Reflection Properties from Dielectric Grating Structures Containing Photonic Crystal Grating

Mitsuhiro Yokota #, Kitaro Yoshizuru *

Department of Electrical and Systems Engineering, University of Miyazaki
1-1, Gakuen Kibanadai Nishi, Miyazaki 889-2192, Japan
m.yokota@m.ieice.org

* Department of Electrical and Electronic Engneering, University of Miyazaki
1-1, Gakuen Kibanadai Nishi, Miyazaki 889-2192, Japan

***Abstract*—The scattered electromagnetic wave from two dimensional dielectric grating structure containing photonic crystal grating (PCG) is analyzed by using the Method of Moments (MoM). PCG is assumed to consist of the dielectric cylinders. Numerical results show the effect of depth of grating, relative permittivity, and dielectric cylinders on reflectance for three kinds of the structure. We think that PCG analyzed in this paper is useful for the design of frequency filter.**

I. Introduction

Photonic Crystal (PC) has the periodic structure with the order of the light wave and the photonic bandgap or electromagnetic bandgap, where the propagation of the electromagnetic wave is forbidden. This PC can be applied to various devices such as the frequency filter, power divider, and couplers [1], [2]. Recently, the reflection and diffraction properties of the PC Grating [3], [4] have been published.

The scattering problem from the periodic structure is one of the important and basic issues of the electromagnetic theory, and has been investigated from the theoretical and numerical viewpoints. When the shape of the scatterer is the circular cylinder or sphere, the eigen-function expansion method can be used in order to examine the scattering properties. On the other hand, the numerical techniques such as the finite element method [5], the differential method [6], the time domain methods [7], [8], etc are used for the arbitrary shape and the complex structures. The Moment of Method (MoM) [9] has been applied to various problems in the electromagnetic and optics fields and is one of the useful numerical methods to obtain the good accuracy.

In this paper, the scattering from the dielectric periodic structure containing the PCG by using the MoM [10], [11] is examined numerically. The lattice sum technique [12] is used in order to speed up the calculation and obtain the solution precisely. The structures of the grating in this paper are the dielectric grating, PCG, and the dielectric grating containing PCG. The reflectance of the effect of the grating depth, relative permittivity, and dielectric cylinders for three kinds of the structures are examined.

II. Formulation

Consider the scattering from a periodic dielectric grating structure along the x-direction with periodicity Λ. It is assumed that the incident plane wave E_i is polarized along the z-axis, which corresponds to an E-polarized wave. In this case, the scattered field at any point of the structure is obtained using the result of scattering by one cylinder [10], and Floquet's theorem. To obtain the form of the matrix equations by MoM, the pulse function as the basis function is used. The reference cylinder is divided in N number of cells and the total electric field and the relative permittivity is assumed to be constant on each cell. The form of the matrix equation is as follows:

$$\sum_{n=1}^{N} C_{mn} E_n = E_m^i \qquad m = 1, \cdots, N \tag{1}$$

The expression of the coefficient C_{mn} is given as follows

$$\begin{aligned} C_{mn} = {} & j\frac{\pi}{2}[\varepsilon_r(n) - 1] k a_n J_1(k a_n) \\ & \sum_{l=-\infty}^{\infty} \exp(-j k_x l d) H_0^{(2)}(k \rho_{mn}^l) \quad (m \neq n) \end{aligned} \tag{2}$$

and

$$\begin{aligned} C_{nn} = {} & 1 + j\frac{\pi}{2}[\varepsilon_r(n) - 1] \left[\left\{ k a_n H_1^{(2)}(k a_n) - \frac{2j}{\pi} \right\} \right. \\ & \left. + k a_n J_1(k a_n) \sum_{l=-\infty,\, l\neq 0}^{\infty} \exp(-j k_x l d) H_0^{(2)}(k \rho_{nn}^l) \right] \quad (m = n) \end{aligned} \tag{3}$$

where $a_n (= \sqrt{\Delta x \Delta y / \pi})$ is the radius of the equivalent circular cell with the same cross-sectional area. The distance ρ_{mn}^l is defined as $\sqrt{(x_m - ld)^2 + y_m^2)}$ and x_m, y_m are the centers of the mth cell with the location of the reference cylinder. The evaluation of the infinite sum of Hankel functions multiplied by trigonometric angular dependencies in Eqs. (2) and (3) is the most time consuming part in the scattering problem of periodic structures. In order to obtain the summation efficiently with less computational time, the lattice sums technique [12] is applied.

The reflection coefficient R_l of lth order of the space

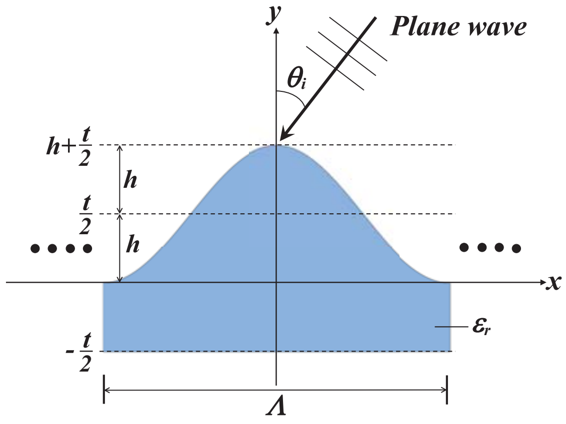

Fig. 1. Geometry of the reflectance from dielectric grating. Model 1

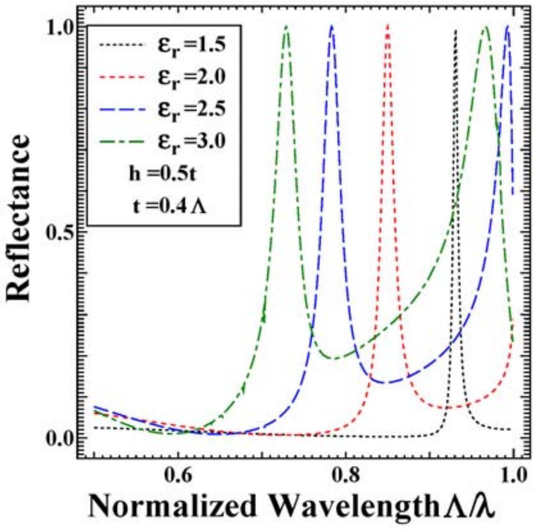

Fig. 2. Reflectance for various relative permittivity. t=0.4Λ, h=0.5t.

harmonics is express as follows:

$$R_l = \frac{-j}{2} \sum_{n=1}^{N} E_n \left[\varepsilon_r(n) - 1\right] ka_n J_1(ka_n) \times \frac{1}{\kappa(k_{x,l})} \exp\left[+jk_{x,l}x_n^0 + j\kappa(k_{x,l})y_n^0\right] \tag{4}$$

where

$$k_{x,l} = k_x + l\frac{2\pi}{\Lambda} \tag{5}$$

$$\kappa(k_{x,l}) = \sqrt{k^2 - k_{x,l}^2}\,, \qquad \mathrm{Im}(\kappa(k_{x,l})) \le 0 \tag{6}$$

III. Numerical Results

In this section, the reflection properties of three kinds of the structure are examined by using the previous procedure. The wavelength of the incident wave is assumed to be 1.55 μm, and the angle of the incidence is $\theta_i = 0$, which corresponds to the normal incidence. The cell size for the MoM calculation is $\Delta x = \Delta y = \lambda/200$.

A. Case of Dielectric Grating

Figure 1 shows the dielectric grating with the periodicity Λ and the depth h, which is expressed by follows:

$$y = t/2 + h\cos Kx \tag{7}$$

where t is the width of the slab, and $K = 2\pi/\Lambda$. The relative permittivity of the background media is free space and ε_r is the relative permittivity of the grating.

Figure 2 shows the reflectance for various relative permittivity ε_r as function of the normalized frequency Λ/λ. The width of the slab and the height is set as t=0.4Λ, h=0.5t, respectively. From this figure, it is found that the peak frequency, which gives the maximum of the reflectance, becomes higher for the smaller relative permittivity, and the full width at half maximum (FWHM) becomes small. Also, there exist two peaks for $\varepsilon_r > 2.5$. In this case, the higher order guided modes can propagate to the x direction in the slab structure, and the fundamental mode and the next mode are coupled with the incident wave.

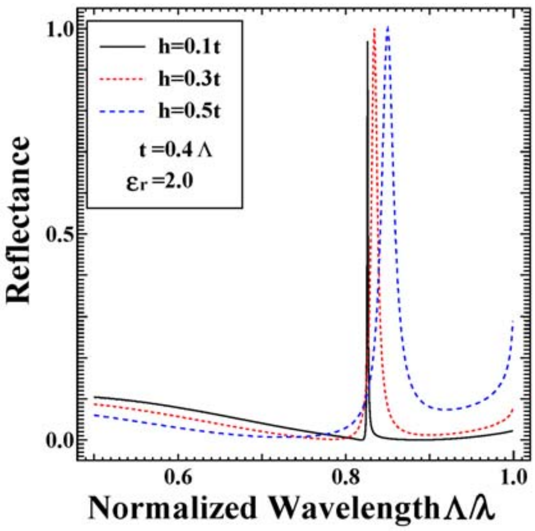

Fig. 3. Reflectance for various depth h of the grating. t=0.4ΛCε_r=2.0

Figure 3 shows the reflectance for various grating depth h as function of the normalized frequency. The width of the slab and the relative permittivity is set as t=0.4Λ, ε_r=2.0, respectively. It can be seen that the peak frequency becomes higher for the larger height, and the reflectance profile becomes sharp. For h=0.1t, it is useful to take out the specific frequency.

The reflectance for various width t of the slab is shown in Fig. 4 as function of the normalized frequency. The height and the relative permittivity of the grating is h=0.5t and ε_r=2.0, respectively. For larger width of the slab, the reflectance profile becomes sharp and there exists the second peak. This is due to the same reason for the case of Fig. 2.

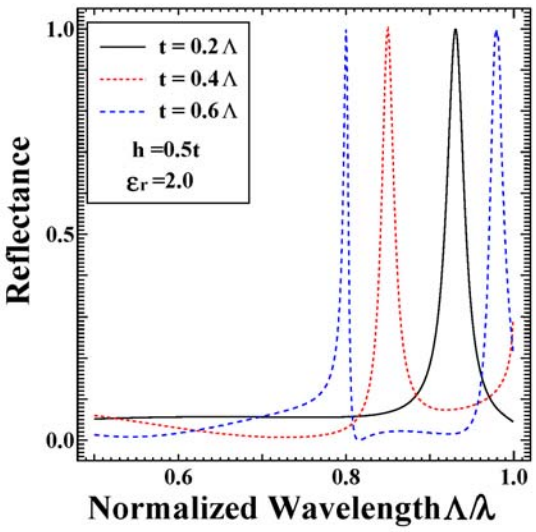

Fig. 4. Reflectance for various width t of the slab.

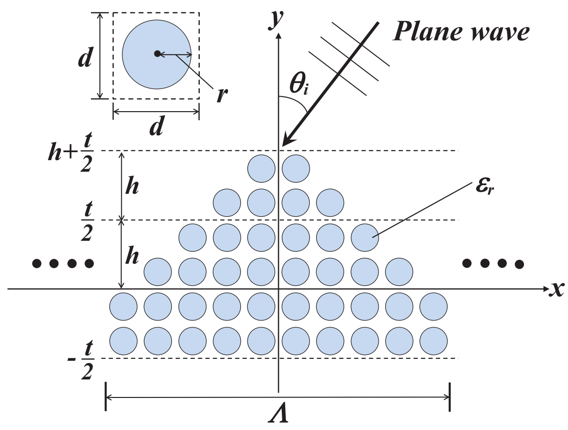

Fig. 5. Geometry of the reflectance from PCG: Model 2

B. Case of PCG

In this subsection, the reflection from PCG as shown in 5 is examined. d is the distance between the cylinders, r is the radius and t is the width of the grating.

Figure 6 shows the reflectance for various grating depth as function of the normalized frequency. The radius of the cylinder, height of the grating, relative permittivity are set as r=0.4d, t=4.0d, and ε_r=2.0. As the grating depth increases, the peak frequency moves to the higher frequency and the FWHM becomes wider. This tendency is almost the same as that of Model 1. However, the side lobe for Model 2 is almost zero in comparison with that of Model 1, then Model 2 is more suitable for the frequency filter.

Next, for the case of t=4.0d, h=2.0d, and ε_r=2.0, the reflectance for various radius of the cylinder is shown in Fig. 7. The reflectance of $r = 0.1$ is almost zero for the range considered in this paper. From this, there is no reflectance even when the material exists. On the other hand, for the case of

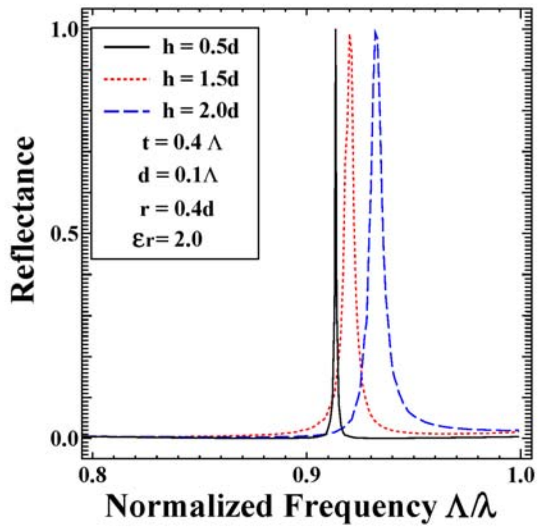

Fig. 6. Reflectance for various depth of the grating. r=0.4dCt=4.0dCε_r=2.0

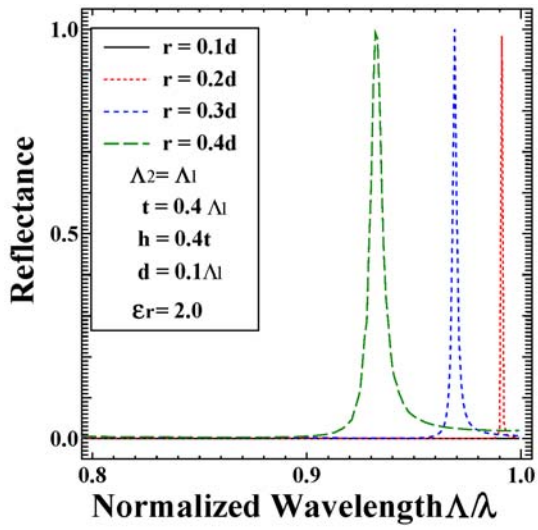

Fig. 7. Reflectance for various radius of the cylinder consist of the grating. t=4.0d, h=2.0d, ε_r=2.0

$r > 0.2$, the peak frequency is lower when the radius of the cylinder becomes larger.

Figure 8 shows the reflectance for various relative permittivity. The radius, width, and height are set as r=0.4d, t=4.0d, and h=2.0d, respectively. As the relative permittivity ε_r increases, the peak frequency becomes small and form is wider. This tendency is almost the same as that of Model 1. In Model 2, there is only one peak frequency in comparison with that of Model 1. This is because the averaged permittivity of the grating is smaller than that of Model 1, and higher mode propagating to x direction can not occur.

C. Dielectric Grating containing PCG

Figure 9 shows the geometry of the reflection by the dielectric grating containing PCG. d is the distance between

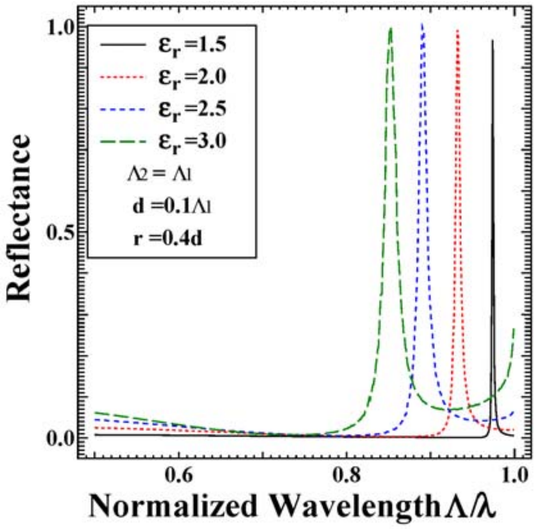

Fig. 8. Reflectance for various relative permittivity. r=0.4dCt=4.0dCh=2.0d

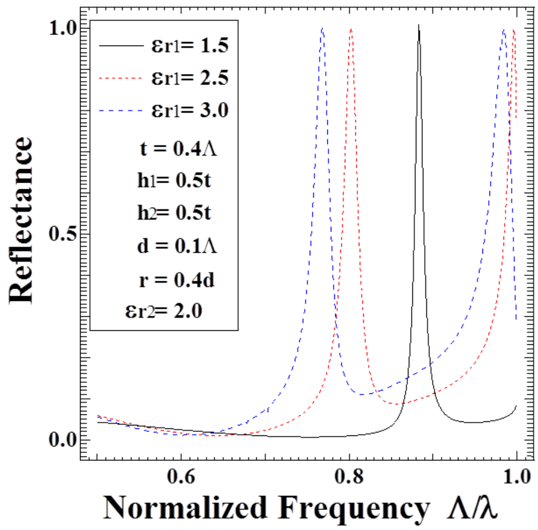

Fig. 10. Reflectance for various relative permittivity ε_{r1} of the dielectric grating.

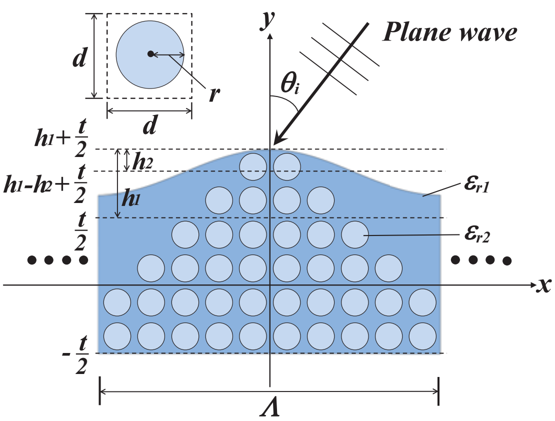

Fig. 9. Geometry of the reflection by dielectric grating containing PCG.

the cylinders, r is the radius of the cylinder, and t is the width of the slab. h_1 and h_2 are, respectively, the height of the PCG and dielectric grating. ε_{r1} and ε_{r2} are the relative permittivity of the slab and dielectric cylinder.

The reflectance for various relative permittivity of the background dielectric grating is shown in Fig. 10. The parameters of the grating are r=0.4d, t=0.4Λ, h_1, h_2=0.5t, ε_{r2}=2.0, respectively. From this figure, it is seen that the peak frequency is lower as the relative permittivity increases, there are two peaks for ε_{r1}=2.5, 3.0. This is due to the reason as that of Models 1 and 2.

IV. Conclusions

In this paper, the reflection properties from the dielectric grating containing PCG have been examined numerically by MoM for the purpose of the design of the frequency filter. PCG has the good properties among the structure considered here.

The bandgap of PCG does not affect the reflection properties for the parameter in this paper. From the numerical results, we think that PCG acts as making the relative permittivity small. The detailed examination about this will be performed. Also, the multilayered structure is one of the candidate for the electromagnetic devices. The examination of this structure is the future work.

Acknowledgement

This work was supported by JSPS KAKENHI Grant Number 23560072.

References

[1] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystal*. Princeton Univ. Press, 1995.
[2] K. Sakoda, *Optical Properties of Photonic Crystals*. Springer, 2001.
[3] A. E. Serebryannikov, "One-way diffraction effects in photonic crystal gratings made of isotropic materials," *Phys. Rev. B 80*, p. 155117, 2009.
[4] A. O. Cakmak, *et. al.*, "Inidirectional transmission in photonic-crystal gratings at beam-type illumination," *Opt. Exp.*, vol. 18, no. 21, pp. 22 280–22 298, 2010.
[5] G. Pelosi, A. Cocchi, and A. Monorchio, "A hybrid FEM-based procedure for the scattering from photonic crystals illuminated by a Gaussian beam," *IEEE Trans. Antennas and Propag.*, vol. 48, pp. 973–980, 2000.
[6] E. Popov and B. Bozhkov, "Differential method applied for photonic crystals," *Appl. Opt.*, vol. 39, pp. 4926–4933, 2000.
[7] M. Koshiba, Y. Tsuji, and M. Hikari, "Time-domain beam propagation method and its application to photonic crystal circuits," *J. Lightwave Technol.*, vol. 18, pp. 102–110, 2000.
[8] H. Ikuno and Y. Naka, *Finite-difference time-domain method applied to photonic crystals*, K. Yasumoto, Ed."Electromagnetic Theory and Applications for Photonic Crystals", Taylor & Francis, 2006.
[9] R. Harrington, *Field Computation by Moment Methods*. IEEE Press, 1993.
[10] A. Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering*. Prentice Hall, 1991.
[11] M. Yokota and M. Sesay, "Two-dimensional scattering of a plane wave from a periodic array of dielectric cylinders with arbitrary shape," *J. Opt. Soc. Am. A*, vol. 25, no. 7, pp. 1691–1696, 2008.
[12] K. Yasumoto and K. Yoshitomi, "Efficient calculation of lattice sums for free-space periodic green's function," *IEEE Trans. Antennas Propag.*, vol. 47, no. 6, pp. 1050–1055, 1999.