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PSO with Restricted Searching Dimensions

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Abstract– The dependence among variables is one of natures that should be considered on the function optimization problems. In general, the restriction of searching dimensions is said to be effective only when the function does not have the dependence among variables. However our previous studies have shown that the one-dimensional search can work well even when the function has the dependence among variables. In this paper, we propose PSO with restricted searching dimensions and examine how our PSO has impacts on the dependence among variables.

1. Introduction

Optimization problem is to search the solution which minimizes the function under the constraint conditions. In case the evaluation function is an unknown multimodal function, it is very difficult to search the optimal solution and the searching may need enormous time. Therefore, the efficient searching is necessary for such problems. Heuristics are well known as the efficient searching and it can search semi-optimal solutions instead of the optimal solution. Heuristics based on more abstract conception are called meta-heuristics which can be utilized broadly in various problems. PSO (Particle Swarm Optimization) [1] is a representative of them.

PSO developed by J. Kennedy and R. C. Eberhart is an algorithm to search the optimal solution. Each particle in PSO moves considering the personal best position (PBP) and the global best position (GBP). At the beginning stage, particles are spread all over the solution space, where the global search is carried out. Particles converge gradually. This means that the global search turns into the local search. Since the update equations consist of only the velocity and the position of each particle, this algorithm is simple and the calculation cost is very small. Therefore, PSO has been applied to various optimization problems in non-linear systems and many PSO models based on the original PSO [1] have been proposed.

Function optimization problems are benchmark ones which are often used to evaluate the performance of PSO. In case of solving function optimization problems by swarm intelligence, the following three points should be considered: the dependence among variables, the number of local minima and the scale of coordinate systems [2]. Especially, we have paid attention to the dependence among variables. In general, the size of the solution space increases exponentially in proportion to the number of dimensions. Therefore, it is inefficient to search the whole solution space if the function is defined in the high dimensional solution space and does not have the dependence among variables [3]. However, as far as we know, we have not found the studies on PSO considering both the dependence among variables and the restriction of searching dimensions.

In our previous work [4], we have proposed a PSO model. A part of particles in this model can search the solution only in one dimension. Also, when the particles converge, they are re-initialized and the searching dimension is changed. We call these particles the one-dimensional searching ones. From the results of numerical simulations we have found the following. When our PSO is applied to the function without the dependence among variables, the one-dimensional searching particles improve the solution precision and reduce calculation cost in comparison with the original PSO [1]. This is a trivial result. On the other hand, when our PSO is applied to the function with the dependence among variables, it cannot find the optimal solution but can improve the solution precision. Therefore, these results mean that the one-dimensional search is not always ineffective even if the function has the dependence among variables.

In this paper, we propose a novel PSO with restricted searching dimensions. We investigate how m(= 1 or 2)-dimensional search affects the solution precision and calculation cost from the viewpoint of "the existence of dependence among variables" and "the shape of function (i.e., unimodal / multimodal)". As a result, we have confirmed that the restriction of searching dimensions works well for function optimization in spite of the existence of the dependence among variables.

2. Original PSO

The dynamics of original PSO [1] is given by

$$v_{i,d}^{t+1} = Wv_{i,d}^{t} + c_1 r_{1\,i,d}^{t}\left(PBP_{i,d}^{t} - x_{i,d}^{t}\right) + c_2 r_{2\,i,d}^{t}\left(GBP_{d}^{t} - x_{i,d}^{t}\right), \quad (1)$$

$$x_{i,d}^{t+1} = x_{i,d}^{t} + v_{i,d}^{t+1}, \quad (2)$$

where $v^t_{i,d}$ and $x^t_{i,d}$ are the d-th dimensional velocity and position of the i-th particle at the t-th iteration respectively. The velocity $\boldsymbol{v}\in\Re^D$ and the position $\boldsymbol{x}\in\Re^D$ are restricted by the domains $\boldsymbol{V}$ and $\boldsymbol{X}$. $\boldsymbol{PBP}^t_i\in\Re^D$ and $\boldsymbol{GBP}^t\in\Re^D$ are the positions with the best evaluation value found by the i-th particle and the swarm until the t-th iteration. $W\in(0,1]$ is an inertia weight coefficient. c_1, $c_2\in[0,2]$ are acceleration coefficients. $r_{1\,i,d}^t$, $r_{2\,i,d}^t\in[0,1]$ are uniform random numbers. Also, it is known that setting the parameters (i.e., W, c_1, c_2) to the following values is good for original PSO [6],

$$\begin{cases} W = 0.729, \\ c_1 = c_2 = 1.49445. \end{cases} \quad (3)$$

3. Proposed method

We have confirmed in our previous work [4] that the one-dimensional search is effective even in optimizing the function with the dependence among variables. But, we have not sufficiently investigated the influence of the restriction of searching dimensions. Therefore in this paper, we propose PSO with restricted searching dimensions. Our proposed PSO searches the whole D-dimensional solution space by the m-dimensional search ($m \le D$).

3.1 Simple design of *m*-dimensional search

This section describes a simple design of the m-dimensional search. Each particle can move only in one m-dimensional subspace selected from the whole D-dimensional space. We call the dimensions given by the subspace 'the restricted searching dimensions'. There are plural subspaces and the number is equal to the combinations of restricted searching dimensions (i.e., ${}_D\mathrm{C}_m$). All particles are divided into ${}_D\mathrm{C}_m$ groups. A group has N_{gp} particles. Each subspace is searched by only one group and all subspaces are searched by ${}_D\mathrm{C}_m$ groups simultaneously.

The update equation of position x and velocity v is given by

$$v_{i,d}^{t+1} = Wv_{i,d}^{t} + c_1 r_{1\,i,d}^{t}\left(PBP_{i,d}^{t} - x_{i,d}^{t}\right) + c_2 r_{2\,i,d}^{t}\left(LBP_{d}^{t} - x_{i,d}^{t}\right), \quad (5)$$

$$x_{i,d}^{t+1} = \begin{cases} x_{i,d}^{t} + v_{i,d}^{t+1}, & \text{if the subspace includes } d, \\ GBP_{d}^{t}, & \text{otherwise.} \end{cases} \quad (6)$$

Particles in each group have the local best position which is best position in a group ($\boldsymbol{LBP}^t \in \Re^D$). In the restricted searching dimensions, the simple design updates the velocity and the position by using ***LBP*** instead of ***GBP*** in Eq.(1). Each particle cannot move in the D–m dimensional subspace. The whole D-dimensional space consists of the m-dimensional subspace and the D–m dimensional subspace. Therefore, the simple design always fixes the position x at ***GBP*** in the D–m dimensional subspace.

There is a problem in the m-dimensional search. Since the simple design searches all subspaces simultaneously, the number of all particles N_{ap} equals to $N_{gp} \times {}_D\mathrm{C}_m$. Therefore, as D and m increase, N_{ap} becomes enormous number. By the above reason, it is essential to decrease the calculation cost by reducing N_{ap}. We propose a method to reduce the calculation cost of the simple design in the next section.

3.2 Low cost design of *m*-dimensional search

PSO with the simple design searches all subspaces simultaneously. But, our proposed PSO with the low cost design searches only a part of subspaces simultaneously. The low cost design re-selects subspaces which are searched simultaneously, and the design repeats the re-selection of subspaces and the judgment of convergence of particles. Therefore, if enough re-selections are executed, it can mostly search all subspaces. We explain the low cost design used in our proposed PSO.

First, we show how to re-select the subspaces. S subspaces are selected at random from all subspaces (${}_D\mathrm{C}_m$) and are searched simultaneously. This re-selection is executed when all particles in a group converge. Also, after the re-selection, the velocity and the position are initialized by the domains $\boldsymbol{V}$ and $\boldsymbol{X}$ (re-initialization). Next, we describe how to judge the convergence. The convergence is measured by the velocity. That is to say, if all particles in a group satisfy $v_{i,d}^{t} < \varepsilon\,(= 10^{-3})$ in all dimensions.

As mentioned above, the low cost design needs enough re-selections to search almost all subspaces. But, Eq.(5) does not always satisfy this demand. Therefore, to obtain the convergence frequently, we modify Eq.(5) as follows:

$$v_{i,d}^{t} = \left(v_{i,d}^{t} \quad \text{given by Eq.(5)}\right) \times \left(1 - \frac{t_{\mathrm{re}\,i}}{T_{\mathrm{re}}}\right), \quad (7)$$

where $t_{\mathrm{re}\,i}$ is the number of iterations after the re-selection. Eq.(7) surely makes particles converge until T_{re} iterations.

4. Experiments

4.1 Experimental conditions

Simulations have been carried out to verify the effectiveness of the restriction of searching dimensions by means of the function optimization problems shown in Table 1.

Table 2 shows PSO models used in these experiments: the original PSO (PSO), the original PSO with the re-initialization (PSO-R), PSO with the one-dimensional search using the simple design (PSO-1S), PSO with the one-dimensional search using the low cost design (PSO-1LC), PSO with the two-dimensional search using the simple design (PSO-2S), and PSO with the two-dimensional search using the low cost design (PSO-2LC). SN stands for the small number of particles and LN does for the large number of particles. Therefore, we can compare the performances of these PSO models from the viewpoint of the swarm size.

In these experiments, The number of search trials in each model is N_{tri} (=100) and the maximum number of iterations in each trial is T (=10000). The successful condition is that $f(\boldsymbol{GBP}) < \alpha$ (=10^{-3}). The evaluation items are the average of global best (GB_{ave}), the average number of iterations in successful trials (ITR_{ave}) and the success rate (SR). The other experimental conditions are shown in Table 3.

Table 1 Function optimization problems.

Function name	Domain $\boldsymbol{V}$ and $\boldsymbol{X}$	Shape	with/without dependence among variables
Rosenbrock (f_1)	$[-5,5]^D$	unimodal	with
Rastrigin (f_2)	$[-5,5]^D$	multimodal	without
RotatedRastrigin (f_3)	$[-5,5]^D$	multimodal	with
Griewank (f_4)	$[-512,512]^D$	multimodal	with

Table 2 PSO models.

	N_{ap}	N_{gp}	# of subspace
PSO(SN)	150	---	---
PSO-R(SN)	150	---	---
PSO-1LC(SN)	150	5	30(=S)
PSO-2LC(SN)	150	5	30(=S)
PSO(LN)	2175	---	---
PSO-R(LN)	2175	---	---
PSO-1S(LN)	2160	72	${}_{30}C_1$
PSO-2S(LN)	2175	5	${}_{30}C_2$

Table 3 Experimental conditions.

N_{tri}	100	T	10000
D	30	T_{re}	1000
S	30	m	1 / 2
ε	10^{-3}	α	10^{-3}
W	0.729	c_1, c_2	1.49445

4.2 Results

We have confirmed the following from the experimental results shown in Tables 4-7.

- The restriction of searching dimensions improves PSO in all functions except f_4. Therefore, it is effective in spite of the existence of dependence among variables.
- PSO-2LC can tend to search better GB_{ave} than PSO-1LC.
- Although particles of the simple design (SD) are 15 times larger than those of the low cost design (LCD), the difference between their solution precisions is small. Moreover, if $N_{ap}\times ITR_{\mathrm{ave}}$ is regarded as the calculation cost, LCD can cut it down drastically. Therefore, we estimate that LCD is more effective than SD.
- In case of f_4 PSO-R had the best result. We think that it is caused by the shape of function. When we see f_4 roughly it looks like unimodal. But, when we see f_4 around the optimal solution finely, it is a multimodal function with the dependence among variables. Therefore, since PSO-R can repeatedly search around the optimal solution by re-initialization, we think that it is more effective than the other PSO models.

5. Conclusions

In this paper, we have proposed PSO with restricted searching dimensions. We have confirmed that the restriction of searching dimensions works well for the function optimization in spite of the existence of the dependence among variables. Moreover, the low cost design of the m-dimensional search can cut down the calculation cost. In the future, we will research the capacity of PSO with restricted searching dimensions, when the number of searching dimensions (m) is larger than two.

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Table 4 RosenBrock (f_1) (N_{tri}=100, T=10000, D=30).

		PSO	PSO-R	PSO-1S	PSO-2S	PSO-1LC	PSO-2LC
SN	GB_{ave}	3.35E+02	2.03E+01	---	---	1.28E+00	4.29E-01
	ITR_{ave}	5.69E+03	---	---	---	3.84E+03	4.51E+03
	SR(%)	8	0	---	---	20	25
LN	GB_{ave}	1.10E+02	1.49E+01	1.20E-03	2.93E-06	---	---
	ITR_{ave}	8.11E+03	---	5.19E+03	3.55E+03	---	---
	SR(%)	11	0	99	100	---	---

Table 5 Rastrigin (f_2) (N_{tri} =100, T=10000, D=30).

		PSO	PSO-R	PSO-1S	PSO-2S	PSO-1LC	PSO-2LC
SN	GB_{ave}	1.52E+02	9.34E+01	---	---	1.53E-13	1.10E-13
	ITR_{ave}	---	---	---	---	4.02E+02	5.35E+02
	SR(%)	0	0	---	---	100	100
LN	GB_{ave}	1.02E+02	6.63E+01	2.05E-13	5.00E-14	---	---
	ITR_{ave}	---	---	4.66E+01	8.37E+01	---	---
	SR(%)	0	0	100	100	---	---

Table 6 RotatedRastrign (f_3) (N_{tri} =100, T=10000, D=30).

		PSO	PSO-R	PSO-1S	PSO-2S	PSO-1LC	PSO-2LC
SN	GB_{ave}	3.25E+02	2.28E+02	---	---	1.03E+02	7.69E+01
	ITR_{ave}	---	---	---	---	---	---
	SR(%)	0	0	---	---	0	0
LN	GB_{ave}	2.19E+02	1.82E+02	1.02E+02	7.76E+01	---	---
	ITR_{ave}	---	---	---	---	---	---
	SR(%)	0	0	0	0	---	---

Table 7 Griewank (f_4) (N_{tri} =100, T=10000, D=30).

		PSO	PSO-R	PSO-1S	PSO-2S	PSO-1LC	PSO-2LC
SN	GB_{ave}	1.12E+01	1.25E-04	---	---	1.50E-02	7.31E-03
	ITR_{ave}	2.66E+02	1.13E+03	---	---	9.04E+01	4.12E+03
	SR(%)	52	100	---	---	47	72
LN	GB_{ave}	1.03E-02	1.11E-17	3.72E-02	3.70E-04	---	---
	ITR_{ave}	5.26E+02	5.23E+02	5.38E+02	1.81E+02	---	---
	SR(%)	59	100	30	100	---	---