# Dielectric Multilayers for Antenna and Cloaking Devices Designed from Transformation Electromagnetics

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Abstract—Transformation electromagnetics provided an effective tool to design devices that could control the electromagnetic wave with desired ray traces. However, the resulted material constitutive parameters for these devices are mostly inhomogeneous and anisotropic, which usually rely on large number of precisely defined sub-wavelength artificial structures, or metamaterials. In this paper, we show that by employing simple linear coordinate transformations, devices for antenna and invisibility cloak applications can be achieved with homogeneous birefringent dielectric media, which can be further constructed with multilayer of normal dielectrics, reducing the complexity of practical realization. Two examples, a planar retrodirective reflector and a ground plane invisibility cloak, have been successfully designed. The functionality has been validated through full-wave simulation, as well as experimental testing on the proto-type.

# I. INTRODUCTION

Recently, based on the form-invariant of Maxwell's equations under certain coordinate transformations, the transformation electromagnetics (TE) proposed in [1, 2] has triggered great interest of applying it to various electromagnetic (EM) device designs due to its potential ability to arbitrarily manipulate the EM wave propagation with the pre-defined distribution of material constitutive parameters implemented through artificial metamaterials. More intensive theoretical and experimental explorations have brought in many interesting results and practical approaches. Besides the invisibility cloak [1-3], EM wave concentrators, rotators, shifter and other interesting EM devices have also been proposed by utilizing the TE method [4-7]. TE has also been utilized to design antenna and related devices. For example, planar antenna and lens have been designed through coordinate transformation, which show similar performance with conventional counterparts, but with lower profile [8-9].

Most devices generated from different coordinate transformations could lead to material with spatially varied and anisotropic constitutive parameters, making the practical realization difficult. For example, to realize the ground plane invisibility cloak in the microwave regime, metamaterial with large number of precisely defined sub-wavelength artificial structures should be used [10]. Such complicated design and fabrication procedure could hinder the TE designed devices to more practical applications. In this presentation, we propose a simple approach to construct material structures and devices designed under the framework of TE. By choosing simple linear coordinate transformations, antenna device and invisibility cloak can be designed with homogeneous birefringent dielectric media, and can be further constructed according to the effective medium theory with multilayer structures of normal dielectrics, reducing the complexity of practical realization. We will show two design examples, one is a planar retrodirective reflector and the other is a ground plane invisibility cloak. Both have been successfully designed through multilayer of normal dielectric slabs. We will demonstrate the performance of these devices through full-wave simulation, as well as the experimental testing on the proof-of-concept proto-type.

# II. PLANAR RETRODIRECTIVE REFLECTOR

A retrodirective reflector is an antenna device that can scatter EM waves in the direction anti-parallel to that of the incoming EM beam. It is quite useful in microwave engineering such as to enhance radar cross sections of ships, airplanes, or in satellite communications, as well as in identification application and military application [11-14]. A conventional retrodirective reflector can be realized by the socalled corner reflector structure which consists of two or three conducting sheet metal or screen surfaces at 90° angles to each other. However, it is quite desirable to construct retrodirective reflectors with planar or low-profile features in many applications.

We employ the TE method to transform a corner reflector to a planar PEC with a dielectric cover structure to form a planar retrodirective reflector. For simplicity, we restrict the problem to a two dimensional (2D) case. We consider a 2D corner reflector with two perfect conducting (PEC) planes AB and BC perpendicular to each other as illustrated in Fig. 1(a). A specifically chosen linear coordinate transformation as schematically shown in Fig. 1, under which only the y coordinate has been transformed, can be described as

$$\begin{cases} x' = x & . \\ y' = \begin{cases} (y+a+x)b/(a+b) & (-a \le x \le 0) \\ (y+a-x)b/(a+b) & (0 \le x \le a) \\ z' = z \end{cases}$$
 (1)



Fig. 1. The original space (a), the transformed space for designing the retrodirective reflector (b), and the multilayer realization of the dielectric cover (c). Comparisons of near-field electric field distributions between the retrodirective reflector (d, e) and corner reflector (f, g) for an incident azimuth angle of  $90^{\circ}$  (d, f) and  $135^{\circ}$  (e, g).

Through such a transformation, the quadrangular free-space area in virtual space (x, y, z) (Fig. 1(a)) is transformed or compressed into a triangular area in the physical space (x', y', z') (Fig. 1(b)), so that the two orthogonal PEC plane AB and AC are converted to the planar PEC surface A'C'. Base on the procedure of TE [5], we could easily calculate the relative permittivity and permeability tensors of the triangular dielectric cover as

$$\hat{\varepsilon}' = \hat{\mu}' = \begin{pmatrix} (a+b)/b & -\operatorname{sgn}(x) & 0\\ -\operatorname{sgn}(x) & 2b/(a+b) & 0\\ 0 & 0 & (a+b)/b \end{pmatrix}.$$
 (2)

As indicated in (2), the advantage of choosing such a linear coordinate transformation is that the resulted dielectric cover becomes homogeneous birefringent material. The dielectric cover on the planar PEC surface could become thinner by using smaller parameter b, resulting in a low profile retrodirective reflector.

To verify the performance of the proposed retrodirective reflector, we have carried out full-wave EM simulation using finite-element method based software. Assume the size of the reflector as  $a = 5b = 15\lambda$ , where  $\lambda$  refers to the free-space wavelength of the incident wave. The near field electric field distributions are compared in Fig. 1 between the retrodirective reflector we designed and the conventional corner-reflector. As clearly indicated in Fig. 1(d) – (g), the impinging EM waves could be reflected back toward the source direction under different incident angles, representing a retrodirective response. The proposed retrodirective reflector has exactly the same scattering properties compared with the conventional corner-reflector, but with a much reduced profile.

For practical realization, we first try to reduce the material parameters for the dielectric cover in the proposed retrodirective reflector to a non-magnetic form while reserve the same EM ray trace similar to the procedure used in [3]. At least for the transverse magnetic (TM) incident waves (with magnetic field along the z coordinate), we could simplify the material parameters by keeping the product of  $\varepsilon_x \mu_z$  or  $\varepsilon_y \mu_z$  unchanged, which become

$$\hat{\varepsilon}' = \begin{pmatrix} (a+b)^2 / b^2 & -sng(x)(a+b) / b & 0\\ -sng(x)(a+b) / b & 2 & 0\\ 0 & 0 & \varepsilon_z \end{pmatrix}, \ \hat{\mu}' = 1 \quad (3)$$

The resulted material can be easily realized through a certain birefringent dielectric with its optical axis rotated by a certain angle  $\theta$  with the x-axis. The principal values of the relative permittivity tensor  $\varepsilon_x^c$ ,  $\varepsilon_y^c$  and  $\varepsilon_z^c$  can be determined through diagonalization of the parameters in (3). As  $\varepsilon_z^c$  is arbitrary for the TM polarization, it can be assumed as  $\varepsilon_z^c = \varepsilon_x^c$ . Therefore, the resulted material can be realized with a birefringent dielectric crystal described by  $\varepsilon_x^c$  and  $\varepsilon_y^c$ . A flexible way to realize the birefringent dielectric has been discussed and successfully utilized to build invisibility cloak in our previous works [15]. The method is based on the effective medium theory (EMT) [16], through which the birefringent dielectric can be represented by multilayer structure of alternating isotropic dielectric 1 and dielectric 2 (as indicated schematically in Fig. 1(c)). The layers are parallel to the *z* axis, and the effective permittivity of the two layers are determined by

$$\varepsilon_{z}^{c} = \varepsilon_{x}^{c} = \frac{\varepsilon_{1} + \eta \varepsilon_{2}}{1 + \eta}, \quad \varepsilon_{y}^{c} = \frac{(1 + \eta)\varepsilon_{1}\varepsilon_{2}}{\eta\varepsilon_{1} + \varepsilon_{2}}, \quad (4)$$

where  $\varepsilon_1$  and  $\varepsilon_2$  represent the relative permittivity of the dielectric 1 and dielectric 2, respectively, and  $\eta = d_1/d_2$  is the thickness ratio of the two layers.

Fig. 2 shows the performance of a proof-of-concept example of the retrodirective reflector working at microwave regime. Assuming the size of the reflector to be a = 1.5b =12.5 cm, we can finally obtain the resulted multilayer dielectric cover structure with  $\eta = 1$ ,  $\varepsilon_1 = 1$  and  $\varepsilon_2 = 33$ , and aligned at  $\theta = 31.7^{\circ}$  or  $\theta = 148.3^{\circ}$  in region I or region II, respectively. These parameters have been obtained for a surrounding medium with permittivity of 2.5, which has been removed in the simulations. It should also be mentioned that the layer thickness must be much less than the wavelength to ensure the validity of the EMT, so we have chosen a reasonable thickness of  $d_1 = d_2 = 0.21$  cm for each layer. The retrodirective reflecting is clearly demonstrate from the near field magnetic field distributions and the far field scattering patterns calculated at 8 GHz in Fig. 2, despite slight specular reflecting at the incident boundary and slight scattering at the other boundary of the cover, which are attributed to the impedance mismatch at the boundaries. It is also found that the retrodirective reflector could have a broadband performance due to the non-dispersive and low loss feature of the multilayer structure of normal dielectric.



Fig. 2. Near-field magnetic field distribution (a), and normalized far-field magnetic field scattering pattern (b) of a multilayer retrodirective reflector with alternating air layer and dielectric layer with relative permittivity of 33 for incident wave at azimuth angle of 123°.

#### III. GROUND PLANE INVISIBILITY CLOAK

To reduce the complexity of practical realization of invisibility cloak, in this section we will show the design, fabrication and performance test of a quasi three-dimensional (3D) ground plane cloak made of normal dielectric in the microwave regime. Here we employ a similar linear coordinate transformation used in the above section for the reflector through which a virtual space (x, y, z) with a triangular cross-section in the x-y plane is squeezed into a region with a quadrilateral cross-section (blue region) in the physical space (x', y', z'), leaving the lower triangle part as the cloaked region. The whole ground plane cloak is a 3D structure stretched infinitely and uniformly along the *z* axis. Supposing the outer surface of the ground plane cloak or the cloaked region has a slope of  $k_1$ , or  $k_2$ , respectively, the transformation can be defined as

$$x' = x$$
  

$$y' = (k_1 - k_2) y / k_1 + k_2 (a + x).$$
 (5)  

$$z' = z$$



Fig. 3. (a) Scheme of the coordinate transformation and the resulted ground plane cloak. (b) Realization of the quasi 3D cloak with multilayer of alternating dielectric 1 and 2. The cloak is covered on a bumped perfect conducting surface enabling a cloaked region underneath.

We could then obtain the material permittivity and permeability tensors for the left block of the ground plane cloak with the standard TE procedures as

$$\hat{\varepsilon}' = \varepsilon_r \mathbf{M}, \quad \hat{\mu}' = \mu_r \mathbf{M}, \tag{6}$$

where

$$\mathbf{M} = \begin{pmatrix} k_1/(k_1 - k_2) & k_1k_2/(k_1 - k_2) & 0\\ k_1k_2/(k_1 - k_2) & k_2^2 k_1/(k_1 - k_2) + (k_1 - k_2)/k_1 & 0\\ 0 & 0 & k_1/(k_1 - k_2) \end{pmatrix},$$
(7)

and  $\varepsilon_r$  or  $\mu_r$  represents the relative permittivity or permeability of the background medium in the virtual space. As a direct consequence of the simple linear coordinate transformation, the resulted material is a spatially uniform medium. For practical consideration, similar to the way we do in the above section, the medium of the cloak cover can be reduced to nonmagnetic for TM wave incidence, and thus can be easily realized through a certain birefringent dielectric with its optical axis rotated by a certain angle  $\theta$  with the *z*-axis. Based on the EMT, the required birefringent dielectric can be further realized through multilayer of alternating dielectric 1 and dielectric 2 (as indicated schematically in Fig. 3(b)).

As a proof-of-concept example of our proposal, we have designed a quasi 3D ground plan cloak at microwave regime. Assuming  $k_1 = 1$ ,  $k_2 = 0.17$ , a = 100 mm, and the cloak is immersed in a background medium of  $\varepsilon_r = 1.75$ . The relative permittivity of the dielectric 1 and 2 composing the carpet cloak is then determined as 4.4 and 1.0, respectively, and the rotation angle  $\theta$  is about 25.8° with respect to *x*-axis. Considering the practical realization, we replace the background medium with free space, which in principle will cause slight reflections at the boundary between the cloak and the background medium due to wave impedance mismatch, but the wave trajectory inside the cloak and thus the cloaking effect will keep unchanged.

To implement the designed cloak, we fabricate the cloak composed of multilayer of FR4 slab and air spacing as depicted in Fig. 4(a). The cloak is aimed to work in the X and Ku band, covering frequency from 8 GHz to 18 GHz. The center frequency corresponds to a free space wavelength of about 23 mm. The whole structure is about 300 mm long (about 13 times of the wavelength) in the z direction, 200 mm wide in the x direction and 100 mm high, creating a cloak region about 200 mm wide and 17 mm high in the centre. To satisfy the TMT, either the FR4 slab or air spacing has a thickness of 1.5 mm, about 1/15 of the centre free space wavelength. The ground plane cloak covers a reflective bump (copper board) on an aluminium perfect conducting board with the size of 300 mm  $\times$  300 mm.

We first checked the functionality of the cloak by full wave simulation using the finite-element method based software. As indicated in Fig. 4(b) and (c), a Gauss beam with TM polarization impinging onto a reflective bump structure will produces considerable diffuse reflection (Fig. 4(b)), while covering the bump with multilayer cloak structure, the bump becomes invisible, with a near field distribution of specular reflection (Fig. 4(c)) mimicking that of a flat reflective sheet.

The performance of the fabricated multilayer cloak is tested in a microwave anechoic chamber. Two standard rectangular horn antennas are used as the transmitter and receiver and linked to a vector network analyzer to measure the scattering field. Both horns can be moved along an arc rail to change their azimuth angles. To obtain a quasi-plane wave of TM polarization, we mainly take advantage of the far field phenomenon of the horns. We fix the incident wave at a certain azimuth angle, and record the scattering field as a function of the azimuth angle from  $10^{\circ}$  to  $80^{\circ}$  throughout the whole X and Ku band. The scattering field amplitude has been normalized to that of the specular reflection by a flat conducting plate of 300 mm × 300 mm.



Fig. 4. (a) Photograph of the fabricated multilayer cloak. Simulated magnetic field distributions of incident EM beam interacting with (b) bare conducting bump, and (c) bump covered by multilayer cloak. (d) The measured far field scattering pattern for the incidence along azimuth angle of  $120^{\circ}$  at 10 GHz. The measurement is carried out from  $10^{\circ}$  to  $80^{\circ}$  with a step of  $5^{\circ}$ .

In Fig. 4(d) we show one result for the case of incidence along the azimuth angle of  $120^{\circ}$ . Both the simulation and the measurement for the far-field scattering indicate that the majority scattering field is along the specular direction at the azimuth angle of  $60^{\circ}$ , verifying the functionality of the ground plane cloak. We notice that a small side lobe appears along the azimuth angle of  $150^{\circ}$ , which is caused by the reflection at the left surface of the cloak due to wave impedance mismatch. More experimental results demonstrate a very broad band performance throughout the whole X and Ku band, which is due to the non-dispersive and low loss feature of the multilayer structure.

## IV. CONCLUSION

We present that through specially chosen linear coordinate transformation, the devices for antenna or cloak applications designed from transformation electromagnetics could require only homogeneous birefringent dielectric media, and can be further constructed according to the effective medium theory with multilayer structures of normal dielectrics, largely reducing the complexity of practical realization.

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