 and it is required that the waves at the $+x$ edge of the slab are matched to the free space. The impedance of a TE$_z$ wave propagating in the $+x$ direction is $Z_{\mathrm{TE}} = \eta_0\sqrt{\mu_{zz}^m/\varepsilon_{yy}^m}$ where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ denotes the wave impedance in free space. After enforcing the matching contision as $Z_{\mathrm{TE}} = \eta_0$, it follows simply that $\chi = R/w$ and

$$\varepsilon_{xx}^m = x^2\frac{w\pi^2}{RL^2}, \quad \varepsilon_{yy}^m = \frac{R}{w}, \quad \mu_{zz}^m = \frac{R}{w} \tag{6}$$

III. SIMULATION

In order to validate the design formula, full-wave simulations are done with COMSOL Multi-Physics Software utilizing the finite-element method (FEM). We consider the following parameters for the geometry $L = 2W = 2R = 0.2$ m and the design is simulated at 4 GHz. A magnetic current line source along the z axis is placed on the $-x$ edge of the converting slab. The instantaneous magnetic field is reported as a colormap in Fig. 2 where one can observe clearly that the metamaterial slab successfully converts the waves due to a magnetic current source to a planar wave. In addition, the normalized far field radiation pattern on the x-y plane is reported in Fig. 3. As expected, the plane wave in the near field yields a directive far-field radiation.

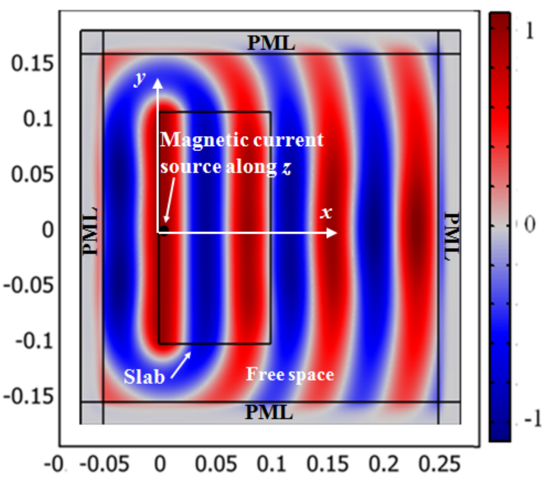

Fig.2. Snap-shot of the time domain magnetic field (polarized along z) distribution of the waves emanating from the magnetic current line source near the origin. Despite the line source excitation, wavefronts are planar.

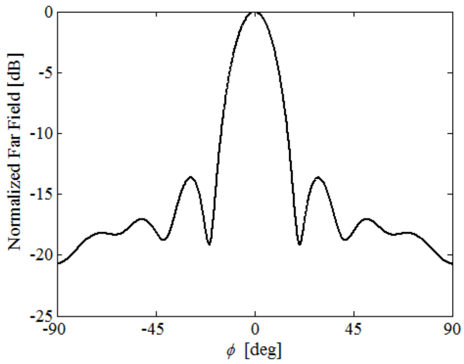

Fig.3. Normalized far-field radiation pattern of the case shown in Fig. 2, showing directivity.

IV. Conclusion

A method based on transformation electromagnetics is reported, consisting in a few steps, and it is applied to transform the electric field emitted by a current line source into a directive radiation pattern. The proposed design method requires only a single spatially varying entry of the permittivity tensor of the metamaterial slab. Full-wave simulation results are in good agreement with the expectations. This technique proves to be a viable way of designing metamaterial slabs for increasing directivity of antennas.

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