Design of External Circuits for Smart Inductive Coupling between Non-Self-Resonant Small Antennas in Wireless Power Transfer Systems

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Abstract—**Wireless power transfer systems of resonance coupling or inductive coupling utilize electrically very small antennas, which are either self-resonant or non-self-resonant. The system with non-self-resonant antennas requires the external circuit for impedance matching and high power transfer efficiency. This paper presents the design formulas for four kinds of external circuits, and discusses their performances. The power transfer efficiency is generally expressed in terms of figure of merit, the product of the inductive coupling coefficient and the system quality factor. The circuit derived from the emulation of a correct equivalent circuit of the self-resonant antenna systems can help the system of inductive coupling to act similarly as the self-resonant systems of resonance coupling.**

I. Introduction

An electrically small antenna used in a wireless power transfer system is not designed to radiate or receive electromagnetic waves against the definition of "antenna" in IEEE standard dictionary[1]. However, the power transfer is realized by the reactive field accompanying the negligibly small radiating field, and not by evanescent field[2]. In this sense, the term "antenna" is used for a loop throughout this paper. The electrically very small antennas used in near field region (NFR) is fundamentally different from those used in far field region (FFR). FFR and NFR are compared and summarized in Table.I. The principal target in the design of the FFR applied electrically small antennas is to lower the Q factor whose limit has been given by L.J. Chu[3]. On the contrary, the NFR applied antennas are desired to be non-radiating and lossless with as high Q value as possible. The FFR applied antennas in Tx and Rx are designed independently from each other, while the NFR applied Tx and Rx antennas are designed together as the coupled antenna systems. The Copernican-like revolution is necessary in the designer's mind.

Small antennas are classified into closed path type (CPT) and open path type (OPT)[4]. The path means the D.C. current path, and the path is formed in closed path type antennas such as loops, and the path is not formed in open path type antennas such as dipoles. A CPT antenna is just an inductor, and an OPT antenna is just a capacitor, at lower frequency. They satisfy the Foster's reactance theorem in good approximation, and their equivalent circuits are derived from the first resonance frequency and the first anti-resonance frequency, together with the low frequency inductance or capacitance. The equivalent circuit for a WPT system with transmitting and receiving small antennas are characterized by four frequencies, at the resonance and the anti-resonance for even and the odd modes, together with the lower frequency inductance or capacitance.

TABLE I
Small Antennas Design in FFR and NFR.

	Q	impedance to be matched	Tx & Rx
FFR	as low as possible	radiation impedance	independent
NFR	as high as possible	image impedance	dependent

A. CPT Self-Resonant Systems and the Equivalent Circuit

Self-resonant small antennas are generally characterized by closely coexisting resonance and anti-resonance frequencies[4]. Foster's reactance theorem[5] together with this property lead directly to their equivalent circuits[6]. The equivalent circuit for the CPT systems are as shown in Fig.1, where both the inductive coupling part and the capacitive coupling part exist, and they are in parallel in the circuit. The capacitive coupling part is dual to the inductive coupling part, and its existence is due to the anti-resonance frequency closely existing to the resonance frequency.

The most commonly adopted equivalent circuit includes only the inductive coupling part and capacitors in series as the external circuit elements. The capacitors cancel the input reactance of the coils and realize the resonance. It is clear that such a simple equivalent circuit cannot predict the real nature of the resonance coupling.

B. Non-Self-Resonant Systems with Series Capacitor Circuit

The most common external circuit configuration in CPT non-self-resonant antenna systems is shown in Fig.2, where capacitors are added in series between the non-self-resonant

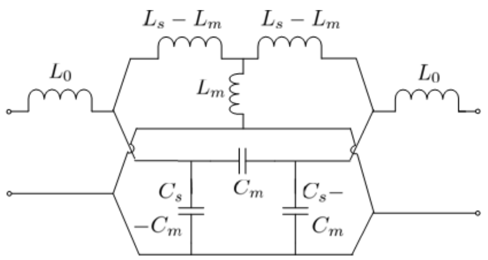

Fig. 1. Equivalent circuit for CPT systems.

antenna (loop) and the Tx or Rx, to which we will refer as sC hereafter. The loss resistances are not included here to simplify the analysis in the next section. The lossless reactive networks can be dealt with in the light of reactance theorem [5].

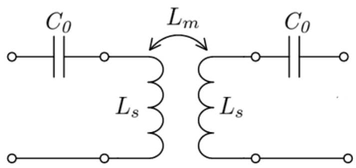

Fig. 2. Series capacitor circuit (sC).

II. External Circuit Design for Non-Self-Resonant Antenna Systems Facilitated by Image Impedance

Various external circuit configurations are possible. If they are lossless, the designs are facilitated by the image impedance concept as shown in this section.

A. Series Capacitor Circuit

The design procedure is illustrated here for the sC circuit configuration. For the circuit of Fig.2, the image impedance is give in terms of angular frequency ω as

$$Z_I(\omega) = \frac{1}{\omega C_0}\sqrt{\left(\frac{\omega^2}{{\omega_{se}}^2} - 1\right)\left(1 - \frac{\omega^2}{{\omega_{so}}^2}\right)}, \tag{1}$$

$$\omega_{se} = 1/\sqrt{C_0 L_s(1+k)}, \tag{2}$$

$$\omega_{so} = 1/\sqrt{C_0 L_s(1-k)}, \tag{3}$$

where $k = L_m/L_s$. Z_I is real for $\omega_{se} < \omega < \omega_{so}$, and pure imaginary elsewhere. Z_I vs. frequency pattern becomes, in this case, a hill pattern with the hilltop at $\omega = \sqrt{\omega_{se}\omega_{so}}$ as shown in Fig.3.

Let us impose two conditions:

$$\omega_0 = \sqrt{\omega_{se}\omega_{so}}, \tag{4}$$

$$Z_I(\omega_0) = R_0, \tag{5}$$

where $\omega_0 = 2\pi f_0$ is the operating angular frequency, and R_0 is the port impedance of Tx and Rx. Two conditions are enough to determine the external capacitance C_0, and the self inductance of the coils L_s as functions of the inductive coupling coefficient k. The results are summarized inside Table II together with the results for other circuit configurations.

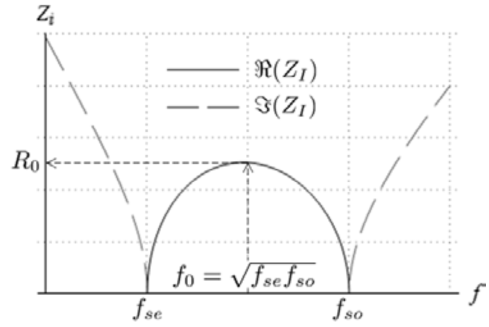

Fig. 3. Design diagram of sC circuit.

B. Parallel Capacitor Circuit

As another external circuit configuration, let us consider first a parallel capacitor circuit as shown in Fig.4. We will refer to this as pC hereafter. The role of the capacitor C_0 is,

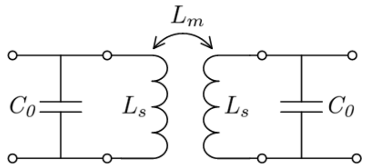

Fig. 4. Parallel Capacitor Circuit (pC).

similarly as in the case of sC circuit, to cancel out the reactance of L_s and to realize the resonance. The design facilitated by the image impedance in this case is illustrated by Fig.5. The results are also shown inside Table II. The difference is clear. The required self inductance L_s is almost proportional to k, while in sC it is almost inversely proportional to k. This result means for smaller k that smaller L_s is optimum in pC, while larger L_s is required in sC.

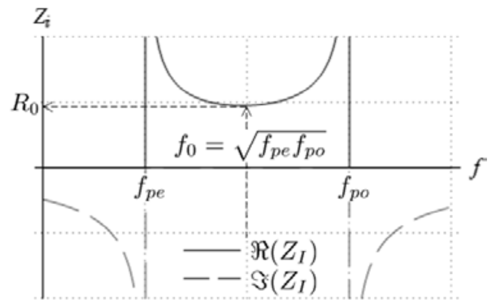

Fig. 5. Design diagram of pC circuit.

C. External Circuit Emulating Self-Resonant Antenna Systems : Series Inductor Parallel Capacitor Circuit

As the next external circuit, it would be natural to consider such a configuration as that emulating the self-resonant antenna system whose equivalent circuit is shown in Fig.1. The capacitive coupling part cannot exist in the smaller and non-self-resonant systems, but the self capacitances of C_0 are included to get the circuit of Fig.6. It should be emphasized that the capacitors are in parallel, and that the circuit includes the external inductance L_0 in series between the Tx or Rx and the coils. We will refer to this circuit as sLpC hereafter. They contribute to the existence of anti-resonance frequencies, f_{po}, f_{pe} together with the resonance frequencies, f_{se}, f_{so}. Generally in the closed path type systems, f_{pe} is the minimum, and f_{so} is the maximum. The image impedance pattern in this

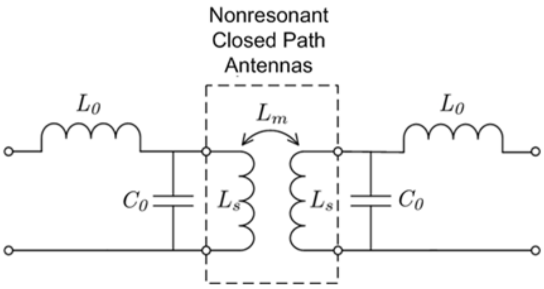

Fig. 6. Series Inductor Parallel Capacitor Circuit (sLpC).

case becomes left to right waterfalls pattern[6]. In designing sLpC, we require two conditions:

$$f_{po} = f_{se} = f_0, \tag{6}$$

$$Z_I(f_0) = R_0. \tag{7}$$

The above two conditions include three equations, and determine the circuit constants $L = 0$,C_0 and the self inductance L_s in terms of k. The results are included in Table II. Similarly as in pC configuration, L_s is inversely proportional to k.

D. Parallel Inductor Series Capacitor Circuit

The fourth circuit configuration is a parallel inductor series capacitor circuit as shown in Fig.7. We will refer to this configuration as pLsC hereafter.

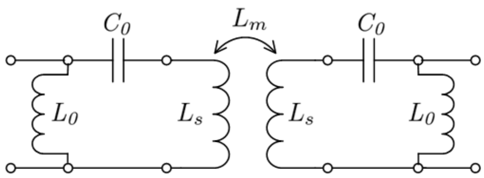

Fig. 7. Parallel Inductor Series Capacitor Circuit (pLsC).

The theoretical treatments are exactly the same as in sLpC. The designed results are included in Table II. Similarly as in sC configuration, L_s is proportional to k.

III. Circuit Constants Formulas and Discussions

The circuit constants obtained are summarized in Table II.

A. Inductive Coupling Coefficient Dependence

In every circuit configurations, the obtained L_0, C_0 and L_s are expressed in terms of the inductive coupling coefficient $k = L_m/L_s$. L_0 contributes to the impedance matching in pLsC and sLpC configurations, and C_0 contributes to the resonance in all cases. L_s is most dependent on k. For smaller

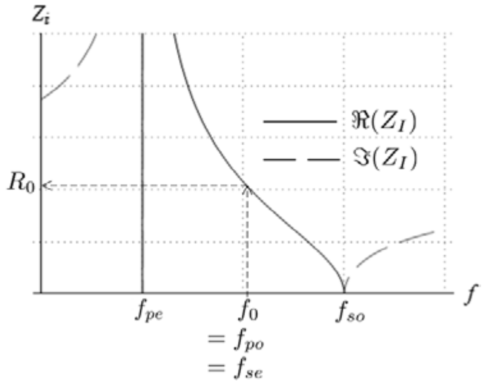

Fig. 8. Design diagram of sLpC and pLsC circuits.

k, L_s should be larger in sC and pLsC, and smaller in pC and sLpC. When the coil size is limited and the inductance is also limited, pC and sLpC configurations are applicable.

B. Power Transfer Efficiency and Figure of Merit

The power transfer efficiency η is defined as

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{ref}} + P_{\text{loss}}}, \tag{8}$$

where P_{in} is the input power from Tx, P_{out} is the output power to Rx, P_{ref} is the reflected power to Tx, and P_{loss} is the loss power absorbed in the wireless structure and the external circuit.

η is equal to $|S_{21}|^2$, where S_{21} is a scattering matrix parameter, and is conveniently calculated from the F matrix (Fundamental matrix) elements as follows.

$$S_{21} = \frac{2}{|F_{11} + F_{12}/R_0 + F_{21}R_0 + F_{22}|}, \tag{9}$$

where R_0 is the port impedance of Tx and Rx.

In sC configuration, it is well-known that η is expressed concisely in terms of figure of merit: $fom = kQ$[7]. We have found that this is the case in all the circuit configurations, and that η is expressed as (10), exactly for sC and pLsC configurations and in good approximation for pC and sLpC configurations. Evaluating P_{loss} by the system quality factor $Q = \omega_0 L_s/R_\ell$, we have derived the expression of $R_\ell/R_0 \cdot fom$ in terms of k as shown in Table II, where R_ℓ is the loss resistance.

$$\eta = |S_{21}|^2 = \frac{1}{|1 + \frac{1}{fom} + \frac{1}{2fom^2}|^2} \tag{10}$$

In cases R_ℓ can be very small, pC or sLpC configurations are applicable and then the coils can be of small size, because of the reasonable fom even for small k.

The sLpC configuration has been derived from the equivalent circuit of self-resonant systems. The preferable property of resonance coupling of CPT self-resonant systems can be deduced from the sLpC type equivalent circuit, but never from the sC type equivalent circuit.

TABLE II
DESIGNED CIRCUIT CONSTANTS, AND FIGURE OF MERIT $\Big(k = \frac{L_m}{L_s}$,
$f(k) = \sqrt{2(\sqrt{1-k^2} - (1-k^2))}/k = 1 - \frac{1}{8}k^2 + \mathrm{O}(k^4)$,
$g(k) = 2/((1-k^2)^(3/4)(\sqrt{1+k} + \sqrt{1-k})) = 1 + \frac{7}{8}k^2 + \mathrm{O}(k^4)\Big)$.

Configuration	$\omega_0 L_0$	$\omega_0{}^2 C_0 L_s$	$\omega_0 L_s$	$\frac{R_\ell}{R_0} fom$
sC	n/a	1	$R_0 \frac{1}{k} \frac{1}{f(k)}$	$\frac{1}{f(k)}$
pC	n/a	$\frac{1}{\sqrt{1-k^2}}$	$R_0 k g(k)$	$k^2 g(k)$
pLsC	R_0	$\frac{1}{1+k}$	$R_0 \frac{1}{2k}$	$\frac{1}{2}$
sLpC	R_0	$\frac{1}{1-k}$	$R_0 \frac{2k}{1-k^2}$	$\frac{2k^2}{1-k^2}$

IV. HYBRID SC-PC CONFIGURATION

It has been assumed so far that the system is symmetric. However, the derived external circuit formulas are valid for the asymmetric systems such as the system with sC for Tx and pC for Rx as shown in Fig. 9. In the hybrid sC-pC configuration of Fig. 9, R_1 and R_2 are loss resistance in Tx and Rx, respectively. The quality factors are defined in each section as $Q_1 = \omega_0 L_{s1}/R_1,\ Q_2 = \omega_0 L_{s2}/R_2$ and the figure of merit is defined as $k\sqrt{Q_1 Q_2}$.

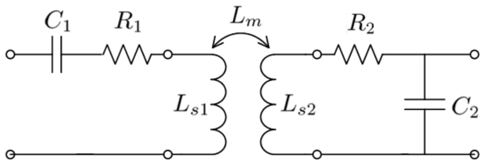

Fig. 9. Series capacitor-parallel capacitor hybrid configuration.

The power transfer efficiency depends mainly on the figure of merit, but also on the ratio of Q_1 to Q_2. Fig. 10 shows η versus fom with $q = (\sqrt{Q_1/Q_2} + \sqrt{Q_2/Q_1})/2$ as a parameter. The parameter q increases monotonically and slowly from 1 to 5 as Q_1/Q_2 increases from 1 to 100, as shown in Fig. 11.

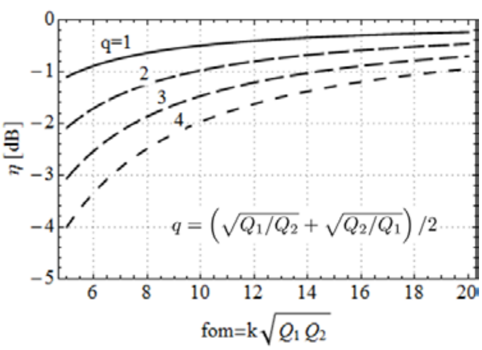

Fig. 10. η vs. fom for the pCsC system with a parameter q.

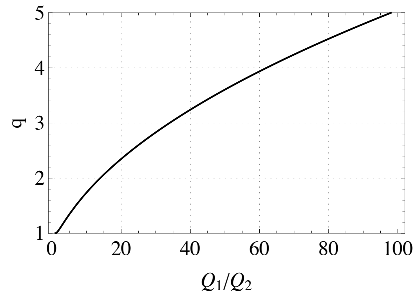

Fig. 11. q vs.Q_1/Q_2.

V. CONCLUSION

Wireless power transfer systems of inductive coupling with non-self-resonant small antennas are investigated in the paradigm of “wireless reactive networks".

Four kinds of external circuit for the impedance matching and for the resonance, sC, pC, pLsC and sLpC, are designed in the light of image impedance, and are discussed in comparative way. The constants are concisely expressed in terms of the inductive coupling coefficient. The power transfer efficiency is expressed, in all cases, by the common equation in terms of the figure of merit. The hybrid sC-pC configuration is also discussed.

The sLpC circuit configuration emulates CPT self-resonant systems which are characterized by four frequencies, even mode and odd mode, resonance and anti-resonance, frequencies. The coexistence of anti-resonance frequency with resonance frequency in both modes is essential in the systems of resonance coupling. The true nature of resonance coupling can only be deduced from the correct understanding of the self-resonant systems. The external circuits connected to the systems of non-self-resonant small antennas can help the systems to act similarly as the self-resonant systems of resonance coupling, if smartly designed.

The choice of the external circuit depends on the operating frequency, the transmission distance, the size of coils, the transmission power, etc. The present study would be helpful for more elaborate studies.

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