

# Surface wave Maxwell fish eye lens

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**Abstract**—The Maxwell fish eye lens has previously been utilised as an imaging device which exhibits sub-wavelength resolution. Here, we propose a new implementation based on surface waves which are excited in a dielectric slab over a metallic ground plane. Although the Maxwell fish eye lens has been shown to possess strong frequency dependence, this paper illustrates that for a single frequency, the proposed surface wave lens is capable of resolving the position of a single source with sub-wavelength resolution.

## I. INTRODUCTION

Perfect imaging, in which features of an object much smaller than the wavelength of the incident wave can be resolved, has been sought for centuries [1-3]. The advent of metamaterials brought about a renewed vigour in the search, as previously unachievable material properties became feasible, and invited a new group of devices to be designed. One such device, the perfect lens, employs the use of a negative refractive index [4]. The latter is able to amplify an evanescent wave, as opposed to the usual decay it suffers in positive index materials, so that an image can be formed that includes a contribution from the evanescent field. While this proposal was very promising, limitations associated with current metamaterial technology restrict the practicality of such a lens since they involve high losses and the material dispersion limits the bandwidth of operation [5].

A new proposal which theoretically circumvents the bandwidth problems of previous perfect imaging lenses was published in 2009 [6]. This lens uses the Maxwell fish eye index profile, with the addition of a mirror around the circumference in order to avoid refractive indices lower than unity. However, this lens presents some further problems with the physical implementation, in that the object and image are located inside the lens. A number of attempts have been made to overcome this problem. The first uses a further transformation of the Maxwell fish eye profile to allow for a region of air in which the object can be placed and the image can be formed [7]. However, in this system, the perfect optical imaging behaviour has been proven to be lost. A second proposal uses a quite different profile where there is a homogeneous imaging region, but if this region is chosen to be air, then the device requires materials with an index less than one [8,9].

Here we propose an implementation of the lens utilising surface wave propagation [10], for which the main advantage is that the source is excited outside of the lens, negating the

necessity of invasive procedures. In this system the source and drain are positioned above, but close to the radially varying dielectric medium. Therefore, surface waves are excited at the interface between the dielectric and air boundary [11].

## II. SURFACE WAVE CHARACTERIZATION

The first step in the design of the Maxwell fish eye lens, is to obtain the propagation properties of surface modes that can potentially be excited in an infinite dielectric slab over a ground plane [12]. For this purpose, the dispersion characteristics of a surface wave excited with a thin dielectric grounded slab and a semi-infinite air region was studied. The employed unit cell for the eigenmode simulations is sketched in Fig. 1(a), and the associated dispersion curves for various values of permittivity of the dielectric slab are plotted in Fig. 1(b).

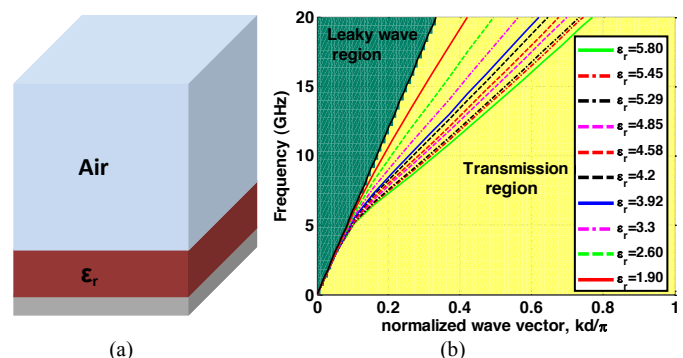


Fig. 1. (a) A sketch of the unit cell for which the dispersion curve has been numerically calculated (b) The dispersion curves for numerous values of relative permittivity.

## III. MAXWELL FISH EYE LENS DESIGN

The permittivity profile of the original space Maxwell fish eye lens is given by [13]

$$\varepsilon(r) = \left( \frac{2}{1+r^2} \right)^2 \quad (1)$$

where  $r$  is the radial coordinate, normalized by the chosen radius of the lens. This profile is illustrated in Fig. 2(a). Although this lens is composed purely of dielectrics, previous studies have shown the device to exhibit frequency dependence [14,15].

If the original operation of the Maxwell fish eye lens is to be retained when surface waves are excited in dielectric slabs over a ground plane, the permittivity of the employed materials have to be changed according to the propagation constant of that surface wave at a particular frequency [10]. This adjusted profile is represented in Fig. 2(b), where the maximum required permittivity in this new map is now increased from 4 to nearly 6. Fig. 2(c) plots the comparison between the original and surface wave permittivity map for the Maxwell fish eye; the relation between these two maps is not linear.

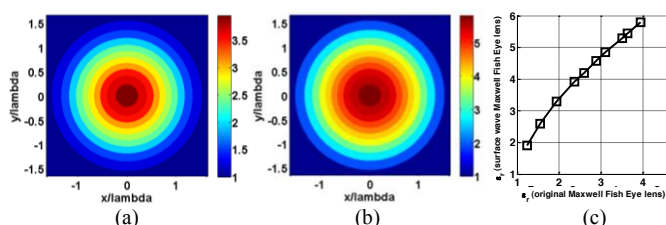


Fig. 2. (a) The permittivity map of the original Maxwell fish eye lens after discretisation. (b) The modified permittivity map suitable for use as a surface wave Maxwell fish eye lens. (c) A comparison of the two permittivity maps given in (a) and (b), where the curve gives the continuous profile, and the squares denote the values of the chosen discrete profile.

#### IV. RESULTS

In the inset of Fig. 3, the proposed configuration is shown, where the source and drain are in the form of a coaxial cables [7,16] in which the inner conductor is exposed by a length that allows operation at the selected frequency. The distance from the source cable to the dielectric is set so that good confinement of the surface wave to the surface is achieved. Furthermore, as in the original conception of the Maxwell fish eye lens [6], a mirror (in the form of a metallic wall) is also employed here. In Fig. 3, the surface wave fish eye lens is shown to be able to produce an image of a source when a drain is positioned at the correct image location. The signal received at the drain is lower than the emitted signal, due to the existence of losses in the graded dielectric slab as well as the fact that some of the emitted power is radiated from the coaxial cables.

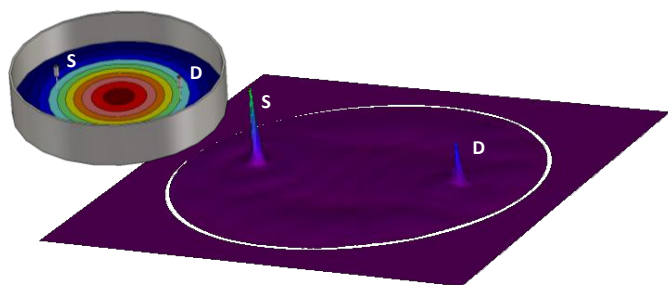


Fig. 3. The electric field distribution when a single emitting source (S), and a single passive drain (D) are positioned above the Maxwell fish eye lens, as illustrated in the inset.

In order to demonstrate the imaging capabilities of this surface wave lens, we study a more complex configuration in which an array of drains is included. In Fig. 4. three drains

have been included to investigate whether, at a single frequency, the source can occupy different positions and be resolved at the correct drain with sub-wavelength resolution. Here, three different simulations were performed, the results of which are given in Fig. 4(a). The red curve are the results when a source is positioned so that drain 1 is at the correct image position, and it can be seen that drain 1 receives a signal higher than at drains 2 and 3. The blue curve is when the source is positioned centrally, and for the green curve, the image should appear in drain 3, both of which are able to be resolved.

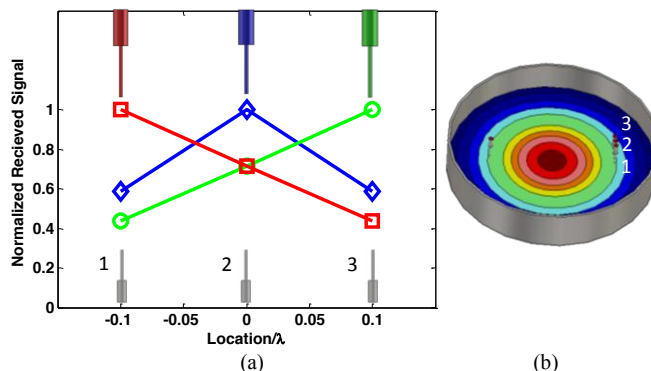


Fig. 4. The Maxwell fish eye lens with a system of three drains, and a single source. (a) The value of the normalized received signal at each output drain for three source positions (red, blue and green). (b) The Maxwell fish eye lens with three drains labelled numerically.

#### V. CONCLUSIONS

In this paper, a modified Maxwell fish eye profile, appropriate for the steering of surface waves at the interface between the dielectric and air boundary has been demonstrated. This implementation of the lens is much more versatile in terms of source and drain arrays than the original version, because no invasive procedures are necessary to embed the antennas/detectors within the dielectric. A single drain and source scenario was used to show that the modified surface wave profile is working effectively and, equivalent to the original Maxwell fish eye studies, this illustrates that an image can be formed if a drain is present. It has also been shown that for a single frequency, the surface wave Maxwell fish eye lens is capable of resolving a single source when three drains are included in the system with sub-wavelength resolution.

#### ACKNOWLEDGMENT

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#### REFERENCES

- [1] J. Cartright, "Defeating diffraction," *Physics World*, May 29, 2012.
- [2] V. Sandoghdar, "Beating the diffraction limit," *Physics World*, 14, 29, 2001.
- [3] N. I. Zheludev, "What diffraction limit?," *Nature Materials* 7, 420-422, 2008.

- [4] J. B. Pendry, "Negative Refraction makes a Perfect Lens," *Phys. Rev. Lett.* **85**, 3966, 2000.
- [5] O. Quevedo-Teruel, W. Tang, and Y. Hao, "Isotropic and Non-Dispersive Planar Fed Luneburg Lens," *Optics Letters*, vol. 37, no. 23, December 1, 2012.
- [6] U. Leonhardt, "Perfect imaging without negative refraction", *New Journal of Physics*, vol. 11 pp.093040, 2009.
- [7] Y. G. Ma, S. Sahebdivan, C. K. Ong, T. Tyc, and U. Leonhardt, "Subwavelength imaging with materials of in-principle arbitrarily low index contrast," *New Journal of Physics* 14, pp. 025001, 2012.
- [8] J. C. Miñano, "Perfect imaging in a homogeneous three dimensional region," *Opt. Express* 14, 9627, 2006.
- [9] T. Tyc, L. Herzanova, M. Sarbort and K. Bering, "Absolute instruments and perfect imaging in geometrical optics", *New Journal of Physics* vol. 13 p.p. 115004, 2011.
- [10] S. Maci, G. Minatti, M. Casaletti, M. Bosiljevac, "Metasurfing: Addressing Waves on Impenetrable Metasurfaces," *IEEE Antennas and Wireless Propagation Letters*, vol.10, no., pp.1499-1502, 2011.
- [11] J. B. Pendry, L. Martin-Moreno, and F. J. Garcia-Vidal, "Mimicking Surface Plasmons with Structured Surfaces," *Science*, vol. 305, no. 5685, pp. 847–848, 2004.
- [12] G. Minatti, F. Caminita, M. Casaletti, S. Maci, "Spiral Leaky-Wave Antennas Based on Modulated Surface Impedance," *IEEE Transactions on Antennas and Propagation*, vol.59, no.12, pp.4436-4444, 2011
- [13] J. C. Maxwell, *The Cambridge and Dublin mathematical journal* vol. 8 p.p.188, 1853.
- [14] T. Tyc, A. Danner, "Frequency spectra of absolute optical instruments", *New Journal of Physics*, vol. 14, pp. 085023, 2012.
- [15] O. Quevedo-Teruel, R. C. Mitchell-Thomas and Y. Hao, "Frequency dependence and passive drains in fish eye lenses", *Physical Review A in press*, 2012.
- [16] Y. G. Ma, S. Sahebdivan, C. K. Ong, T. Tyc and U. Leonhardt, "Evidence for subwavelength imaging with positive refraction", *New Journal of Physics* vol. 13 p.p.033016, 2011.