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Time Delay Unit Quantization at the Subarray Level for Large, Wideband Linear Arrays

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***Abstract*—Time delay units become larger and more expensive as the number of bits and physical size of the bits increases. The goal is to reduce the number of large bits in the array by placing them back in the feed network where they are shared my many elements. As the bits are moved back in the feed network, time delay quantization increases and produces errors in the array factor. This paper presents trade-offs between reducing the number of time delay bits and maintaining a desirable array pattern.**

I. INTRODUCTION

Large, wideband arrays need time delay units (TDUs) to receive/transmit signals with minimal distortion. The maximum time delay needed is a function of the size of the array, the array bandwidth, and the maximum scan angle. Some important issues regarding time delay units in an array are given in [1][2][3]

This paper explains how the distribution of the time delay bits in the corporate feed affects the array pattern in terms of beam pointing and quantization lobes. Section II presents the array factor model for the linear array of subarrays and shows the possible positions for placing time delay in the subarray architecture. Section III demonstrates how to calculate the time delay at the subarray level given the time delay at the element. Examples are provided to show the trade-offs associated with the placement of the time delay.

II. LINEAR ARRAY FORMULATION WITH SUBARRAYS

Fig. 1 is a diagram of a linear array with a corporate feed. A phase shifter or time delay unit is at each element. Time delay may also be placed after any power combiner/splitter as shown by the square blocks. The time delay units have discrete line lengths assigned to data lines or bits. In order to save costs, time delay should be placed at the highest tolerable level in the feed network.

The array factor for a uniform linear array of N elements along the x-axis is given by

$$AF(\theta) = \sum_{m=1}^{M} \sum_{n=1}^{N} e^{-j\omega \mathrm{T}_{mn}} e^{j\delta_n} e^{jkx_n \sin\theta} \qquad (1)$$

where

ω = radial frequency
k = wave number
x_n = location of element n
θ = angle from boresight

As shown in Fig. 1, there are $M-1$ combining (subarray) levels that can have time delay units. In order to steer the beam to θ_s, the time delay is in the T_{mn} matrix, while the phase shift at the elements in δ_n.

$$\mathrm{T}_{mn} = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} & \tau_{14} & \tau_{15} & \tau_{16} & \cdots & \tau_{1N} \\ \tau_{21} & \tau_{21} & \tau_{22} & \tau_{22} & \tau_{23} & \tau_{23} & \cdots & \tau_{2N/2} \\ \vdots & & & & & & \ddots & \vdots \\ \tau_{M1} & \tau_{M1} & \tau_{M1} & \tau_{M1} & \tau_{M1} & \tau_{M1} & \cdots & \tau_{M2} \end{bmatrix} \qquad (2)$$

$$\delta_n = -kx_n \sin\theta_s \qquad (3)$$

The maximum time delay needed is given by

$$\tau_{\max} = Nd \sin\theta_s / c \qquad (4)$$

where c is the speed of light. The most significant bit (MSB) must be at least half the maximum time delay.

$$\tau_{MSB} \geq \tau_{\max} / 2 \qquad (5)$$

while the least significant bit (LSB) is given by

$$\tau_{LSB} = \tau_{MSB} / 2^{N_{bits}-1} \qquad (6)$$

where N_{bits} is the number of time delay bits.

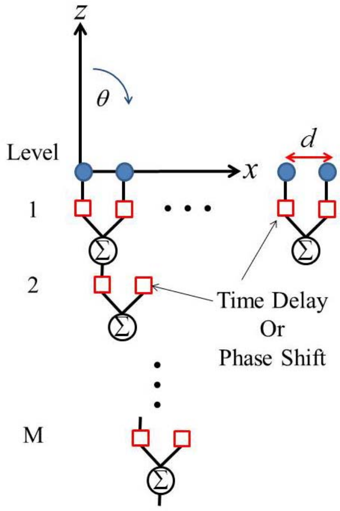

Fig. 1. Linear array with corporate feed network. Possible locations for time delay and/or phase shift are indicated by the small squares.

III. TIME DELAY UNITS IN THE CORPORATE FEED

As an example, consider an N = 64 element array with a maximum scan angle of 60º. The array operates from 8 to 10 GHz and has an element spacing of 0.52λ. The maximum time delay across the aperture is 2.0675 ns. Assume the time delay units have 9 bits with a most significant bit (MSB) of 1.0337 ns and a least significant bit (LSB) of 0.0040 ns. If all 9 bits are placed at the element, then Fig. 2 shows the array factors at 8, 9, and 10 GHz. There is no beam pointing error or undesirable sidelobes.

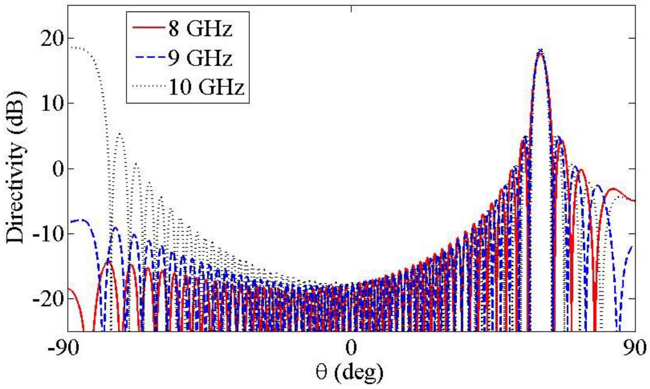

Fig. 2. Array factor steered to 60º with all the time delay unit bits at the elements.

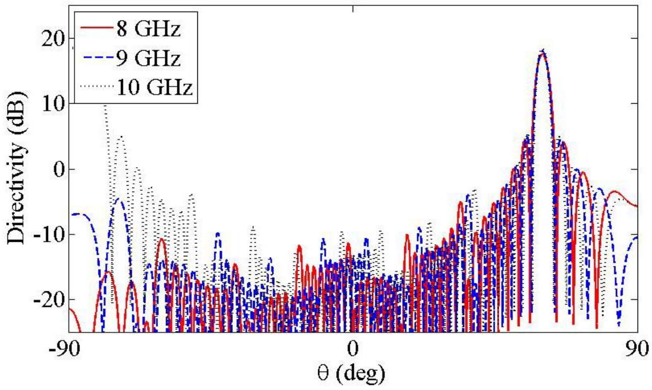

Fig. 3. Array factor steered to 60º with 4 time delay unit bits at the elements and one time delay bit at each of the remaining 5 levels. The MSB is at level 6.

Time delay units are expensive, and the large bits will not fit behind the elements, so they must be placed farther back in the feed network. Locating the large bit of the time delay units has a tremendous influence on the sidelobe level and main beam. Fig. 3 shows the array factor when the 4 LSBs are placed at the element, and 1 bit placed at each level with the MSB at level 6. Fig. 4 shows the array factor when the 4 LSBs are placed at the element, and all the remaining bits are placed at level 2. Note that Fig. 3 and Fig. 4 are identical. If the 5 MSBs are moved back one level to Level 3, then the resulting array factors appear in Fig. 5. In this case, the quantization of the time delay results in large quantization lobes that have a gain that is larger or on the order of the main beam. Suppressing large, undesirable quantization lobes requires larger bits to be placed closer to the elements.

Fig. 6 shows plots of the total time delay at each element for Fig. 2 (solid blue line), Fig. 4 (dotted black line), and Fig. 5 (dashed red line). The cases for Fig. 2 and Fig. 4 are very close, so there is very little perturbation to the sidelobes. On the other hand, the solid blue line shows distinct quantized levels that produce the quantization lobes in Fig. 5. Plateaus in the total time delay must be avoided in any practical array system.

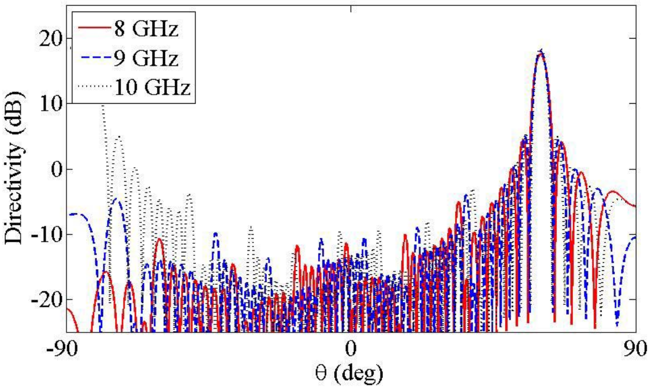

Fig. 4. Array factor steered to 60º with 4 time delay unit bits at the elements and 5 bits at Level 2 (identical to Fig. 3).

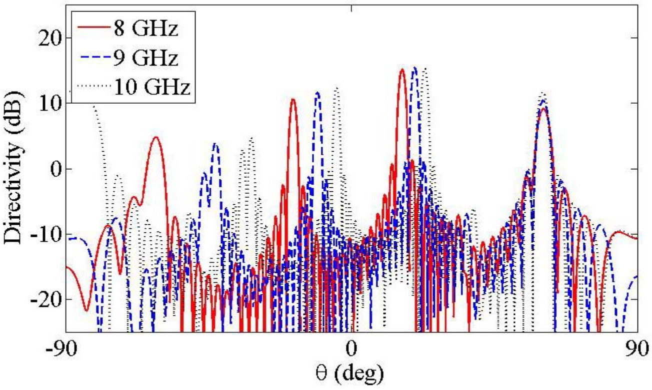

Fig. 5. Array factor steered to 60º with 4 time delay unit bits at the elements and 5 bits at Level 3.

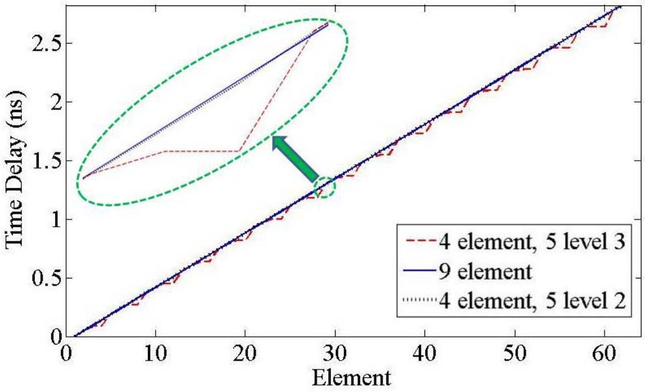

Fig. 6. The total time delay experienced by a signal at each element. Solid blue line has all bits at the elements. Dotted black line has 4 bits at the element and 5 bits at level 2. Dashed red line has 4 bits at the element and 5 bits at level 3. The upper left corner shows a blow up of the plot near elements 28 and 29.

IV. CONCLUSIONS

Time delay units are needed to steer the beam of a large, wideband array. Some of the time delay bits must be placed back in the corporate feed network to minimize the cost of the time delay units and the fact that the large bits will not fit in an area near the element. If all the time delay bits are placed at the element, then the beam pointing and sidelobe levels are minimally perturbed. Large bits need to be carefully place in the subarray structure in order to avoid large quantization lobes that are of the magnitude of the main beam. At least one bit must appear at each level between the element and the highest level, or quantization lobes appear.

REFERENCES

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