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Metasurfaces designed by coordinate transformations

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Abstract—**Metasurfaces are thin metamaterial layers characterized by unusual reflection properties of plane waves and/or dispersion properties of surface/guided waves. Properly modulated metasurfaces can be used to transform an incoming surface wave field into a different wavefield configuration with desired properties. This work proposes a systematic approach to design a large number of metasurface-based devices based on a metasurface transformation theory. This approach represents an extension of the Transformation Optics method to control the wavefront of surface waves through the use of modulated metasurfaces.**

I. INTRODUCTION

Metamaterials are artificial media which can be engineered to achieve electromagnetic behaviours which can not be found in nature. These artificial media can be formed by periodically arranging many small inclusions in a dielectric host environment. This concept can be extended to two dimensional lattices on a surface, thus realizing thin metamaterial layers characterized by unusual reflection properties of plane waves and/or dispersion properties of surface/guided waves. This surface version of a metamaterial is referred to in the literature as "metasurface" [1].

Metasurfaces can be realized at microwave frequencies by printing a dense periodic texture of small elements on a grounded slab, with or without shorting vias. The basic assumption in the conventional analysis of metasurfaces is the periodical distribution of the constituent elements with a period small in terms of the wavelength. Under this condition, the metasurface can be accurately characterized in terms of an equivalent surface impedance tensor relating the tangential components of the average electric and magnetic fields [2]. By introducing a modulation of the equivalent surface impedance it is possible to engineer the interaction of a given incoming field with the metasurface, thus realizing a large class of devices [3]. For instance, through the application of the holographic concept, modulated metasurfaces can be used to produce leaky-wave radiation [4-5]. Moreover, metasurfaces can be designed to achieve a prescribed equivalent refractive index profile, thus realizing planar lenses [6], or horns with increased directivity [7]. Many other applications are feasible, provided one is capable of defining the equivalent impedance pattern needed to obtain a desired wavefield transformation. This work proposes a systematic approach to metasurface design based on an extension of the Transformation Optics concept.

Transformation Optics is a systematic approach that makes use of coordinate transformations to design electromagnetic devices capable of controlling the propagation of electromagnetic waves [8]. This control is achieved on the basis of macroscopic equivalent constitutive tensors of a volumetric anisotropic material. The TO methodology has been applied, for instance, to design invisibility cloaks, i.e. shells of anisotropic materials capable of rendering any object within their interior cavities invisible to detection from outside [9, and references therein]. However, the technological difficulties in controlling the variation of the equivalent constitutive tensors of volumetric metamaterials, together with anisotropy and extreme values of the parameters, complicate the engineering implementation of TO in practical devices.

This work shows how the TO approach can be extended to control the propagation of design surface waves (SW) through properly designed modulated metasurfaces, with a significant increase in technological simplicity.

II. METASURFACE DESIGN BASED ON A COORDINATE MAPPING

In order to extend TO theory to metasurfaces, let us consider two half-spaces: a "virtual" one, and a "real" one. Both half-spaces are filled by *free space*, however, they have two *different boundary conditions*. We assume that the virtual space possesses boundary conditions described by a *uniform* scalar reactive surface-impedance jX_S.

We define a coordinate transformation from the real to the virtual space. This transformation leaves unchanged the coordinate orthogonal to the surface impedance, so that it can be actually described by a 2D mapping. Similarly to what happens in the TO approach, the wavefronts of a SW propagating in the virtual space would be distorted in the real space according to the aforementioned coordinate mapping. This effect can be obtained by imposing in the real space boundary conditions described by a modulated anisotropic reactive impedance $j\underline{\underline{\mathbf{X}}}_{eq}$, whose entries are univocally related to the coordinate transformation. This relationship is obtained by matching the metasurface local dispersion equation to the one associated with the transformed wavefront.

It is noted that the direct application of a two dimensional version of TO would imply a variable compression of the medium in the direction normal to the surface, with a consequent inhomogeneity of the surrounding medium. In order to simplify the practical implementation we have instead assumed *a priori* the presence of free space above the impedance surface, and we have compensated this assumption by changing the value of the reactance tensor. The process suggested here is not exact; however, it is possible to identify the parameters of the coordinate transformation that reveal the accuracy of the approximation and to define the condition to avoid radiation losses.

A. Conformal Mappings

In the proposed approach, conformal mappings result in an isotropic equivalent surface impedance. In these cases, it is possible to identify an equivalent refractive index and to apply an effective Fermat's principle formulation to determine the ray paths. A similar approach can be also used for quasi-conformal mappings [10].

Isotropic metasurfaces can be realized at microwave frequencies by printing a dense periodic texture of small symmetric elements on a grounded slab. The modulation is obtained by gradually changing the geometry of the elements in contiguous cells. Figure 1 illustrates examples of practical realizations of modulated metasurfaces, consisting of square or circular patches with different sizes printed on a grounded slab.

Due to the small dimensions of the unit cell, the impedance variation can be assumed to be almost continuous. The value of the equivalent surface impedance at a given unit cell is obtained by considering the relevant constituent element as embedded in a uniform periodic structure which locally matches the geometry.

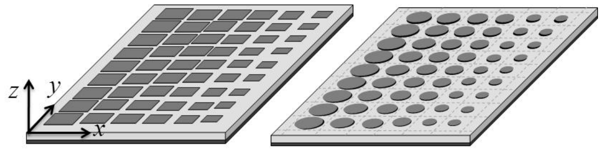

Fig. 1 Modulated metasurfaces at microwave frequencies consisting of small patches with variable sizes printed on a grounded slab.

B. Non Conformal Mappings

In the proposed approach, non-conformal mapping s lead to anisotropic equivalent impedance tensors. A specific anisotropic behaviour of the metasurface can be obtained by using asymmetric constituent elements. These elements typically exhibit two non-dimensional parameters, which primarily affect the principal values and the principal axes of the impedance tensor, respectively. The first parameter is the ratio between a characteristic length of the patch, and the side length of the periodic cell. Increasing this parameter implies increasing the magnitude of the principal values of the impedance tensor. The second parameter is the orientation angle and it mostly influences the orientation of the principal axes of the tensor. Figure 2 shows the entries of the equivalent impedance tensor for a screw-head patch as a function of the two aforementioned parameters. These maps have been obtained by assuming the patch embedded in a periodic Cartesian lattice and applying a periodic Method of Moment (MoM) analysis for a limited but sufficiently high number of prameter pairs.

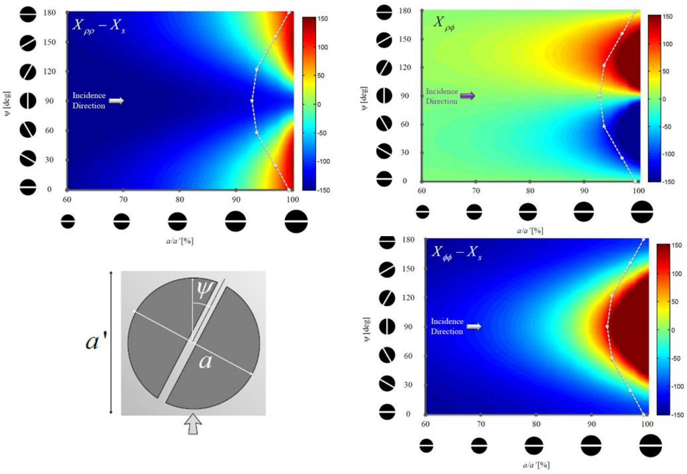

Fig. 2: Maps (in ohms) of the reactance tensor components $X_{ee}-\overline{X}_{ee}, X_{hh}-\overline{X}_{hh}, X_{eh}$; where $\overline{X}_{ee}, \overline{X}_{hh}$ are average values around 300 Ohms for the circular slotted element in the inset (results from [5]).

III. CONCLUSION

A systematic approach based on a generalization of the transformation optics concept has been presented for the design of metasurface based devices. As the outputs of the conventional TO approach are the metamaterial constitutive parameters able to perform a certain modification of the ray-field path, here the outcomes are the components of the equivalent impedance tensor capable to create a prescribed curved wavefront surface wave.

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